

An Improved Optimal Sliding Mode control for Structural Stability

Leila Fatemi, Morteza Moradi, Azadeh Mansouri

Abstract—In this paper, the modified optimal sliding mode control with a proposed method to design a sliding surface is presented. Because of the inability of the previous approach of the sliding mode method to design a bounded and suitable input, the new variation is proposed in the sliding manifold to obviate problems in a structural system. Although the sliding mode control is a powerful method to reject disturbances and noises, the chattering problem is not good for actuators. To decrease the chattering phenomena, the optimal control is added to the sliding mode control. Not only the proposed method can decline the intense variations in the inputs of the system but also it can produce the efficient responses respect to the sliding mode control and optimal control that are shown by performing some numerical simulations.

Keywords—Structural Control; optimal control; optimal sliding mode controller; modified sliding surface

I. INTRODUCTION

DURING decades, development of the technology has led to higher accurate control systems to overcome nonlinearity and uncertainty of the systems. Moreover, reliability is a main character of a control system. So, to increase such properties, new methods should be employed to design a controller. Structural systems are vulnerable against wind and earthquake if no reliable control systems is used in the structures. Structures with inappropriate control systems have performed poorly in the recent earthquake. The semi-active control using a variable friction damper was proposed by [1]. The friction damper provides a structure with the damping effect in response to the displacement. Conventional PID controller is a simple and reliable method to control different systems but it could not produce desirable responses in the presence of intense variations. [2] proposed a sliding mode control for a structural control system and compares the sliding mode and PID controller to show the efficient performance of the sliding mode control.

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Tuned mass damper, consisting of mass, damping and a spring, is an effective structural vibration control device that commonly attached to a system. Different methods were proposed to tune the coefficients [3, 4]. A maximum principle was proposed by [5]. A robust method to control a structure against earthquake was proposed by [6]. The 4-story structure was considered where for each floor one controller was used. But, the main problem of its robust method is that the maximum domain of the inputs is too large that is very hard to produce in practice. Also, [7] proposed the applications of the LQR, H_2 and H_∞ in a structural system then compared the performances of the methods.

To improve the performance of the optimal method, this method was combined with the sliding surface [8]. Also, stability of the closed loop system and an application of the conventional optimal sliding mode control for a nonlinear system was proposed by [9]. Because of the method that used to design sliding surface, the proposed optimal sliding mode control can not be employed in the structural system.

In this paper, we propose a new sliding surface and then solely employ it to control the structural system and also combine it with the optimal control to produce the optimal control signal that can mitigate the variations of the structure in the presence of the earthquake. To show the performance of the proposed method some simulations are done. Results show that the proposed method has an outperformance respect to the other methods.

II. SYSTEM MODEL

The structural system can be described by Lagrange's equations. The dynamics of a linear structural system is as follow [6, 7]:

$$M\ddot{q} + C\dot{q} + Kq = f \quad (1)$$

Where q is a displacement vector, M is a mass matrix, C is a damping coefficient matrix, K is a stiffness coefficient matrix and f is a vector of external force that includes earthquake and control forces. Defining $X = [q, \dot{q}]^T$ and doing some manipulation, Eq (1) in state space model is as follow:

$$\dot{X} = AX + BU \quad (2)$$

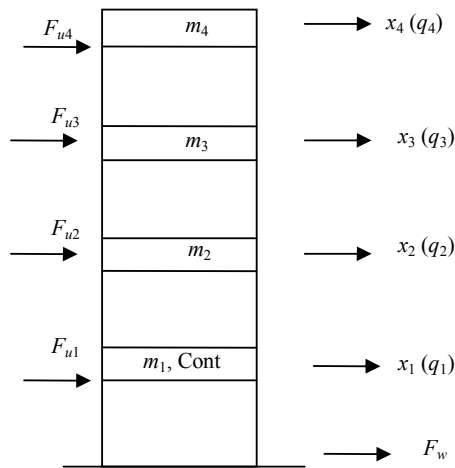


Fig. 1 Structural System

Where

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \quad (3)$$

The system consists of a controller in each floor. Fig (1) shows the model of the structure.

III. OPTIMAL CONTROLLER

By using LQR method, the feedback gain can be obtained as:

$$k_f = -R^{-1}BP \quad (4)$$

Where R is an arbitrary positive definite matrix and P is computed from following equation:

$$PA + A^T P - PBB^T P + Q = 0 \quad (5)$$

Where Q is an arbitrary positive definite matrix. At result, the input of the system can be written as:

$$U = -K_f e \quad (6)$$

Where $e = r - X$. As far as the desired response for the outputs of the system is zero we have $r = 0 \Rightarrow e = -X$. In this paper, for simulation part, $Q = 200I_{8 \times 8}$ and $R = 10^{10}I_{4 \times 4}$ are chosen.

IV. SLIDING MODE CONTROL

Before we proposed our method, we consider the conventional method to design sliding surface. From Eq(2), following switching surface can be designed:

$$S = \{X \in R^8, S(X) = HX = 0\} \quad (7)$$

Where H is chosen to make $HB \neq 0$ [8]. We define u_1 that can be computed by imposing $\dot{S} = 0$ as:

$$\begin{aligned} \dot{S} &= H\dot{X} = HAX + HBU = 0 \\ \Rightarrow u_1 &= -(HB)^{-1}HAX \end{aligned} \quad (8)$$

By using Eq (8), the switching control u_s is designed as:

$$u_s = -(HB)^{-1}M_s \operatorname{sgn}(S) \quad (9)$$

Where $M_s > 0$ is a constant diagonal matrix. The whole control law is:

$$U = u_1 + u_s \quad (10)$$

But, to use this control law for the system (2), there are two problems:

- 1) We have one free parameter C to design u_1 while both H and H^{-1} exist in the equation. So, a time consuming process should be done till the good value of H is chosen.
- 2) In a structural system, the elements of A matrix are big while elements of B matrix are small that result an input with large domain. In practice such an input cannot be produced.

To solve the problems, a new sliding surface is proposed as follow:

$$S = HX + H(L - A) \int_0^t X dt \quad (11)$$

Where L is a matrix that is chosen by designer. By employing the method (8) into Eq(11) we have:

$$\begin{aligned} \dot{S} &= HAX + HBU + HLX - HAX = HBU + HLX = 0 \\ \Rightarrow u_{1n} &= -(HB)^{-1}HLX \end{aligned} \quad (12)$$

Combining Eq(12) with Eq(9), we have:

$$U = u_{1n} + u_s \quad (13)$$

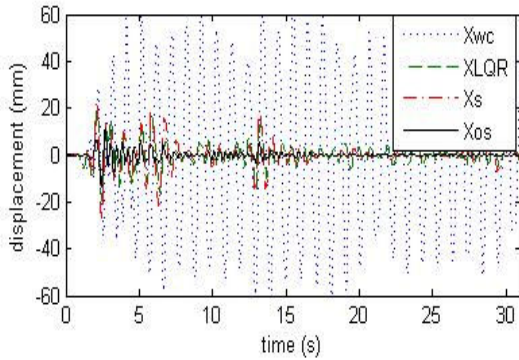


Fig. 2 Response of the closed loop system

This new controller will be used in the simulations. In the new method, we have L matrix instead of A matrix. L is adjustable and is chosen by designer. In this case, we are able to tune the maximum domain of the inputs. To prove the stability of the closed loop system, following Lyapunov function is selected:

$$V = S^T S \quad (14)$$

Derivation along Eq(4) results:

$$\dot{V} = S^T \dot{S} = S^T (HBU + HLX) \quad (15)$$

By using Eq(13), (12) and (9) into Eq(15) we have:

$$\begin{aligned} \dot{V} &= S^T (HBu_{in} + HBu_s + HLX) \\ \dot{V} &= -S^T M_s \operatorname{sgn}(S) = -M_s |S| \leq 0 \end{aligned} \quad (16)$$

To design the sliding mode controller, $L = 2I_{8 \times 8}$, $M_s = I_{4 \times 4}$ and $H = 10[I_{4 \times 4}, I_{4 \times 4}]$ are chosen.

V. OPTIMAL SLIDING MODE CONTROL

To design the optimal sliding mode control, following equation is used as the main input:

$$U = V_{op} - MV_s \quad (17)$$

Where V_{op} is the optimal control that is produced based on Eq (6) and $V_s = \operatorname{sgn}(S)$. Sliding surface is designed as:

$$S = S_o + \varphi \quad (18)$$

Where S_o is the sliding surface that is defined by Eq(11) and φ is auxiliary variable that is the solution of the differential

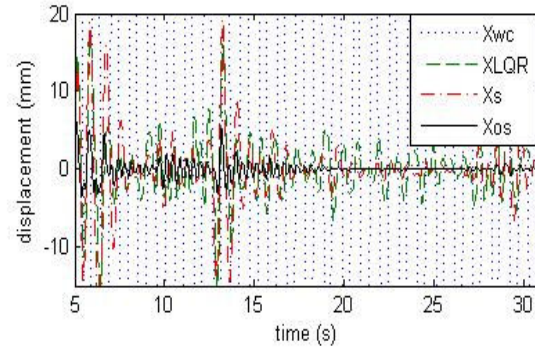


Fig. 3 Focus in the response of the closed loop system

equation:

$$\dot{\varphi} = -H(LX + BV_{op}) \quad (19)$$

To prove the stability of the closed loop system, consider following lyapunov function

$$V = S^T S \quad (20)$$

Derivation along Eq(20), we have

$$\dot{V} = S^T \dot{S} = S^T (\dot{S}_o + \dot{\varphi}) \quad (21)$$

By substituting Eq (19) into Eq (21) and computing \dot{S}_o

$$\begin{aligned} \dot{V} &= S^T (HAX + HBU + HLX - HAX - HLX - HBV_{op}) \\ &= S^T (HBU - HBV_{op}) \end{aligned} \quad (22)$$

By using Eq(17)

$$\begin{aligned} \dot{V} &= S^T (HBV_{op} - HBM_s V_s - HBV_{op}) \\ &= -S^T M_{ss} \operatorname{sgn}(S) = -M_{ss} |S| \leq 0 \end{aligned} \quad (23)$$

Where $M_{ss} = CBM_s$. The values of the parameters for this part are chosen as

$$L = A + 0.01I_{8 \times 8}, \quad H = 1000[I_{8 \times 8}, I_{8 \times 8}]$$

$$M_s = 5 \times 10^5 I_{4 \times 4}$$

VI. SIMULATION

The data for simulating the system was gained from [6]. To simulate the closed loop system with three controllers, following notice is considered:

- We chosen the above parameters values till the inputs of the closed loop system in all cases for three controllers approximately have same

maximum domain. In this way, the comparison of the results is more valuable.

Fig. 2 shows the response of the three controllers. As the figure shows, the LQR and sliding mode control approximately have same response. The response of the combined optimal sliding mode control is suitable and smaller than the two other controllers. For a better consideration, the focus in time span [5, 31] was shown in Fig 3. The domain of the output variations for response of the optimal sliding mode controller is less than 0.1 mm in time span [17, 27]. The maximum domain for the LQR controller is 8.8×10^5 , for the sliding mode control is 7.5×10^5 and for the optimal sliding mode control is 9.4×10^5 . The domains show that with the same constraint on the input of the system, the optimal sliding mode control has a better performance. Also, if we compare this result with [6], our proposed method has outperformance while in [6] the domain of the controller is 19×10^6 that is too much large. In next simulation, we consider the response of the proposed controller in the presence of the uncertainty. Suppose that $M = M + 0.5M$. Fig 4 demonstrates the response of the closed loop system for the three controllers. Fig 5 shows the responses more precisely. Although the response of the optimal sliding mode control has some fluctuations the maximum domain in time span [17, 27]s is less than 0.5 mm. The figures show that the proposed method can be reliable and produce an optimum output.

VII. CONCLUSION

Optimal sliding mode control was the main controller that was employed in this paper. But, there was a problem to design the sliding surface for the structural system. To obviate this issue the new sliding surface was proposed and the stability of the closed loop system was proved by using lyapunov method. The proposed sliding surface was combined with the optimal control and employed in the closed loop system. The simulations were shown that the proposed method is stable and powerful enough to reject the earthquake disturbance and overcome on the uncertainty in the dynamic of the system.

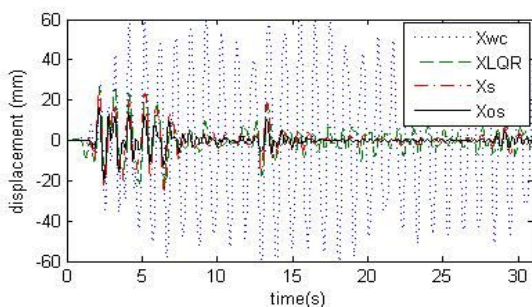


Fig. 4 Response of the closed loop system in the presence of uncertainty

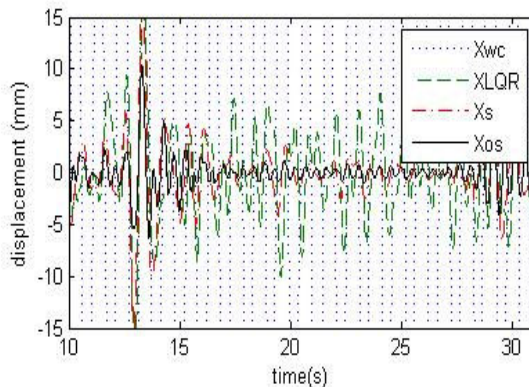


Fig. 5 Focus in the response of the closed loop system in the presence of the uncertainty

REFERENCES

- [1] O. A. Nishitani, et al, "Semi-active structural control with variable friction dampers", Proceeding of the American Control Conference, California, 1999, pp. 1017-1021.
- [2] R. Guclu, "Sliding mode and PID control of a structural system against earthquake", Mathematical and Computer Modeling, Vol. 44, 2006, pp. 210-217.
- [3] C. L. Lee, et al, "Optimal design theories and applications of tuned mass dampers", Engineering Structures, Vol. 28, 2006, pp. 43-53.
- [4] S. Chakraborty, B. K. Roy, "Reliability based optimum design of tuned mass damper in seismic vibration control of structures with bounded uncertain parameters", Probabilistic Engineering Mechanics, Vol. 26, 2011, pp. 215-221.
- [5] J. M. Sloss, et al, "Optimal control of structural dynamic systems in one space dimension using a maximum principle", Journal of Vibration Control, Vol 11, 2005, pp. 245-261.
- [6] S. G. Wang, et al, "Simulation of the robust control for uncertain structural system against earthquake", Proceeding of the 4th World Congress on Intelligent Control and Application, China, 2002, pp. 1681-1686.
- [7] G. E. Stavroulakis, et al, "Robust active control against wind-induced structural vibrations", Journal of Wind Engineering and Industrial Aerodynamics, Vol. 94, 2006, pp. 895-907.
- [8] R. Xu, "Optimal sliding mode control and stabilization of underactuated systems", Dissertations, Ohio State University, 2007.
- [9] C. Pukdeboon, A. S. I. Zinober, "Optimal sliding mode controllers for attitude tracking of spacecraft", 18th IEEE International Conference on Control Applications, Saint Petersburg, 2009, pp. 1708-1713.