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An Integrated Mixed-Integer Programming Model to Address Concurrent Project Scheduling and Material Ordering

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***Abstract*—**Concurrent planning of project scheduling and material ordering can provide more flexibility to the project scheduling problem, as the project execution costs can be enhanced. Hence, the issue has been taken into account in this paper. To do so, a mixed-integer mathematical model is developed which considers the aforementioned flexibility, in addition to the materials quantity discount and space availability restrictions. Moreover, the activities duration has been treated as decision variables. Finally, the efficiency of the proposed model is tested by different instances. Additionally, the influence of the aforementioned parameters is investigated on the model performance.

***Keywords*—**Material ordering, project scheduling, quantity discount, space availability.

I. INTRODUCTION

THE resource constraint project scheduling problem is ordinarily treated as an independent issue in the project management planning phases [1]. Likewise, the procurement planning is taken into consideration as another field of research [2]. However, it was shown in the recent studies that the simultaneous consideration of project scheduling and materials procurement can lead to the improvement of the projects management with respect to the reduction in unnecessary execution costs [3]. Consequently, the purpose of this research is to deal with the concurrent planning of project scheduling and material ordering in order to enhance the project-associated costs. The essence of concentration on the issue can be justified with regard to the fact that separate planning could result in non-optimal solutions, as the orders are set along with the project schedule. Whereas, the orders can be put into practice in the course of activities execution scheduling to prevent incurring surplus costs. In other words, the costs such as ordering, holding, and activities completion may increase if the orders are set in a further phase [4].

The synchronized consideration of project scheduling and material ordering, i.e., non-renewable resources, was first presented by [5]. They developed the concept under the framework of an integrated critical path method and material requirement planning. Afterwards, [6] followed up the improvement of [5] by a heuristic solution based on the slack rule. They also entered both renewable and non-renewable resources in their proposed model formulation. Afterwards, the aforementioned problem was pursued for fixed duration of project activities [7].

All studies assumed that the activity duration is known, in advance, and can be efficiently represented by a constant value, up to the previously mentioned papers. Additionally, the trade-offs between inventory holdings, completed activities holding, material ordering, and project delays had been taken into consideration. However, [8] accounted for other factors in order to enhance total project costs and increase the schedule flexibility. The new factors consisted of variable activity duration, variable project worth, rewards for early completion of the project than its due date, and discounts in material procurement. Sheikh Sajadieh et al. [9] developed Dodin and Elimam's model [8] by presenting a genetic algorithm to solve larger size instances, as the resource constraint project scheduling problems belong to the NP-hard categories.

The consideration of reward/penalty for early/late completion of the project may influence the procurement conditions with respect to the time and size aspects. The same issue can be tracked for the circumstance where the activity completion can be considered under compression status. Moreover, materials procurement methods can be followed as another important issue which can be reflected in terms of quantity discounts presence. On the other hand, the limitation on the available space to hold and store the required materials can affect the project execution, in practical instances. Thus, the authors addressed the synchronized project scheduling and material ordering considering accessibility amount to the holding space and present quantity discounts. The rest of the paper has been organized as follows. The proposed mathematical formulation is presented in Section II. Section III pertains to the numerical instances, accompanied by the influence of the aforementioned issues on the obtained results. Finally, conclusions and future research interests are presented in Section IV.

II. MATHEMATICAL MODEL

The mixed-integer programming model is introduced in this section. It assumes that the requirement amount to the materials for activities completion does not pertain to the activity duration. On the other hand, the activities may require m different materials to be completed. The applied indices, parameters, and decision variables are mentioned first, as follows.

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TABLE I
INDICES

Parameters	Random distribution functions
$j = 1,..., N$	Index of project activities.
$m = 1,..., M$	Set of required materials (non-renewable resources).
$k = 1,..., K$	Discount quantity ranges.
$t = 1,..., H$	Set of time units.

TABLE II
PARAMETERS

Parameters	Random distribution functions
P_j	Set of activities preceding activity *j*.
P_0	Set of activities with no precedent activity.
b_j	Compression cost of activity *j*.
c_j	Compression cost of activity *j* per one time unit.
e_j	Earliest completion time of activity j assuming that the duration of each activity equals its compressed time.
l_j	Latest completion time of activity j assuming that the duration of each activity equals its compressed time and the project latest completion time equals *H*.
u_j	Upper bound on activity *j* duration (normal completion time).
v_j	Lower bound on activity *j* duration (compressed completion time).
J_t	Set of activities that can be completed in period *t*.
α_{mk}	Limit on the quantity range *k* of material *m*.
δ_{mk}	Cost of material *m* in quantity range *k*.
G_m	Ordering cost of material *m*.
h_m	Holding cost per one unit of material *m*.
K_m	Number of discount ranges for material *m*.
L_m	Lead time of material *m* in time periods.
R_{jm}	Unit amount of material *m* to process activity *j*.
d	Project due date after which the given penalty is incurred to the contractor.
C	Materials holding capacity.
S_m	Space volume that material *m* occupies to store.
H	Maximum length of the planning horizon to deliver the project.
p	Penalty cost of delivering the project beyond its due date per time period.
r	Reward of delivering the project before its due date per time period.
s	Percentage of the activity's worth representing the holding cost of completed activities per time period.

Model Formulation

Now, the mathematical model formulation can be presented through (1)-(12), as follows.

TABLE III
DECISION VARIABLES

Parameters	Random distribution functions
x_{jt}	1 if activity *j* is completed in time period *t* and 0, otherwise.
λ_{mkt}	1 if material *m* is ordered in quantity range *k* in time period *t* and 0, otherwise.
I_{mt}	Inventory level for material type *m* at the end of time period *t*.
q_{mkt}	Quantity amount of material type *m* ordered in quantity range *k* time period *t*.
w_{jt}	Activity *j* worth completed by the end of time period *t*.
W_t	Project worth by the end of time period *t*.
z_j	Activity *j* duration.

$$MinZ = [\sum_{j=1}^{N}[b_j - c_j(z_j - v_j)] - \sum_{t=e_N}^{d} r(d-t)x_{Nt} + \sum_{t=d}^{H} p(t-d)x_{Nt} + \sum_{t=1}^{H-1} sW_t + \sum_{m=1}^{M}\sum_{k=1}^{K_m}\sum_{t=1}^{H-L_m} G_m\lambda_{mkt} + \sum_{m=1}^{M}\sum_{k=1}^{K_m}\sum_{t=1}^{H-L_m}\delta_{mk}q_{mkt} + \sum_{m=1}^{M}\sum_{t=1}^{H-1} h_m I_{mt}] \tag{1}$$

S.t:

$$\sum_{t=e_i}^{l_i} tx_{it} + z_j \le \sum_{t=e_j}^{l_j} tx_{jt}, \quad \forall i \in P_j$$
$$z_j \le \sum tx_{jt}, \quad \forall i \in P_0 \tag{2}$$

$$v_j \le z_j \le u_j, \quad \forall j = 1,..., N \tag{3}$$

$$\sum_{t=e_j}^{l_j} x_{jt} = 1, \quad \forall j = 1,...,N \tag{4}$$

$$w_{jt} \geq \sum_{j=1}^{N} [b_j - c_j(z_j - v_j)] - b_j(1 - x_{jt})],$$
$$\forall j = 1,...,N, \quad \forall t = 1,...,H \tag{5}$$

$$W_t \geq W_{t-1} + \sum_{j \in J_t} w_{jt}, \quad \forall t = 1,...,e_N$$
$$W_0 = 0 \tag{6}$$

$$w_t \geq W_{t-1} + \sum_{j \in J_t} w_{jt} - SB \sum_{e_N}^{t} x_{Nt},$$
$$\forall t = e_N + 1,...H \tag{7}$$
$$(SB = \sum_{j=1}^{N} b_j)$$

$$I_{mt} = I_{m(t-1)} + \sum_{k=1}^{K_m} q_{mk(t-L_m)} - \sum_{j=1}^{N} R_{jm} x_{jt}$$
$$I_{m0} = 0, \quad \forall t = 1,...,H \quad \forall m = 1,...,M \tag{8}$$

$$\sum_{m=1}^{M} I_{mt} S_m \leq C, \quad \forall t = 1,..., H \tag{9}$$

$$\sum_{k=1}^{K_m} \lambda_{mkt} \leq 1, \quad \forall t = 1,...,H \quad \forall m = 1,...,M \tag{10}$$

$$\alpha_{m(k-1)t} \lambda_{mkt} \leq q_{mkt} \leq \alpha_{mkt} \lambda_{mkt},$$
$$\forall t = 1,...,H \quad \forall m = 1,...,M \quad \forall k = 1,...,K_m \tag{11}$$

$$x_{jt},\ \lambda_{mkt} \in \{0,1\},\ I_{mt},\ q_{mkt},\ w_{jt} \geq 0 \tag{12}$$

The objective function (OF) purpose is to minimize the project costs, shown by (1). It consists of compression cost, reward for early and penalty for delay in project delivery, activities worth, material ordering cost, material purchase cost, and material holding cost, respectively. However, if the project is completed earlier than its due date, the assigned reward is abstracted from the aforementioned costs. Equation (2) corresponds to the precedence relations, in which activity j cannot be completed until its precedent, i.e., activity *i*, is completed. Equation (3) states that the activities duration is bounded by the lower and upper values, respectively. The activities completion time is addressed by (4) that must be met within the earliest and latest completion times. Equations (5) and (6) denote the activity and project worth, respectively, in which the project worth can be calculated in time period t through the cumulative sum of the completed activities worth at the time. It should be noted that the activity worth concept refers to the fact that a given project cannot be exploited unless it is completed. Hence, a holding cost can be imagined for the completed activities. Moreover, (7) has been added to the model in order to prevent continuously accumulating the project worth beyond its completion time. Equation (8) corresponds to the inventory balance status, in which the inventory of each material can be monitored over the planning horizon. The space availability to store the materials is also considered by (9). Equation (10) represents that the ordered quantity of each material is restricted by a lower and upper limit, respectively. Equation (11) stipulates that the activity requirement to the materials must be satisfied at most in terms of a single order. Finally, (12) shows the nature of the decision variables.

III. Computational Study

This section deals with the computational experiments to show the applicability and efficiency of the mathematical model. In this regard, different problem categories have been taken into account to provide the comparison possibility for the presence of storing space availability and quantity discounts, respectively. Therefore, three separate states are investigated to represent the aforementioned conditions. In the first state, the problem just provides the synchronized consideration of project scheduling and material ordering under storing space limitation. In the second state, the problem takes the quantity discount into account. Finally, the third state investigates both the limitation on material holing and both the potential to procure the resources in different prices. Data generation method is demonstrated in Table IV, as follows. The performance of the model has been tracked for each of the states and the relevant results are shown in Tables V-VII, respectively.

GAMS 22.1 solver has been used to solve the instances and all calculations are run on a Core i3 PC with 2.0 GHz CPU and 4 GB of RAM.

It should be noted that holding cost of completed activities has been considered equal to 0.01 of W_t period, for the sake of simplicity. Moreover, different values are regarded for the project planning horizon and due date for each of the aforementioned states to let the model function appropriately. Moreover, *L* and *U* stand for limited and unlimited space to store the required materials, respectively.

According to Table V, it can be found that the possibility to increase the storing space can lead to the project costs improvement. In fact, more available space can yield to better balance between holding and ordering costs. The results of Table VI also denote higher flexibility to the project execution for the circumstances with more quantity discounts. It can be understood that the procurement costs have noticeably reduced in the quantity discounts. Finally, Table VII depicts the results of the solutions under the space availability and quantity discount conditions. According to the obtained solutions, the mathematical formulation has achieved the final scheduling and ordering decisions with respect to the contingencies which implies the applicability of the model for real problems. As stated earlier, such problems belong to the NP-Hard ones and need a more flexible solution method to reach the solution in a reasonable times. The issue can be better reflected for considering the problem under space limitation and discount utilization possibility, in particular. Hence, application of more efficient solution methodologies can be highlighted for large-sized projects.

TABLE IV
DATA GENERATION METHOD

Parameters	Random distribution functions
u_j	~ U[3, 5]
v_j	~ U[1, 3]
c_j	~ U[300, 800]
R_{jm}	~ U[50, 150]
a_j	~ U[1000, 1500]
b_j	~ U[1500, 2500]
δ_{mk}	~ U[15, 25]
α_{mk}	~ U[50, 300]
G_m	~ U[100, 200]
h_m	~ U[3, 6]
L_m	~ U[1, 3]
C	~ U[100, 300]
S_m	~ U[1, 3]

TABLE V
MODEL PERFORMANCE UNDER PRESENCE OF STORING SPACE LIMITATION

J	*M*	*C*	Solution Elapsed Time (Sec.)	OF Value
7	2	*L*	5	34106
7	3	*L*	9	38815
7	2	*U*	6	33876
10	2	*L*	95	41157
10	3	*L*	116	43154
10	2	*U*	91	40486
15	2	*L*	404	57482
15	3	*L*	446	58450
15	2	*U*	398	57215
17	2	*L*	851	60181
17	3	*L*	933	61772
17	2	*U*	819	58876
20	2	*L*	1388	82783
20	3	*L*	1592	87554
20	2	*U*	1458	80667

TABLE VI
MODEL PERFORMANCE UNDER PRESENCE OF QUANTITY DISCOUNTS

J	*M*	*K*	Solution Elapsed Time (Sec.)	OF Value
7	2	2	6	33516
7	3	3	8	36335
7	2	4	6	32786
10	2	2	105	38848
10	3	3	142	41066
10	2	4	118	38790
15	2	2	465	53381
15	3	3	495	57315
15	2	4	434	52954
17	2	2	1090	60044
17	3	3	1154	61772
17	2	4	934	57184
20	2	2	1548	79185
20	3	3	1613	82985
20	2	4	1739	78226

IV. CONCLUSIONS

The authors proposed a mixed-integer programming model in this paper to deal with simultaneous planning of project scheduling and material procurement under the possibility of space availability and quantity discounts. The consideration the aforementioned issues could enhance the model flexibility which was tested by different numerical instances. However, the mathematical formulation did not incorporate the potential uncertainty in the structural parameters which may result in unrealistic solutions in the real world problems. Consequently, the entrance of uncertain parameters can be an interesting future research direction. Moreover, the application of efficient heuristic approaches can be mentioned as another research interest such that the formulation obtains the final solution for larger networks, as well.

TABLE VII
MODEL PERFORMANCE UNDER PRESENCE OF STORING SPACE LIMITATION AND QUANTITY DISCOUNTS

J	*M*	*C*	*K*	Solution Elapsed Time (Sec.)	OF Value
7	2	L	2	6	33716
7	3	U	3	8	36335
7	2	L	4	6	32945
10	2	U	2	105	38848
10	3	L	3	146	41576
10	2	U	4	118	38790
15	2	L	2	477	54868
15	3	U	3	495	57315
15	2	L	4	461	54502
17	2	U	2	1090	60044
17	3	L	3	1213	61883
17	2	U	4	934	57184
20	2	L	2	1692	80247
20	3	U	3	1613	82985
20	2	L	4	1947	79331

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