

# Analysis of Codebook Based Channel Feedback Techniques for MIMO-OFDM Systems

Muhammad Rehan Khalid, Ahmed Farhan Hanif and Adnan Ahmed Khan

**Abstract**—This paper investigates the performance of Multiple-Input Multiple-Output (MIMO) feedback system combined with Orthogonal Frequency Division Multiplexing (OFDM). Two types of codebook based channel feedback techniques are used in this work. The first feedback technique uses a combination of both the long-term and short-term channel state information (CSI) at the transmitter, whereas the second technique uses only the short term CSI. The long-term and short-term CSI at the transmitter is used for efficient channel utilization. OFDM is a powerful technique employed in communication systems suffering from frequency selectivity. Combined with multiple antennas at the transmitter and receiver, OFDM proves to be robust against delay spread. Moreover, it leads to significant data rates with improved bit error performance over links having only a single antenna at both the transmitter and receiver. The effectiveness of these techniques has been demonstrated through the simulation of a MIMO-OFDM feedback system. The results have been evaluated for 4x4 MIMO channels. Simulation results indicate the benefits of the MIMO-OFDM channel feedback system over the one without incorporating OFDM. Performance gain of about 3 dB is observed for MIMO-OFDM feedback system as compared to the one without employing OFDM. Hence MIMO-OFDM becomes an attractive approach for future high speed wireless communication systems.

**Keywords**—MIMO systems, OFDM, Codebooks, Channel Feedback

## I. INTRODUCTION

**R**ADIO spectrum is one of the major limiting factors in wireless communications due to its scarcity. MIMO technology utilizes antenna arrays at both the transmitter and receiver to increase the spectral efficiency of the system [1]. A MIMO system takes advantage of the spatial diversity [2] that is obtained by spatially separated antennas in a dense multipath scattering environment. The use of multiple antennas at the transmitter and receiver is by now a well recognized technique to achieve high data rates in wireless communications [3, 4]. The radio spectrum is scarce and expensive resource, hence efficient channel utilization techniques are required to support high data rates. The multipath characteristics of the environment cause the MIMO channels to be frequency selective. For frequency selective deep fading, MIMO system remains ineffective. OFDM, a multicarrier transmission scheme, is well recognized for its potential for attaining high rate transmission over frequency selective channels [5]. It can transform such

a frequency-selective MIMO channel into a set of parallel frequency-flat channels. Implementing space resources based on OFDM i.e., MIMO-OFDM can provide higher data rates [6]. Thus the combination of the two powerful techniques, MIMO and OFDM, is very attractive and promising broadband wireless communications scheme [7].

Wireless channels are constantly changing due to multipath, movement of the receiver, and also the changes in environment. Having knowledge of the channel prior to the transmission is very valuable. It enables the transmitter to exploit the channel in the best possible way. CSI is fed back to the transmitter as CSIT [8]. A multitude of different transmission techniques have been proposed in literature, especially for the special cases of full CSIT and no CSIT. The throughput can be significantly improved if CSIT is available. CSIT helps in adapting the transmitted signal to the time varying channel. Obtaining CSIT for MIMO systems require considerably higher amount of bandwidth than SISO systems to transmit CSI back to the transmitter, as there are multiple antennas and a large amount of CSI has to be sent back to the transmitter. Since CSI is estimated at the receiver so it has to be communicated to the transmitter using as few bits as possible, thus utilizing the minimum possible bandwidth. Hence low rate channel feedback schemes are required [9]-[14]. Also, the channel is continuously varying and CSIT becomes continuously outdated, therefore a continuous feedback is required from the receiver to maintain a specific performance level.

In order to send the CSI back to the transmitter in MIMO systems, several techniques have proposed. Low rate channel feedback techniques, see for example [9]-[14], include Channel Norm feedback techniques [15] and Codebook based channel feedback techniques. The scope of this paper is to discuss codebook based feedback techniques. The work presented here is based on the work of Tung and Hanif [16]-[18], and is extended to MIMO Feedback systems combined with OFDM.

The paper is organized as follows. In Section II, the overall system model of MIMO-OFDM feedback system is introduced. Then we highlight codebook based channel feedback techniques including Decorrelating Linear Transformation and Feedback Index Selection in Section III. SVD technique for MIMO-OFDM subcarriers is demonstrated in Section IV. Section V presents comprehensive simulation results and comparisons to highlight the performance gain. Finally concluding remarks are presented in Section VI.

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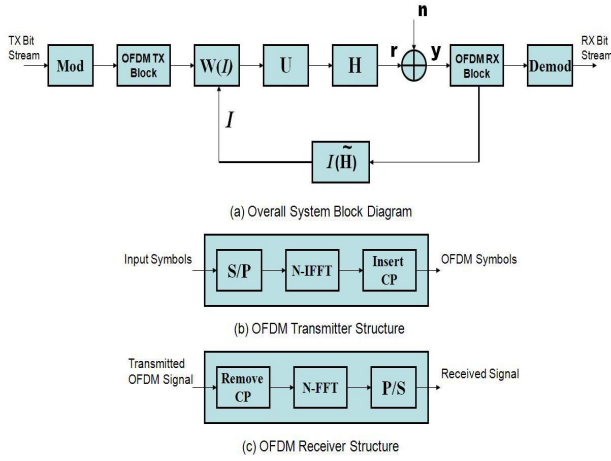


Fig. 1. Overall Block Diagram

## II. SYSTEM MODEL

Fig. 1 represents the overall system block diagram of a generalized MIMO-OFDM Feedback system with  $N_t$  transmit and  $N_r$  receive antennas. The complex valued channel matrix is denoted by  $H \in \mathbb{C}^{N_r \times N_t}$ , where  $N_t$  and  $N_r$  represent the number of transmit and receive antennas respectively.  $W_i$  represents the precoding matrix which comes from the codebook  $W_i$ .  $U$  represents a unitary matrix and  $I \in \{0, \dots, 2^K - 1\}$  is the index of the precoding matrix, where  $K$  is the number of feedback bits. The OFDM symbols transmitted vector is represented by  $x$ , and the components of white noise  $n$  are complex Gaussian with zero mean and unit variance. The system model can be written in the form

$$y = HUW_i x + n \quad (1)$$

If we consider the case of perfect channel state information at the transmitter and receiver, we can decompose the MIMO channel on each tone into parallel non-interfering SISO channels using singular value decomposition (SVD).

$$H_i = U_i S_i V_i^* \quad (2)$$

where  $U_i$  and  $V_i$  are unitary matrices, and  $S_i$  is the diagonal matrix of singular values of  $H_i$ . Note that the operator  $(\cdot)^*$  is the conjugate transpose operator.

The OFDM symbols can be expressed in complex baseband notation as

$$s_k = \sum_{i=0}^{N-1} d_i \exp(j \frac{2\pi i k}{N}); \quad 0 \leq k \leq N-1 \quad (3)$$

where  $d_i$  is the data symbol corresponding to each subchannel and  $N$  is the number of subchannels. It can be seen that  $s_k$  is equivalent to the inverse fourier transform operations of  $d_i$ . Similarly OFDM symbols for  $M_t$  transmitters can be represented as

$$S_m = \sum_{k=0}^{N-1} s_k; \quad 1 \leq m \leq M_t \quad (4)$$

The  $N$ -point FFT requires only on the order of  $N \log N$  multiplications rather than  $N^2$  as in straightforward computation. Due to this fact, an OFDM system typically requires fewer computations per unit time than an equivalent system with equalization. Transmission of data in frequency domain using an FFT, as a computationally efficient orthogonal linear transformation, results in robustness against ISI in the time domain. The cyclic prefix is added to an OFDM symbol in order to combat the effects of multipath. Intersymbol interference is avoided between adjacent OFDM symbols by introducing a guard period in which the multipath components of the desired signal are allowed to die out, after which the next OFDM symbol is transmitted. A useful technique to help reduce the complexity of the receiver is to introduce a guard symbol during the guard period. Specifically, this guard symbol is chosen to be a prefix extension to each block. However, in order for this technique to work, the guard interval should be greater than the channel delay spread. Thus, we see that the relative length of the cyclic prefix depends on the ratio of the channel delay spread and OFDM symbol duration.

The codebook based techniques, proposed by Tung [16], combine long-term and low-rate short-term CSI over correlated MIMO channels which increases the expected rate as compared to techniques using only short-term CSI or only long-term CSI. The long-term CSI is collected in the form of an estimated covariance matrix of the Rayleigh fading channel statistics and the eigenvectors of the resulting covariance matrix are used as the unitary matrix.

The feedback index is selected based on the Shannon Capacity equation. This scheme was used in [16, 17], and the codebooks have been designed for this scheme. In this method, the Index corresponding to precoding matrix  $W_i$  which gives the highest expected rate is denoted by  $I$ . Channel Capacity is calculated for all the transmit covariance matrices  $Q_i$  in a particular codebook. The index of the matrix which gives the best performance for that sample instant is sent back to the transmitter. The respective transmit covariance matrix  $Q_i$  is used in the next transmission.

## III. CODEBOOK BASED FEEDBACK TECHNIQUES

Codebook based techniques represent a class of CSI feedback schemes in which transmitter and receiver have the same set of codebooks available. The two feedback techniques considered in this paper are described in [16]. The codebooks contain precoding matrices  $W_i$  and transmit covariance matrices  $Q_i$ , where  $i$  is the feedback index. The size of the codebook depends on the number of feedback bits. The codebooks are optimized using convex optimization techniques. The training algorithm for generation of codebooks is demonstrated in [16]. Table 1 shows the summary of Feedback Techniques, and notation used throughout the paper.

TABLE I  
FEEDBACK TECHNIQUES

	First Technique	Second Technique
<b>Long Term Statistics</b>	Yes	No
<b>Short Term Statistics</b>	Yes	Yes
<b>Unitary Matrix <math>U</math></b>	$U = U_{Estimated}$	$U = U_I$
<b>Feedback Bits</b>	1, 2 or 4	1, 2 or 4
<b>Codebook Type</b>	VQ Trained	VQ Trained

#### A. Decorrelating Linear Transformation (DLT)

The first technique uses the long-term channel statistics to perform a unitary transformation of the channel coefficients. DLT is based on estimating the unitary matrix from the long-term channel statistics. This linear transformation will decorrelate the channel coefficients which results in a more efficient codebook. The main point of DLT is that the transformed CSI can be transmitted back to the transmitter with low overhead. It decouples the channel and increases the capacity of the channel. The Covariance matrix  $R^{Tx}$ , estimated from long-term channel statistics, is given by

$$R^{Tx} = \frac{1}{N} \sum_{n \in N} H^H(n)H(n) \quad (5)$$

where  $H$  is the MIMO channel matrix. Singular value decomposition of  $R^{Tx}$  gives  $U^{Tx}$  and  $D$  according to the following relation.

$$U^{Tx} D^{Tx} (U^{Tx})^H = R^{Tx} \quad (6)$$

where  $U^{Tx}$  is the decorrelating matrix calculated using the available channel statistics.

The second feedback technique uses only the low-rate short-term CSI. It is dependent on the feedback obtained from the receiver. Unlike the first technique, it only depends on the instantaneous CSI. It does not rely on the long-term statistics of the channel like the first technique. Here we do not need to estimate the decorrelating unitary matrix. Hence this technique is not computationally as rigorous as the first one. Though this technique bears shortcomings in terms of throughput as compared to the first one.

#### B. Index Selection

Two different methods can be used for the selection of the feedback index described in [16, 17]. First one is the mutual information based index selection, and second one is based on channel norm. Mutual information based index selection is a useful method when using adaptive modulation and coding. While channel norm based index selection is useful when fixed modulation and coding is used.

In Mutual Index Selection (MIS), the index is selected based on the Shannon Capacity equation. This scheme was used in [16, 17], and the codebooks have been designed for this scheme. In this method, the Index corresponding to precoding matrix  $W_i$  which gives the highest expected rate is denoted by

TABLE II  
OFDM PARAMETERS

<b>Number of Subcarriers</b>	64
<b>OFDM symbol time</b>	64 symbol periods
<b>Guard Interval</b>	16 symbol periods

I. The following relation shows the feedback index selection in MIS

$$I = \arg \max_i \det(I_{N_r} + H U Q_i U^H H^H) \quad (7)$$

where

$$Q_i = W_i W_i^H \quad (8)$$

Channel Capacity is calculated for all the transmit covariance matrices  $Q_i$  in a particular codebook. The index of the matrix which gives the best performance for that sample instant is sent back to the transmitter. The respective transmit covariance matrix  $Q_i$  is used in the next transmission.

#### IV. SVD OF MIMO SUBCARRIERS

Channel matrix singular value decomposition (SVD) method is employed in MIMO-OFDM systems in order to overcome subchannel interference. It allows to allocate the transmitted bit and power through the subchannels in an optimum manner. The SVD technique can be directly applied to the MIMO channel decomposition when perfect CSI knowledge is available at the transmitter side. The MIMO channel can be decomposed into a set of parallel SISO channels by applying a unitary pre-filtering and post-filtering matrix to the transmitted and received signal respectively [19]. In the notation of matrices, the matrix  $H$  has the singular value decomposition:

$$H_i = U_i S_i V_i^* \quad (9)$$

Now, if we use a transmit precoding filter  $V_i$  and a receiver shaping filter  $U_i$ , the equivalent MIMO channel between the IFFT and FFT blocks decomposes into parallel subchannels. Therefore, we can use each parallel subchannel's SNR to specify the number of bits and energy allocated. In general, each precoder and shaping matrix will be different for different subchannels. The optimization problem will be larger for MIMO channels than in SISO case, but the decomposition has allowed us to proceed without any changes to the optimization algorithm [19, 20].

#### V. RESULTS

##### A. Assumptions and Simulation Parameters

For the scope of this work, six different signal constellations are used,  $M \in \{0, 1, 2, 4, 6, 8\}$ . We assume that the feedback channel is perfect. Although a noisy or imperfect channel could be assumed, see for example [16]. Here we mainly focus on perfect feedback with  $K \in \{1, 2, 4\}$ , where  $K$  is the number of feedback bits. The parameters used for OFDM throughout the simulations are listed in Table 2.

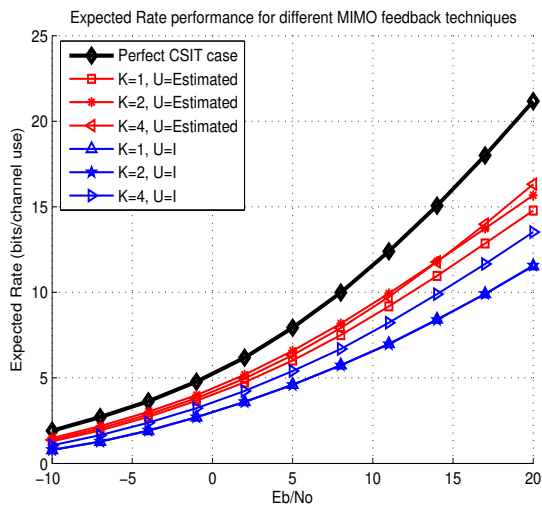


Fig. 2. Performance results for 4x4 MIMO system corresponding to 1, 2 and 4 bit feedback

### B. Simulation Results for 4x4 MIMO channels

Fig. 2 shows the expected rate performance for the 4x4 MIMO system with codebook based channel feedback over a series of SNRs. The results are simulated for the cases of perfect CSIT, 1, 2 and 4-bit feedback both with the first feedback technique (where Unitary matrix is estimated from long-term channel statistics) and the second one (based only on instantaneous channel information). Fig. 3 shows the expected rate performance for the same system with the incorporation of OFDM. The results are simulated for all the cases highlighted in Fig. 2. The results clearly highlight that MIMO-OFDM feedback system outperforms the MIMO feedback system without the implementation of OFDM.

Fig. 4 highlights the performance improvement acquired with the incorporation of OFDM in the MIMO feedback system. Performance comparisons are evaluated for the cases of perfect CSIT and 1-bit feedback with both feedback techniques. Careful investigation of the performance curves indicate that MIMO-OFDM feedback system curves approach the ideal curve of perfect CSIT case.

To emphasize the performance improvement, Fig. 5 shows the performance comparisons evaluated for the cases of perfect CSIT and 4-bit feedback with both feedback techniques. Careful investigation of the performance curves indicate that MIMO-OFDM feedback system curves approach much closer to the ideal curve of perfect CSIT case. Fig. 6 shows the performance gain achieved with the implementation of OFDM in the MIMO feedback system. The results indicate that the MIMO-OFDM feedback system provides substantial performance improvement over the same system without employing OFDM. A substantial gain of about 3 dB is observed with the incorporation of OFDM in the MIMO feedback system, making the proposed arrangement promising for future high speed wireless communication systems.

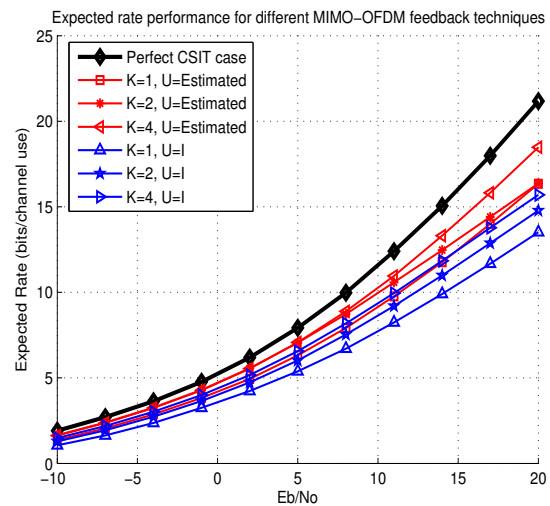


Fig. 3. Performance results for 4x4 MIMO-OFDM system corresponding to 1, 2 and 4 bit feedback

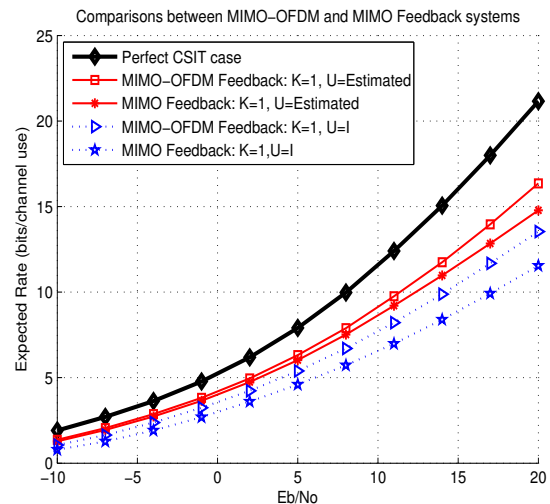
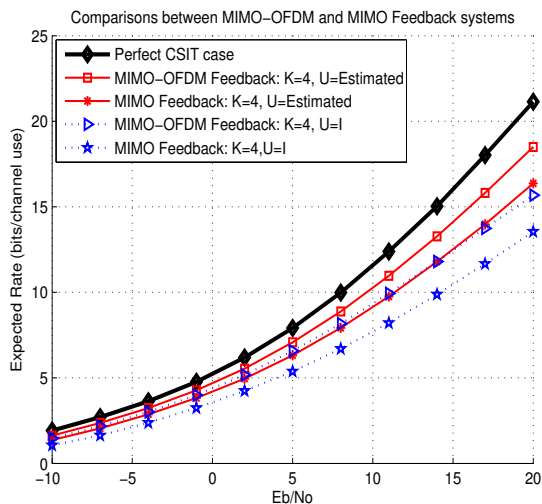
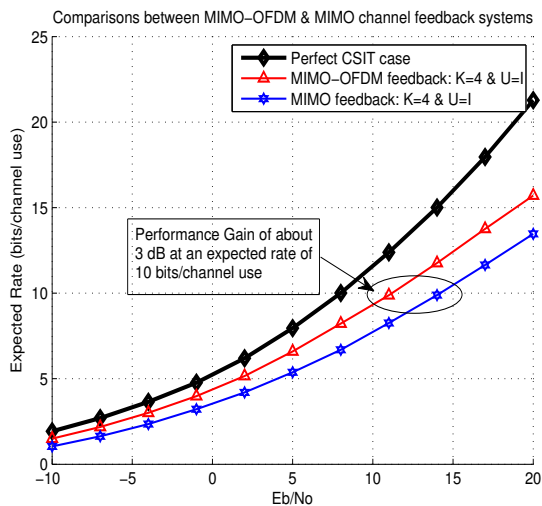


Fig. 4. Performance comparisons for both feedback techniques:  $k = 1$

## VI. CONCLUSION

In this paper, two low rate channel feedback schemes were used for MIMO systems combined with OFDM. MIMO-OFDM system combines the advantages of both the techniques i.e., MIMO and OFDM to simultaneously increase spectral efficiency, and eliminate the effects of delay spread in a multipath fading environment. The evaluations were performed over 4x4 MIMO channels. The results show that MIMO-OFDM feedback system outperforms the MIMO feedback system in terms of spectral efficiency over a range of SNRs. Comprehensive analysis and comparisons of simulation results show that MIMO-OFDM channel feedback system provides considerable improvement as compared to the same system

Fig. 5. Performance comparisons for both feedback techniques:  $k = 4$ Fig. 6. Performance Comparisons for 4x4 MIMO-OFDM Feedback system:  $k=4$ 

without incorporating OFDM. A substantial gain of about 3 dB is observed with the incorporation of OFDM in the MIMO feedback system, making the proposed arrangement promising for future high speed wireless communication systems.

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