

# An Improved Model for Prediction of the Effective Thermal Conductivity of Nanofluids

K. Abbaspoursani, M. Allahyari, and M. Rahmani

**Abstract**—Thermal conductivity is an important characteristic of a nanofluid in laminar flow heat transfer. This paper presents an improved model for the prediction of the effective thermal conductivity of nanofluids based on dimensionless groups. The model expresses the thermal conductivity of a nanofluid as a function of the thermal conductivity of the solid and liquid, their volume fractions and particle size. The proposed model includes a parameter which accounts for the interfacial shell, brownian motion, and aggregation of particle. The validation of the model is verified by applying the results obtained by the experiments of  $\text{TiO}_2$ -water and  $\text{Al}_2\text{O}_3$ -water nanofluids.

**Keywords**—Critical particle size, nanofluid, model, and thermal conductivity.

## I. INTRODUCTION

HEAT transfer plays an important role in numerous applications where the thermal energy transfer between fluid through some heat transfer devices; such as, heat exchangers, evaporators, condensers, and heat sinks. Increasing the heat transfer efficiency of these devices is desirable, because by increasing efficiency, the space occupied by the device can be minimized, which is important for applications with compactness requirements. There are several methods to improve the heat transfer efficiency. Some methods are utilization of extended surfaces, application of vibration to the heat transfer surfaces, and using of micro channels which came an increase in heat transfer area and therefore increasing the size of device. In the recent decade some researches purposed heat transfer efficiency can also be improved by increasing the thermal conductivity of the working fluid and shown that the nanofluids have great potential for heat transfer enhancement [15, 16]. Commonly used heat transfer fluids such as water, ethylene glycol, and engine oil have relatively low thermal conductivities, when compared to the thermal conductivity of solids. High thermal conductivity of solids can be used to increase the thermal conductivity of a fluid by adding solid particles in nano size to that fluid. In order to justify of anomalous thermal conductivity enhancement of nanofluid some researches have explained several reasons such as;

Brownian motion of nanoparticles, clustering of nanoparticles, liquid layering around nanoparticles, and ballistic phonon transport in nanoparticles. In order to model the thermal conductivity based on the mentioned mechanisms several equations presented by researchers based on Brownian motion which developed for the determination of thermal conductivity of nanofluids based on the Brownian motion of nanoparticles [4, 9-10]. Xie et al. [6] studied the effect of the interfacial nano-layer on the enhancement of thermal conductivity with nanofluids. According to the model the enhanced thermal conductivity of nanofluid by considering the effects of the nano-layer thickness, nanoparticle size, volume fraction, and thermal conductivity ratio of particle to fluid. But the densification of the adsorption layer affects further the compatibility of the particle surface to the surrounding liquid. There are many other theoretical models that take the effect of liquid layering around nanoparticles into account [7-8]. Xuan et al. [11] studied the thermal conductivity of nanofluids by considering Brownian motion and clustering of nano particles. An equation was proposed to predict the thermal conductivity of nanofluids. It should be noted that the equation is not non-dimensional, which is an indication of a mistake in the analysis. Hezaveh et al. [13] present a new equation for predicting effective thermal conductivity for  $\text{Al}_2\text{O}_3$  nanoparticles dispersed in ethylene glycol (EG) and water based on shape factor and volume fraction of nano-particles is developed. It has been proved that assuming  $\text{Al}_2\text{O}_3$  particles as spherical molecules is not acceptable. Also, according to morphology of  $\text{Al}_2\text{O}_3$ , a new shape factor for these nanoparticles which is more realistic, is given. Finally, theoretical results have been compared with experimental data and it is shown that shape factor value applied for  $\text{Al}_2\text{O}_3$ -Water system cannot be used for  $\text{Al}_2\text{O}_3$ /EG system. Therefore, EG molecules cover  $\text{Al}_2\text{O}_3$  nanoparticles differently and base fluids molecular structure must be considered in nanoparticle shape factor. Moghadassi et al. [21], A model for the effective thermal conductivity of nanofluids based on dimensionless groups has been presented. This model indicate that increase the thermal conductivity of nanofluid with particle size decreasing. it is in good agreement with experimental data for  $\text{Al}_2\text{O}_3$ -water and  $\text{Al}_2\text{O}_3$ -EG nanofluids in some range of particle size and particle volumetric fraction. Explanations and analyze the heat transfer mechanisms in nanofluids and previously developed correlations have not completely predicted the anomalous increase in the thermal conductivity. It is clear that more research want to arrive at comprehensive and suitable theories. In this present study, Through the use of appropriate dimensionless groups and experimental data, a new expression to estimate the effective thermal conductivity of nanofluids will be derived. Further studies were performed to understand how some factors affect the thermal conductivity of nanofluids. Attention to get factors

K. Abbaspoursani is with the Faculty of Mechanical Engineering, Islamic Azad University, Takestan Branch, Takestan, Iran, 5<sup>th</sup> Km Hamaden Motorway, Shamishop Cross, Takestan (phone: 0282-5270157; fax: 0282-5270145; e-mail: a.abbaspour@tiau.ac.ir).

M. Allahyari is correspondence author and he is an M.S. student in the Faculty of Mechanical Engineering, Takestan Azad University, Takestan Branch, Takestan, Takestan (e-mail: md.allahyari@yahoo.com).

M. Rahmani is the Member of Young Researchers Club, Islamic Azad University, Takestan Branch, Takestan, (e-mail: Meisamrahmani68@yahoo.com).

shows important of effective parameter that influences the thermal conductivity of nanofluids.

## II. FORMULATION OF THE MODEL

In this section by applying the theory of dimensional analysis to the heat transfer phenomena of a nanofluid, a new model to estimate the effective thermal conductivity of nanofluids will be derived. For obtaining a relatively accurate relation between thermal conductivity and other effective thermo-physical properties of a nanofluid, this process should be analyzed with a specific nanofluid under some pre evaluated conditions. In the present work the formulation of the proposed model for the effective conductivity in derived by the several experimental data [5, 14, 17] for TiO<sub>2</sub>-water and [19-20] for Al<sub>2</sub>O<sub>3</sub>-water based on the technique of dimensional analysis. By assuming the  $k_{eff}$ , the effective conductivity of nanofluid is a function of the thermal conductivities of the base fluid ( $k_f$ ), the solid particle ( $k_p$ ), the interfacial shell ( $k_i$ ), the particle diameter ( $d_p$ ), the volume fraction of the particle ( $\phi$ ), the interfacial shell thickness ( $t$ ), the temperature of nanofluid ( $T$ ), and half of the base fluid boiling temperature ( $T_c$ ), it can be written as:

$$k_{eff} = f(k_f, k_p, k_i, d_p, \phi, T, T_c) \quad (1)$$

By using the theorem of dimensional analysis, the Eq.(1) is expressed in a modified form in terms of dimensionless variables as follow:

$$\frac{k_{eff}}{k_f} = f_1\left(\frac{k_i}{k_p}, \frac{t}{d_p}, \phi, \frac{T}{T_c}\right) \quad (2)$$

Usually For ananofluid,  $k_{eff} > k_f$  which yields:

$$\left(\frac{k_{eff}}{k_f}\right) > 1 \Rightarrow \left(\frac{k_{eff}}{k_f}\right) = 1 + R + R' + R'' + \dots \quad (3)$$

Where R, R', and R'', are the enhancement factors and expressed as follows:

$$R = \left[\left(\frac{k_i}{k_p}\right)^a \left(\frac{t}{d_p}\right)^b \phi^c \left(\frac{T}{T_c}\right)^d\right] \quad (4)$$

Combining Eqs. (8) and (9), it yields a general form for the relative effective thermal conductivity as follows:

$$\frac{k_{eff}}{k_f} = 1 + \left[\left(\frac{k_i}{k_p}\right)^a \left(\frac{t}{d_p}\right)^b \phi^c \left(\frac{T}{T_c}\right)^d\right] + R' + R'' + \dots \quad (5)$$

For a particular nanofluid system, where  $k_p$ ,  $k_i$  and  $t$  are constant, Eq. (9) simplifies into:

$$\frac{k_{eff}}{k_f} = 1 + m \frac{\phi^\alpha}{(d_p)^\beta} \left(\frac{T}{T_c}\right)^\delta + R' + R'' + \dots \quad (6)$$

Where m is a factor that depends on the properties of the solid particle, base fluid and interfacial shell, while  $\alpha$  and  $\beta$  are empirical constants determined from experimental data, and R', R'' are the parameters which depends on the nanofluid properties and experiment conditions and thus can be estimated. A careful examination of the experimental data of [5,14,17] yields a correlation for R' as follows:

$$R' = \phi \left(1 - \frac{d_c}{d_p}\right) \left(\frac{T}{T_c}\right) \quad (7)$$

$$R'' = R''' = \dots = 0 \quad (8)$$

Then the Eq. (6) reduce to:

$$\frac{k_{eff}}{k_f} = 1 + m \frac{\phi^\alpha}{(d_p)^\beta} T^\delta + n \phi \left(1 - \frac{d_c}{d_p}\right) T \quad \text{for } d_p \geq d_c \quad (9)$$

Where n is a factor that depends on the properties of the base fluid.

## III. RESULTS AND DISCUSSIONS

The model for prediction of effective thermal conductivity of nanofluid which discussed in previous section, applied and tested in TiO<sub>2</sub>-water and Al<sub>2</sub>O<sub>3</sub> - water systems. The model parameters have been defined for these systems and results are summarized in Table I. The model validation and its accuracy were performed by enormous experimental data published in literature and outperform previously derived models [5, 14, 17]. The comparative results of the model predictions are shown in Fig.1-a to Fig.1-c for TiO<sub>2</sub>-water system and Fig.2-a to Fig.2-c for Al<sub>2</sub>O<sub>3</sub>-water system, respectively. Careful examinations of Figs.1-a to Fig.1-c it reveals that the model is in good agreement with experimental data and outperforms previously derived models when applied to examined nanofluids.

TABLE I  
MODEL PARAMETERS FOR TiO<sub>2</sub>-WATER AND Al<sub>2</sub>O<sub>3</sub>- WATER SYSTEMS

| Parameter | Quantity                                    |   |
|-----------|---|---|
|           | TiO <sub>2</sub> -water system <sup>a</sup> | Al <sub>2</sub> O <sub>3</sub> -water system <sup>b</sup> |
| m         | 515   | 10  |
| $\alpha$  | 0.912                                       | 0.350   |
| $\beta$   | 1.9   | 2.6   |
| n         | 4   | 0.0121  |
| $d_c$     | 20  | 21  |
| $\delta$  | 0   | 1   |

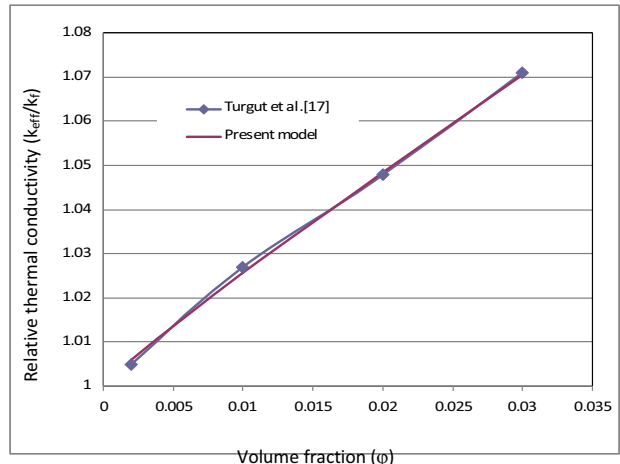
<sup>a</sup>Based on data of [5, 14, 17] and <sup>b</sup>Based on data of [19-20]

Furthermore, the proposed model shows a non-linear relation between the effective thermal conductivity and nanoparticle volume fraction, while the previous models deviate from experimental data of nanofluids, this may be arises from the fact that these models haven't consider the critical diameter which it shows a specific size, that there is a braid to thermal conductivity of nanofluid after this size and the present model show this realization. This fact is due to the nanoparticle motion mechanism of dilute suspensions such as Brownian motion and effective of liquid layering around nanoparticles. Because of decreasing the nano-particle size, the effective Brownian motion and effective surface increasing; that if ( $d_p < d_c$ ) so the effect of this fact in the heat transfer of nanofluid is much more as shown by Murshed et al. [18]. The other fact is the clustering of nanoparticles, which depends on the particle volume fraction. As clarified from Figs.1-a and Fig.1-b, the gradient of thermal conductivity enhancement for the case of ( $\phi < 1\%$ ) is much more than for ( $\phi > 1\%$ ) and therefore for the former case the effective of clustering of nanoparticles decreases.

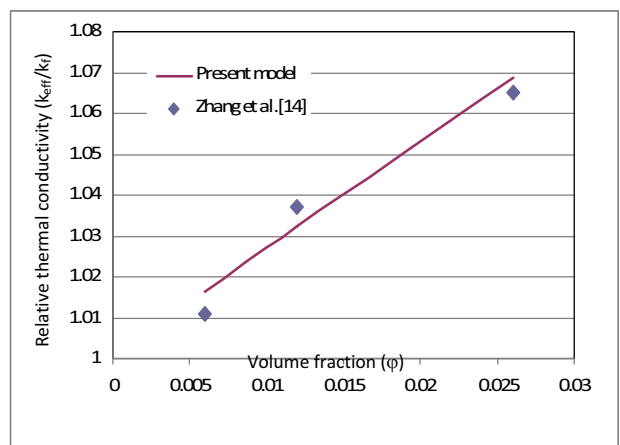
IV. CONCLUSIONS

In this paper, we developed an empirical model for prediction of the effective thermal conductivity of nanofluids, based on the dimensionless variables of ( $k_p/k_f$ ),  $\phi$ , ( $t/d_p$ ), and ( $T/T_c$ ). The proposed model accounts for the interfacial shell, brownian motion, and aggregation of particle. The validation of the model is verified by applying the results obtained by the experiments of  $TiO_2$ -water and  $Al_2O_3$ -water systems. The model parameters have been calculated by applying it to data of both systems obtained by experiments and published in literature.

Comparing the model predictions with the conventional linear models, and also experimental data it shows an improvement in thermal conductivity of nanofluid which show the effect of Brownian motion, nano-layer (interfacial shell), and clustering of nanoparticles. The model could be applied for range of  $d_p < d_c$ .

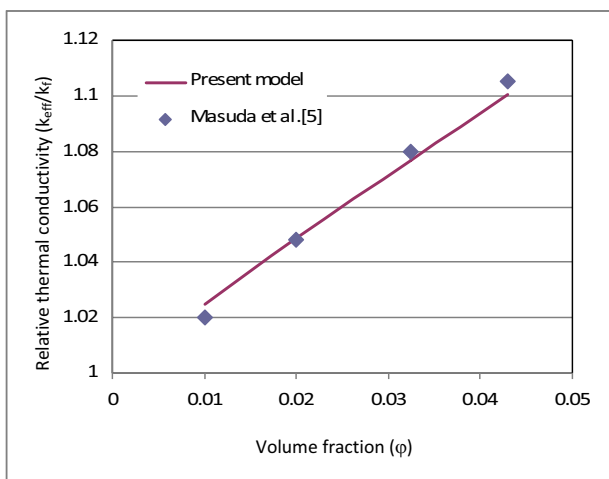


b)  $TiO_2$ -water system with particle size of  $d_p = 21$  nm

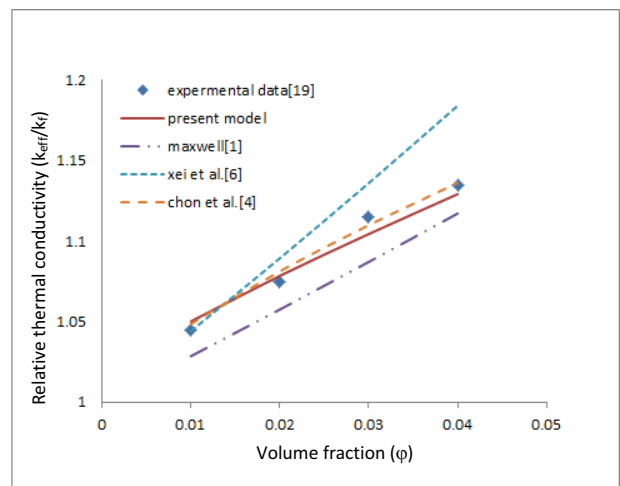


c)  $TiO_2$ -water system with particle size of  $d_p = 40$  nm

Fig. 1 Variation of relative thermal conductivity ( $k_{eff}/k_f$ ) with volume fraction ( $\phi$ ) for  $TiO_2$  - water system and comparison of model prediction with experimental results for various particle size



a)  $TiO_2$ -water system with particle size of  $d_p = 27$  nm



a)  $Al_2O_3$ -water system with particle size of  $d_p = 45$  nm

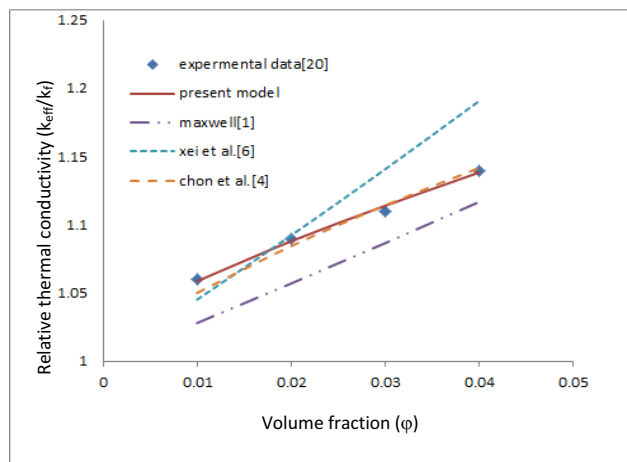
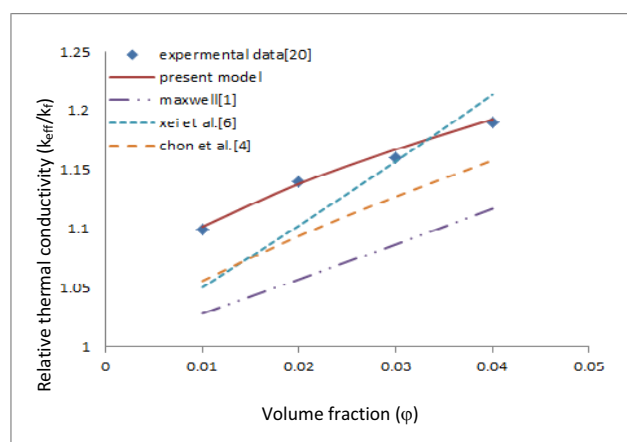
b) Al<sub>2</sub>O<sub>3</sub>-water system with particle size of  $d_p = 40$  nmc) Al<sub>2</sub>O<sub>3</sub>-water system with particle size of  $d_p = 30$  nm

Fig. 2 Variation of relative thermal conductivity ( $k_{eff}/k_f$ ) with volume fraction ( $\phi$ ) for Al<sub>2</sub>O<sub>3</sub>-water system and comparison of model prediction with experimental results for various particle size

## REFERENCES

- [1] J. C. Maxwell, "A Treatise on Electricity and Magnetism, 2nd Ed., Oxford Univ. Press, Cambridge, UK 1904, p. 435.
- [2] B. Wang, L. Zhou, and X. Peng, "A Fractal Model for Predicting the Effective Thermal Conductivity of Liquid with Suspension of Nanoparticles," *Int. J. Heat Mass Tran.*, 46(14), 2003, pp. 2665-2672.
- [3] P. Keblinski, S. R. Phillpot, S. U. S. Choi, and J. A. Eastman, "Mechanisms of Heat Flow in Suspensions of Nano-Sized Particles (Nanofluids)," *Int. J. Heat Mass Tran.*, 45(4), 2002, pp. 855-863.
- [4] C.H. Chon, K.D. Kihm, S.P. Lee, and S.U.S. Choi, "Empirical Correlation Finding the Role of Temperature and Particle Size for Nanofluid (Al<sub>2</sub>O<sub>3</sub>) Thermal Conductivity Enhancement", *Appl. Phys.*, 2005.
- [5] H. Masuda, A. Ebata, K. Teramae, and N. Hishinuma, *NetsuBussei (Japan)* 7, 227, 1993.
- [6] H. Xie, M. Fujii, and X. Zhang, "Effect of Interfacial Nanolayer on the Effective Thermal Conductivity of Nanoparticle-Fluid Mixture", *Int. J. Heat Mass Tran.*, 48(14), 2005, pp. 2926-2932.
- [7] W. Yu, and S. U. S. Choi, "The Role of Interfacial Layers in the Enhanced Thermal Conductivity of Nanofluids: A Renovated Maxwell Model", *J. Nanopart. Res.*, 5(1), 2003 pp. 167-171.
- [8] B. Wang, L. Zhou, and X. Peng, "A Fractal Model for Predicting the Effective Thermal Conductivity of Liquid with Suspension of Nanoparticles", *Int. J. Heat Mass Tran.*, 46(14), 2003, pp. 2665-2672.
- [9] J.Koo and C. Kleinstreuer, "A New Thermal Conductivity Model for Nanofluids", *J. Nanopart. Res.*, 6(6), 2004, pp. 577-588.
- [10] J. Xu, B. Yu, M. Zou, and P. Xu, "A New Model for Heat Conduction of Nanofluids Based on Fractal Distributions of Nanoparticles", *J. Phys. D: Appl. Phys.*, 39(20), 2006, pp. 4486-4490.
- [11] Y. Xuan, Q. Li, and W. Hu, "Aggregation Structure and Thermal Conductivity of Nanofluids" *AIChE Journal*, 49(4), 2003, pp. 1038-1043.
- [12] b-X. Wang, W.-Y. Sheng, and X.-F. Peng, "A Novel Statistical Clustering Model for Predicting Thermal Conductivity of Nanofluid", *Int. Thermophys.* 30, 2009, pp. 1992-1998.
- [13] H. Hezaveh1, M. Keshavarz Moraveji, "Modeling Effective Thermal Conductivity of Al<sub>2</sub>O<sub>3</sub> Nanoparticles in Water and Ethylene Glycol Based on Shape Factor", *Int. J. of Chem. Eng. and Appl.*, Vol. 2, No. 1, 2011, ISSN: 2010-0221
- [14] X. Zhang, H. Gu, and M. Fujii, *Int. J. Thermophys.* 27, 2006, p 569.
- [15] X. Wang, X. Xu, and S. U. S. Choi, "Thermal Conductivity of Nanoparticle-Fluid Mixture" *J. Thermophys. Heat Transfer*, 13(4), 1999, pp. 474-480
- [16] Y. Xuan and Q. Li, "Heat Transfer Enhancement of Nanofluids", *Int. J. Heat Fluid Flow*, 1(21), 2000, pp. 58-64.
- [17] A. Turgut, I. Tavman, M. Chirtoc, H.P. Schuchmann, C. Sauter, and S. Tavman, "Thermal Conductivity and Viscosity Measurements of Water-Based TiO<sub>2</sub> Nanofluids"
- [18] S.M.S. Murshed, K.C. Leong, and C. Yang, *Int. J. Therm. Sci.* 44., 2005, p 367.
- [19] D.-W. Oh, A. Jain, J.K.E. Goodson, and J.S. Lee, "Thermal Conductivity Measurement and Sedimentation detection of Al<sub>2</sub>O<sub>3</sub> Nanofluids by Using the 3W Method", *Int. J. Heat Fluid Flow*, 29, 2008, p 1456.
- [20] B.S.A. Shin, "Minimum Quantity Lubrication (MQL) Grinding Using Nanofluid", The University of Michigan, [http://wumrf.engin.umich.edu/resdarch/file/advmach\\_files/mqlm](http://wumrf.engin.umich.edu/resdarch/file/advmach_files/mqlm)
- [21] A. R. Moghadassi, S.M. Hosseini, D. Henneke, and A. Elkamel, "A Model of Nanofluids Effective Thermal Conductivity Based on Dimensionless Groups", *J. of Thermal Analysis and Calorimetry*, Vol. 96, 2009, pp 81-84.