Analyses of Natural Convection Heat Transfer from a Heated Cylinder Mounted in Vertical Duct

H. Bhowmik, A. Faisal, Ahmed Al Yaarubi, Nabil Al Alawi

Abstract—Experiments are conducted to analyze the steady-state and the power-on transient natural convection heat transfer from a horizontal cylinder mounted in a vertical up flow circular duct. The heat flux ranges from 177 W/m² to 2426 W/m² and the Rayleigh number ranges from 1×10⁴ to 4.35×10⁴. For natural air flow and constant heat flux condition, the effects of heat transfer around the cylinder under steady-state condition are investigated. The steady-state results compare favorably with that of the available data. The effects of transient heat transfer data on different angular position of the thermocouple (0°, 90°, 180°) are also reported. It is observed that the transient heat transfer around the cylinder is strongly affected by the position of thermocouples. In the transient region, the rate of heat transfer obtained at 90° and 180° are higher than that of stagnation point (0°). Finally, the dependence of the average Nusselt number on Rayleigh number for steady and transient natural convection heat transfer are analyzed, and a correlation equation is presented.

Keywords—Steady-state, transient, natural convection, Rayleigh number, Nusselt number, Fourier Number.

I. INTRODUCTION

STEADY-STATE and time dependent natural convection heat transfer between cylindrical surface and surrounding air is very important in many engineering applications. These kinds of problems are encountered in the heat transfer from tube heating like air conditioning systems, steam heated coils, electronic immersion heaters, air heating systems, energy storage systems, solar thermal receiver, laptops, cell phones, etc. Cooling of such components can reduce the losses of electric power and can improve their performance. Thus, the analysis of natural convection heat transfer is becoming important for the researchers. The steady-state natural convection heat transfers from horizontal cylinder under constant temperature or constant heat flux conditions have widely been studied [1]-[3]. They studied the effect of orientation of the cylinder and the distance between cylinders for the best overall convection heat transfer. Morgan [4] published a review on the natural convection from smooth, circular cylinders and reported experimental and numerical results under different flow and heating conditions, and with many correlations. Kuehn and Goldstein [5] reported the effect of heat transfer on angular and radial velocity around a cylinder in air, for Rayleigh number ≤10⁷. Saitoh et al. [6] and Wang et al. [7] also investigated numerically the convection heat transfer from a cylinder and provided a solution for predicting heat transfer at different circumferential positions. Kuehn and Goldstein [5], Qureshi and Ahmed [8], and Chouikl et al. [9] observed that the heat transfer is maximum at the bottom of the cylinder (stagnation point) and decreases at the top of the cylinder. The decrease in heat transfer attributes to an increase of thermal boundary layer thickness and heat transfer at the top.

The time dependent natural convection study was reported by some researchers for different experimental conditions and with different geometric profiles [10]-[18]. Most of the available studies are based on time dependent natural convection heat transfer from flat surfaces in vertical orientation or enclosures surfaces. The studies by Goldstein and Briggs [10], Genceli [11], Atayilmaz [12], Sadeghipour and Kannani [13], Sammakia et al. [14], Gupta and Pop [15] are only a few examples of transient studies on cylinders, wires or curved surfaces. Gupta and Pop [15] extended the results of Elliott [16], by analyzing the effects of curvature of cylinder on transient natural convection and stated that the curvature increases the skin friction and heat transfer from the cylinder. Kuehn and Goldstein [5], Merkin [17], and Farouk and Guceri [18], analyzed only the steady state heat transfer from horizontal cylinders and ignored the transient effects. Parsons and Mulligan [19] reported transient natural convection study from suddenly heated horizontal wire and reported a shape factor correlation to estimate the transient free convection heat transfer.

It is observed in the literature that many studies are based on the natural convection heat transfer in an enclosure or cavity or on a flat wall, and most of the analyses are one or two dimensional numerical. However, they neglect the effect of three-dimensional heat transfer, which is very important in natural convection. Some studies are related to transient natural convection heat transfer from thin wire, thin rod, which shows limited applications and cannot meet with the real situation. In fact, the heat transfer situation is changed with the circumference of the cylinder; especially at the top of the cylinder surface where two boundary layers merge to form the buoyant plum without flow separation, which is very important for heat transfer from horizontal cylinders. In addition, insufficient heating produces no penetration in the temperature field. However, heating must be done to overcome the viscosity of the surrounding fluid. These important issues were not addressed carefully in the previous studies and there was no correlation reported to predict the transient heat transfer around the horizontal cylinder. The
author reported some analyses for sudden power on transient natural convection and pumping power off transient forced convection heat transfer from discrete heaters placed in vertical rectangular channel [20]. However, the present experimental work investigates the cooling of a heated cylinder placed horizontally in a vertical circular duct. In this work, the transient buoyancy driven flow adjacent to a horizontal cylinder is investigated experimentally in the range of Rayleigh number, $1 \times 10^4 < Ra_D < 4.35 \times 10^4$, in air of Prandtl number around 0.72. The flow transient occurs by sudden switch on the heater power that allows the gradual increase of cylinder surface temperature. The variation of average Nusselt number with time dependent Fourier number around the cylinder is presented. The steady-state and time dependent results are compared with the available results of similar conditions. The transient heat transfer data at different thermocouple positions (0°, 90° and 180°) are analyzed, and the correlation equations are represented to predict the heat transfer data.

II. EXPERIMENTAL ANALYSES

A. Setup

The experimental setup consists of a heated cylinder of outside diameter 10 mm, length 70 mm, mounted at the top of the vertical circular duct of diameter 70 mm, as shown in Fig. 1 (a). The cylinder, made of copper, is placed in such a way that the entry effect on the fluid boundary layer is negligible and the flow is fully developed before the cylinder. A film resistive heater is an internal heater placed inside the hollow cylinder that provides uniform surface temperature. The heater power is varied by using a variable voltage transformer. The conduction heat transfer toward the circumferential direction depends on the film thickness and it provides a non-uniform convective heat flux. One K-type thermocouple is attached to the mid-way along the cylinder wall to measure the temperature of the cylinder surface. The thermocouple is flushed to the cylinder surface in such a way that it will not disturb the flow of air. One more thermocouple is attached at the upstream of the cylindrical duct to measure the inlet air temperature of the duct. The thermocouple has the ability to measure the temperature in the range 0 to 133 °C with resolution, 0.1 °C. The mounting arrangement allows the cylinder to rotate about its horizontal axis, as shown in Fig. 1 (b). The thermocouples are connected with a PC based data acquisition system to record the steady-state and time dependent temperature.

B. Procedure

The experiments are conducted to study the thermal behavior of horizontal cylinder under suddenly applied heat flux in natural air flow. The power supplied to the heated cylinder is varied and measured with the data acquisition system. The voltage regulator, ammeter and digital voltmeter are used to vary the input power of the heater attached to the cylinder. The design of the cylinder mounting arrangement allows to minimize the loss of heat by conduction. The heating element (heater) is rated to produce 100 W nominally at 24 V DC into the heated cylinder. The maximum temperature of the heating element is limited to 120 °C that ensures a minimum radiation heat loss. The apparatus is allowed to record the time dependent temperature data after the heater power is switched-on.

![Fig. 1 Schematic diagram of the test section and section of the cylinder](image)

The Biot number varies here from 0.0025 to 0.003, and it ensures the isothermal condition on the cylinder. The heated cylinder is then rotated to measure the temperatures at different angular positions, $\theta = 0^\circ, 90^\circ, 180^\circ$, as shown in Fig. 1 (b).

C. Data Analyses

The thermophysical properties of air are taken from the property table of air, based on average temperature of fluid inlet and cylinder surface temperature and at atmospheric pressure [21]. The steady and unsteady natural convection parameters are calculated as

Nusselt number

$$Nu_D = \frac{Q_{in}D}{kA(T_i - T_j)} \quad (1)$$

Grashof number as

$$Gr_D = \frac{g\beta(T_i - T_j)D^2}{\nu^2} \quad (2)$$

Rayleigh number as

$$Ra_D = Gr_D Pr = \frac{g\beta(T_i - T_j)D^3}{\nu\alpha} \quad (3)$$

Fourier number as
The Biot number as

$$\text{Bi} = \frac{hD}{k_e}$$  \hspace{1cm} (5)

D. Uncertainty Analyses

The uncertainty analyses are based on the procedures reported in [22]. The heat loss due to conduction is minimized by the design arrangement of the cylinder. The surface of the cylinder is coated with heat resistant paint which provides an emissivity around unity. To test the reliability of the experimental setup, the data were collected by a single person or same measuring instruments under the same conditions, and in different time, and observed around 2% variations in temperature readings. The uncertainty of the temperature measurements is measured and found to be less than 1% that provides the uncertainties of $q^*$, $\text{Nu}_D$, $\text{Ra}_D$ and $\text{Fo}$ are about 5.0%, 6.5%, 3.5% and 4.0%, respectively. The uncertainty of the fluid properties is assumed to be negligible.

III. RESULTS AND ANALYSES

A. Steady-State Results

Firstly, the steady-state analysis was carried out and these results were verified before conducting the transient experiments. The wall temperature of the cylinder was allowed to reach steady-state, typically in about 2 hours, after setting the heater power. The steady state results were obtained for heat flux ranging from 20 W/m² to 500 W/m² that provides the Rayleigh number variations from $2.1 \times 10^3$ to $1.8 \times 10^4$, the Prandtl number is around 0.72. The inlet air temperature was maintained constant, around 24 °C. For the steady-state analyses, the total number of experiments was carried out as 15. The present steady-state data are modified as $\text{Nu}_D/\text{Ra}_D^{1/4}$ to compare with the results of Kuehn and Goldstein [1] and Akagi [4] for $\text{Pr} \approx 0.7$, and in the Rayleigh number ranges, $2.1 \times 10^3$ to $1.8 \times 10^4$, shown in Fig. 2. It is observed that the present results are qualitatively similar to the reported results, and the present data are about 15% higher than those of reported data. It is also seen that the present steady state results agreed well within θ < 90° and agreed favorably with similar slope at θ > 90° indicating the region of wider thermal plume that increases with angle.

It can also be seen from the circumferential temperature distribution, at the bottom, stagnation point (θ = 0°) of the cylinder that the fluctuation of temperatures is generally high and minimum at θ = 180°. These fluctuations create turbulence in the buoyancy flow that leads to increase the heat transfer at θ = 0°, this observation is similar to the numerical observation of Qureshi and Ahmad [6] for horizontal cylinder. Again, the boundary layer is very thin at bottom or stagnation point, θ = 0°, the boundary layer thickness increases from bottom, and reached a thick layer at the top of the cylinder, θ = 180°, and the flow becomes steady laminar. This laminar flow leads to decrease the temperature gradient and heat transfer at the top of the cylinder.

It can also be seen in Fig. 2, the decrease of heat transfer is not very significant up to θ = 90°, after that decreases sharply toward the top of the cylinder. This may be due to the development of buoyant plume above the cylinder, where two boundaries layers merge. The plume creates insulation and resists the bulk fluid to flow and reduce the heat transfer. These observations are consistent with the results of similar conditions [1]-[3]. The comparison of the local and average Nusselt number values with the results of Kuehn and Goldstein [5], Saitoh et al. [6], Wang et al. [7], and Chouikh et al. [9], is given in Table I. It is seen that the present data compare favorably at θ = 0° and 90°, and at θ = 180°, the result is not good. It can also be observed in the literature that Rayleigh number and Nusselt number are related to each other for steady-state natural convection heat transfer [1]-[9]. However, the steady-state equation can be written as the following form.

Fig. 2 Influence of $\text{Ra}_D$ on local $\text{Nu}_D$

Fig. 3 Dependence of average Nusselt number on Rayleigh number
Transient period of 190 secs, the variation of average NuD with time and recorded in one-second intervals. In the transient temperature of the cylinder was increased sharply and the total number of test run was conducted as 15. The average RaD is presented in Fig. 3. A sharp decay of NuD is sensitive on the cylinder at θ = 90°, because of the high flow number. It is also observed that the convection motion is observed by Parsons and Mulligan [22], at low Rayleigh significantly with increasing time. This behavior was also thermal boundary layer at the beginning that increases increasing time. This may be due to the small thickness of the observed at the initial period of time and remains steady with time. The flow pattern is almost similar at 90° and 180°, while the maximum NuD value is obtained at 180°, around 2% higher than 90°. The transient heat transfer data are now used for the development of correlation equations at 0°, 90°, and 180°. It was observed that transient natural convection heat transfer is strongly influenced by NuD, RaD and time dependent Fo [7], [19], [20]. Thus, the correlation equation can be proposed here as:

\[ \text{Nu}_D = C (\text{Ra}_D)^m \]  \hspace{1cm} (6)

The value of C and m are obtained from the linear fit analysis, and the quality of fit result is obtained from the value of statistical indicator, R-square value [23]. The value of C and m at different angular positions of the thermocouple and the corresponding R-squared value for all fitted curves are obtained. For the overall data, the values of C= 0.7 and m= 0.3 are obtained from the fit line. The R-squared value for the fit lines is obtained here as 0.90, which is close to 1, ensures a good linear fit.

### B. Transient Results

<table>
<thead>
<tr>
<th>R(_{ad})</th>
<th>Local Nu(_D) at θ (deg)</th>
<th>Average Nu(_D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^4)</td>
<td>4.739 4.639 3.176 4.34</td>
<td></td>
</tr>
<tr>
<td>(2.1\times10^4)</td>
<td>5.836 5.896 4.018 5.82</td>
<td></td>
</tr>
<tr>
<td>(5.0\times10^3)</td>
<td>7.512 7.012 5.321 6.79</td>
<td></td>
</tr>
<tr>
<td>Present results</td>
<td>7.681 7.333 5.49 6.97</td>
<td></td>
</tr>
<tr>
<td>(1.3\times10^4)</td>
<td>8.048 7.732 6.093 7.49</td>
<td></td>
</tr>
<tr>
<td>(1.8\times10^4)</td>
<td>6.24 5.64 1.46 4.94</td>
<td></td>
</tr>
<tr>
<td>Kuehn &amp; Goldstein [5]</td>
<td>5.995 5.41 1.534 4.826</td>
<td></td>
</tr>
<tr>
<td>Saitoh et al. [6]</td>
<td>6.03 5.56 1.5 4.86</td>
<td></td>
</tr>
<tr>
<td>Wang et al. [7]</td>
<td>6.023 5.433 1.539 4.831</td>
<td></td>
</tr>
<tr>
<td>Chouikh et al. [9]</td>
<td>5.0\times10^3 6.836 5.896 4.018 4.34</td>
<td></td>
</tr>
<tr>
<td>(9.2\times10^3)</td>
<td>8.048 7.732 6.093 7.49</td>
<td></td>
</tr>
<tr>
<td>(2.1\times10^3)</td>
<td>4.739 4.639 3.176 4.34</td>
<td></td>
</tr>
</tbody>
</table>

The heater power-on transient tests were conducted in five different heat fluxes: 177, 500, 987, 1617, and 2426 W/m\(^2\), and the total number of test run was conducted as 15. The transient temperature of the cylinder was increased sharply with time and recorded in one-second intervals. In the transient period of 190 secs, the variation of average Nu\(_D\) with average Ra\(_D\) is presented in Fig. 3. A sharp decay of Nu\(_D\) is observed at the initial period of time and remains steady with increasing time. This may be due to the small thickness of the thermal boundary layer at the beginning that increases significantly with increasing time. This behavior was also observed by Parsons and Mulligan [22], at low Rayleigh number. It is also observed that the convection motion is sensible on the cylinder at θ = 90°, because of the high flow velocity at the side of the cylinder or θ = 90°, that leads to increase the convection motion as well as the heat transfer. The flow pattern is almost similar at 90° and 180°, while the fluid flow at bottom (θ=0°) and top (θ=180°) is almost stagnation. It is to be noted here that the beginning of convection flow occurs at 0°, and reaches almost maximum at 90° when the flow field and boundary layers are fully developed. This is due to the fact that increasing Ra\(_D\) tends to increase the buoyancy driven flow near the surface of the cylinder that decreases with Fourier number. It is also observed in Fig. 3 that at small time (around 20 s), and Ra\(_D\) < 42500, the Nu\(_D\) is independent of time, representing a transient conduction regime.

Again, at small time the upward fluid motion near the cylinder surface tends to form circulating flow region around the cylinder. This region grows with time and approaches from bottom to the top of the cylinder. This may be due to the development of isotherms, which is attached strongly at the bottom of the cylinder and move toward the top of the cylinder with increasing time. At the beginning of transient heat transfer, the thermal boundary layer is very thin, and the transfer of heat is only by conduction and all the curves coincide with each other, as observed in Fig. 3. At Rayleigh number Ra\(_D\) > 42500, the steeper free convection regime corresponds to the steady Nusselt number. Finally, the curves show that the free convection regime is steeper than those of conduction regime. It can also be observed, the fluctuation of temperatures at different θ is generally high and decreases with increasing time and angle (θ). The present Nu\(_D\) data at 90° and 180° are about 27% and 29% higher than that of 0°, while the maximum Nu\(_D\) value is obtained at 180°, around 2% higher than 90°. The transient heat transfer data are now used for the development of correlation equations at 0°, 90°, and 180°. It can be proposed here as:

\[ \text{Nu}_D = C (\text{Ra}_D)^m \]  \hspace{1cm} (7)

To determine the exponent n of Ra\(_D\), the results in Fig. 3 are re-plotted by modifying the variation of [Nu\(_D\)/Ra\(_D\)] with Fo for different values of n, ranging from −0.25 to +0.25, and obtained the corresponding R-square values to verify the linear fit lines. The R-squared values for different n are plotted in Fig. 4 for θ = 0°, 90° and 180°. It is observed that the maximum R-squared value obtained at n = 0.1 for 0°, and at n = 0 for 90° and 180°. Thus, (7) can be simplified by putting the values of n as:
The circumference of the cylinder is investigated and observed that transient operations, the effect of heat transfer on the available results in the literature. During the heater power-on convection heat transfer agreed favorably with that of the cylinder. The steady state experimental results of natural unsteady natural convection heat transfer from a horizontal cylinder in air, that might be applicable under understanding of the steady-state and time dependent thermal behavior of horizontal cylinder in air, that might be applicable for the design of tube type heat exchange devices.

VI. CONCLUSIONS

The experiments are conducted to analyze the steady and unsteady natural convection heat transfer from a horizontal cylinder. The steady state experimental results of natural convection heat transfer agreed favorably with that of the available results in the literature. During the heater power-on transient operations, the effect of heat transfer on the circumference of the cylinder is investigated and observed that the average Nu data increased with the angle (θ), of about 27% from 0° to 90°, and 2% from 90° to 180°. All the experimental data are presented by correlation equations. In the correlation equation (7), the appropriate value of the exponent, n of Ra, for θ = 0°, 90°, and 180° are obtained, the unique values of the exponents are, n = 0.1 for θ = 0°, and n = 0 for θ = 90°, 180°. Thus, the reported correlations for the present conditions are shown in (8) and (9), and the values of C and m are given in Table I.

NOMENCLATURE

Greek Symbols

- \( \mu \) Dynamic viscosity, kg/m·s
- \( \rho \) Mass density, kg/m³
- \( \theta \) Angular position of thermocouple, Deg
- \( \beta \) Thermal expansion co-efficient, 1/K
- \( \nu \) Kinematic viscosity, \( \mu/\rho \), m²/s
- \( \alpha \) Thermal diffusivity, m²/s

REFERENCES


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