

An Exact Solution of Axi-symmetric Conductive Heat Transfer in Cylindrical Composite Laminate under the General Boundary Condition

M.kayhani, M.,Nourouzi , and A. Amiri Delooei

Abstract—This study presents an exact general solution for steady-state conductive heat transfer in cylindrical composite laminates. Appropriate Fourier transformation has been obtained using Sturm-Liouville theorem. Series coefficients are achieved by solving a set of equations that related to thermal boundary conditions at inner and outer of the cylinder, also related to temperature continuity and heat flux continuity between each layer. The solution of this set of equations are obtained using Thomas algorithm. In this paper, the effect of fibers' angle on temperature distribution of composite laminate is investigated under general boundary conditions. Here, we show that the temperature distribution for any composite laminates is between temperature distribution for laminates with $\theta = 0^\circ$ and $\theta = 90^\circ$.

Keywords—exact solution, composite laminate, heat conduction, cylinder, Fourier transformation.

I. INTRODUCTION

TODAY, usage of composite materials in aeronautic industries, submarines, automotive engineering, sport equipments and etc has been noticeably progressed. This remarkable usage of these kinds of materials is because of its high strength and having high module with low density. Therefore, in many applications, use of these materials is commodious compare to isotropic materials and these materials are preferable. So far, a lot of researches have been carried out about mechanical and thermo mechanical behavior of composite laminates while very few works are available about heat transfer of these materials [1]-[3]. Primary research in this field has been carried out on anisotropic crystals [4],[5]. Ma and Chang [6] studied analytical heat conduction in anisotropic multilayer media. They changed anisotropic problem to a simple isotropic problem by using a linear coordinate transformation. There are some accomplished researches about heat transfer in composite materials that are reviewed briefly. Kulkarani and Brady [7] presented a thermal mathematical model for heat transfer in laminated carbon-carbon composites. This model was based on volumetric

percentage of matrix and fibers and using of this model also heat transfer coefficient indirection of fibers and perpendicular to fibers has been estimated. Johansson and Lesnic [8] showed applications of MFS methods for transient heat conduction in layered materials and developed this method for numerical estimation of heat flux in these materials. Sun and Wichman [9] presented a theoretical solution for transient heat transfer in a one-dimensional three-layer composite slab and compared obtained resultants with finite element solution. Karageorghis and Lesnic [10] introduced a solution for heat conduction in laminated composite material that its conduction coefficient was dependence to temperature and boundary condition consisted of convection and radiation. Haji-sheikh *et al.* [11] obtained a mathematical formulation for steady-state heat conduction and temperature distribution in multi-layer bodies. They affirmed that if layers are homogenous, eigenvalues will be real numbers but for orthotropic state these values can be imaginary numbers. Guo *et al.* [12] studied temperature distribution in thick polymeric matrix laminates and compared it with results of numerical solution. They solved transient heat transfer in polymeric matrix composite laminates using finite element method. They considered the internal energy generation due to chemical reactions in the heat transfer equation. Singh *et al.* [13] obtained an analytical solution for conductive heat transfer in multilayer polar coordinate system in radial direction. Bahadur and Bar-Cohen [14] presented analytical solution for temperature distribution and heat flux in a cylindrical fin with orthotropic conductive coefficient and compared its results with obtained results from finite element solution. Onyejekwe [15] obtained an exact analytical solution for conductive heat transfer in composite media using boundary integral theory.

Tarn and Wang [16],[17] studied conductive heat transfer in cylinders that are made of functional graded material (FGM) and composite laminates. Furthermore, many studies about conductive heat transfer have been carried out in nano-composites [18],[19]. One of the applications of composite materials is in manufacturing super conductive materials. Cha *et al.* [20] investigated inverse temperature distribution and heat generation in super conductor composite materials.

Kayhani *et al.* [21] studied analytically the conductive heat transfer of cylindrical composite laminates in radial and angular directions (r, ϕ) . This solution is only valid for

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composite pipes and vessels with large ratio of longitudinal to radial dimension which is related to special case of very long pipes and vessels.

In this paper, an exact solution for conductive heat transfer in cylindrical composite laminates is presented. This analytical solution can be used to analyze the conductive heat transfer, thermal stresses and strains in composite pipes and vessels. Fig. 1 shows the geometry of the composite laminate in current research. According to the figure, the fibers are wound around the cylinder and the direction of fibers in each lamina can be differed from another layers. Unlike the work of Kayhani *et al.* [21], we focus on axi-symmetric heat transfer in cylindrical composite laminates by considering the heat conduction in longitudinal and radial dimensions (r, z).

This analytical solution is also obtained for general linear thermal boundary condition which covers combined effect of the heat conduction, convection and radiation at boundaries. Finding the most generalized analytical solution based on the complicated boundary conditions is one of the main innovation of current work. For this purpose, an appropriate Fourier transformation has been derived using the Sturm-Liouville theorem. We used this Fourier transformation to change the partial differential equation of heat transfer in cylindrical composite laminates to an ordinary equation. Due to the difference of the fibers direction in each layer, a set of equations of Fourier series coefficients is obtained based on the boundary conditions at inner and outer of the cylindrical laminate and temperature and heat flux continuity at boundaries located between the layers. The solution of these equations is obtained using the recessive Thomas algorithm.

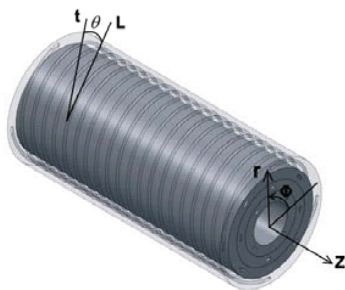


Fig.1 Direction of fibers in a cylindrical laminate

II. MODELING AND GOVERNING EQUATIONS

In this research, the fibers in each layer have been winded in specific directions around the cylinder and steady conductive heat transfer in a cylindrical composite laminate has been studied. Fig. 1 shows a cylindrical laminate according to describes condition. In this figure, r, φ and z are elements of off-axis coordinate system (reference coordinate system). If L is tangent line on cylinder in direction of fibers and t is tangent line on cylinder in

direction of φ , hence; angle of fibers (θ) is angle between L and t .

The Fourier relation in cylindrical coordinate system for orthotropic material is as follows[21]:

$$\begin{Bmatrix} q_r \\ q_\varphi \\ q_z \end{Bmatrix} = - \begin{bmatrix} \bar{k}_{11} & \bar{k}_{12} & \bar{k}_{13} \\ \bar{k}_{21} & \bar{k}_{22} & \bar{k}_{23} \\ \bar{k}_{31} & \bar{k}_{32} & \bar{k}_{33} \end{bmatrix} \begin{Bmatrix} \frac{\partial T}{\partial r} \\ \frac{1}{r} \frac{\partial T}{\partial \varphi} \\ \frac{\partial T}{\partial z} \end{Bmatrix}. \quad (12)$$

By using the balance of energy for a cylindrical element, conductive heat transfer equation is obtained as follows relation:

$$\begin{aligned} & \bar{k}_{11} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \bar{k}_{22} \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} + \bar{k}_{33} \frac{\partial^2 T}{\partial z^2} + (\bar{k}_{12} + \bar{k}_{21}) \frac{1}{r} \frac{\partial^2 T}{\partial \varphi \partial r} \\ & + (\bar{k}_{13} + \bar{k}_{31}) \frac{\partial^2 T}{\partial r \partial z} + \frac{\bar{k}_{13}}{r} \frac{\partial T}{\partial z} + (\bar{k}_{23} + \bar{k}_{32}) \frac{1}{r} \frac{\partial^2 T}{\partial \varphi \partial z} = \rho c \frac{\partial T}{\partial t}. \end{aligned} \quad (13)$$

Where conductive heat coefficient (\bar{k}) are defined as below [21].

$$\begin{aligned} \bar{k}_{11} &= k_{22} \\ \bar{k}_{22} &= m_l^2 k_{11} + n_l^2 k_{22} \\ \bar{k}_{33} &= n_l^2 k_{11} + m_l^2 k_{22} \\ \bar{k}_{12} &= \bar{k}_{21} = 0 \\ \bar{k}_{13} &= \bar{k}_{31} = 0 \\ \bar{k}_{23} &= \bar{k}_{32} = m_l n_l (k_{11} - k_{22}) \end{aligned} \quad (14)$$

With substituting (14) in (13), the heat transfer equation in cylindrical composite laminate is obtained:

$$\begin{aligned} & k_{22} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + (m_l^2 k_{11} + n_l^2 k_{22}) \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} \\ & + (n_l^2 k_{11} + m_l^2 k_{22}) \frac{\partial^2 T}{\partial z^2} + 2m_l n_l (k_{11} - k_{22}) \frac{1}{r} \frac{\partial^2 T}{\partial \varphi \partial z} = \rho c \frac{\partial T}{\partial t}. \end{aligned} \quad (15)$$

In this research, steady conductive heat transfer in direction of r and z is studied, so; relation (15) can be simplified as below:

$$k_{22} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + (n_l^2 k_{11} + m_l^2 k_{22}) \frac{\partial^2 T}{\partial z^2} = 0. \quad (16)$$

Fig.2 shows layers in a cylindrical laminate. In this figure, if $r = r_i$ is boundary between two layers i and $i+1$, then regarding to temperature continuity and heat flux continuity two below relations are obtained:

$$T^{(i)} = T^{(i+1)}. \quad (17a)$$

$$-k_{22} \frac{\partial T^{(i)}}{\partial r} = -k_{22} \frac{\partial T^{(i+1)}}{\partial r}. \quad (17b)$$

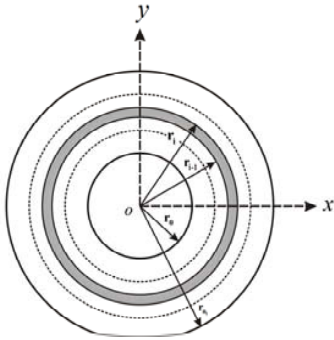


Fig.2 Arrangement of layers in a cylindrical laminate

III. ANALYTICAL SOLUTION OF HEAT CONDUCTION FOR GENERAL BOUNDARY CONDITIONS

In this section, analytical solution for (16) is obtained using Fourier transformation. Relation (16) can be rewritten as follows:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{\mu^2} \frac{\partial^2 T}{\partial z^2} = 0. \quad (18)$$

The general thermal boundary conditions are:

$$a_1 T(r, 0) + b_1 \frac{\partial T(r, 0)}{\partial z} = f_1(r). \quad (19a)$$

$$a_2 T(r, L) + b_2 \frac{\partial T(r, L)}{\partial z} = f_2(r). \quad (19b)$$

$$c_1 T(r_{in}, z) + d_1 \frac{\partial T(r_{in}, z)}{\partial r} = g_1(z). \quad (19c)$$

$$c_2 T(r_{out}, z) + d_2 \frac{\partial T(r_{out}, z)}{\partial r} = g_2(z). \quad (19d)$$

Where $f_1(r)$, $f_2(r)$, $g_1(z)$, $g_2(z)$ are arbitrary functions.

In these relations, μ is given by below relation:

$$\mu = \sqrt{\frac{k_{22}}{n_l^2 k_{11} + m_l^2 k_{22}}}. \quad (20)$$

Regarding to general boundary conditions, it is necessary to use Sturm-Liouville theorem to find suitable Fourier transformation of arbitrary function $f(z)$: [27]

$$F(f) = \frac{\int_a^b s(z) f(z) \varphi_n(z) dz}{\int_a^b s(z) \varphi_n^2(z) dz} \quad (21)$$

Where $s(z)$ is weighting function and $\varphi_n(z)$ is eigenfunction that achieved from homogenous boundary conditions for z direction. Inverse Fourier transformation defines as below:

$$f(z) = \sum_{n=0}^{\infty} F(f) \varphi_n(z) \quad (22)$$

Using the separation of variables method for solving (18) and by considering homogenous boundary conditions, following equation for z direction has been achieved:

$$\frac{\partial^2 Z(z)}{\partial z^2} + \lambda^2 Z(z) = 0. \quad (23)$$

$$a_1 Z(0) + b_1 \frac{\partial Z(0)}{\partial z} = 0. \quad (24a)$$

$$a_2 Z(L) + b_2 \frac{\partial Z(L)}{\partial z} = 0. \quad (24b)$$

By solving (23) respect to boundary conditions (24), the eigenfunction for this problem is achieved:

$$\phi_n = (a_1 \sin(\lambda_n z) - b_1 \lambda_n \cos(\lambda_n z)). \quad (25)$$

where λ_n is derived as follow:

$$(a_1 b_2 - a_2 b_1) \lambda_n \cos(\lambda_n L) + (a_2 a_1 + b_1 b_2 \lambda_n^2) \sin(\lambda_n L) = 0 \quad (26)$$

Regarding to the Sturm-Liouville theorem and (23), the weighting function is constant. By substituting these relations to Sturm-Liouville relation (21), suitable Fourier transformation for this problem is obtained (F):

$$F(f) = \frac{4\lambda_n}{A_n^2} \int_0^L f(z) \left(a_1 \sin(\lambda_n z) - b_1 \lambda_n \cos(\lambda_n z) \right) dz. \quad (27)$$

$$\text{where, } \frac{\left((-a_1^2 + (b_1 \lambda_n)^2) \sin(2\lambda_n L) + 2a_1 b_1 \lambda_n \cos(2\lambda_n L) \right)}{\sqrt{+2\lambda_n (a_1^2 + (b_1 \lambda_n)^2) L - 2a_1 b_1 \lambda_n}} \quad (28)$$

According to definition of Fourier transformation second order derivations compare to z is :

$$F(f) = \left\{ \left(\frac{2\lambda_n}{A_n} \right)^2 \left[\frac{-a_1}{(a_2 \cos(\lambda_n L) - b_2 \lambda_n \sin(\lambda_n L))} f_2(r) \right] + f_1(r) \right\} \quad (29)$$

If this Fourier transformation applies on (18) and boundary conditions in r direction (19c, 19d), then below relations will be obtained:

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} - \left(\frac{\lambda_n^2}{\mu} \right) U = \left(\frac{2\lambda_n^2}{A_n^2 \mu} \right) \times \left[\frac{a_1}{(a_2 \cos(\lambda_n L) - b_2 \lambda_n \sin(\lambda_n L))} f_2(r) - f_1(r) \right] \quad (30)$$

$$c_1 U(r_0, n) + d_1 \frac{\partial U(r_0, n)}{\partial r} = G_1(n). \quad (31a)$$

$$c_2 U(r_{n_i}, n) + d_2 \frac{\partial U(r_{n_i}, n)}{\partial r} = G_2(n). \quad (31b)$$

In these above relations:

$$U(r, n) = F(T(r, z)). \quad (32a)$$

$$G_1(n) = F(g_1(z)). \quad (32b)$$

$$G_2(n) = F(g_2(z)). \quad (32c)$$

If the right hand side of (30) is equal to $h(r, n)$, general solution of this equation in each composite laminate is as follow:

$$U^{(i)}(r, n) = a_n^{(i)} I_0 \left(\frac{\lambda_n}{\mu_i} r \right) + b_n^{(i)} K_0 \left(\frac{\lambda_n}{\mu_i} r \right) + w^{(i)}(r, n). \quad (33)$$

In relation (33), $w(r, n)$ is non-homogenous answer for (30) and in general form is as below:

$$w^{(i)}(r, n) = I_0 \left(\frac{\lambda_n}{\mu_i} r \right) \times \int_{r_{in}}^{r_{out}} \left[r \times K_0 \left(\frac{\lambda_n}{\mu_i} r \right) \times h(r, n) \right] dr + K_0 \left(\frac{\lambda_n}{\mu_i} r \right) \times \int_{r_{in}}^{r_{out}} \left[r \times I_0 \left(\frac{\lambda_n}{\mu_i} r \right) \times h(r, n) \right] dr. \quad (34)$$

In these relations, I_0 and K_0 are modified Bessel function of the first kind and the second of order zero respectively. Finally, by acting inner and outer boundary conditions on direction of r and regarding to temperature continuity and heat flux continuity in boundary between layers, coefficients of a_n and b_n are obtained. So, according to relation (31a):

$$a_n^{(1)} \left[c_1 I_0 \left(\frac{\lambda_n}{\mu_1} r_0 \right) + d_1 \frac{\lambda_n}{\mu_1} I_1 \left(\frac{\lambda_n}{\mu_1} r_0 \right) \right] + b_n^{(1)} \left[c_1 K_0 \left(\frac{\lambda_n}{\mu_1} r_0 \right) - d_1 \frac{\lambda_n}{\mu_1} K_1 \left(\frac{\lambda_n}{\mu_1} r_0 \right) \right] = G_1(n) - w^{(1)}(r_0, n). \quad (35a)$$

Similarly, for boundary conditions of (31b):

$$a_n^{(n_i)} \left[c_2 I_0 \left(\frac{\lambda_n}{\mu_{n_i}} r_{n_i} \right) + d_2 \frac{\lambda_n}{\mu_{n_i}} I_1 \left(\frac{\lambda_n}{\mu_{n_i}} r_{n_i} \right) \right] + b_n^{(n_i)} \left[c_2 K_0 \left(\frac{\lambda_n}{\mu_{n_i}} r_{n_i} \right) - d_2 \frac{\lambda_n}{\mu_{n_i}} K_1 \left(\frac{\lambda_n}{\mu_{n_i}} r_{n_i} \right) \right] = G_1(n) - w^{(n_i)}(r_{n_i}, n). \quad (35b)$$

Here, boundary conditions of relations (17a) and (17b) act on common part of two adjacent layers, according to relation (17a) for temperature continuity; below relation is obtained:

$$U^{(i)}(r_0, n) = U^{(i+1)}(r_0, n) \Rightarrow a_n^{(i)} I_0 \left(\frac{\lambda_n}{\mu_i} r_i \right) + b_n^{(i)} K_0 \left(\frac{\lambda_n}{\mu_i} r_i \right) - a_n^{(i+1)} I_0 \left(\frac{\lambda_n}{\mu_{i+1}} r_i \right) + b_n^{(i+1)} K_0 \left(\frac{\lambda_n}{\mu_{i+1}} r_i \right) = w^{(i+1)}(r_i, n) - w^{(i)}(r_i, n). \quad (35c)$$

Regarding to relation (17b), for heat flux continuity below relation is obtained:

$$a_n^{(i)} \frac{\lambda_n}{\mu_i} I_1 \left(\frac{\lambda_n}{\mu_i} r_i \right) + b_n^{(i)} \frac{\lambda_n}{\mu_i} K_1 \left(\frac{\lambda_n}{\mu_i} r_i \right) - a_n^{(i+1)} \frac{\lambda_n}{\mu_{i+1}} I_1 \left(\frac{\lambda_n}{\mu_{i+1}} r_i \right) + b_n^{(i+1)} \frac{\lambda_n}{\mu_{i+1}} K_1 \left(\frac{\lambda_n}{\mu_{i+1}} r_i \right) = \frac{\partial w^{(i+1)}(r_i, n)}{\partial r} - \frac{\partial w^{(i)}(r_i, n)}{\partial r}. \quad (35d)$$

where I_1 and K_1 are modified Bessel function of the first kind and the second of order one respectively. Here, a set of equations consisting of (35a), (35b), (35c) and (35d) is solved to determine coefficients of $a_n^{(i)}$ and $b_n^{(i)}$. Fortunately This set of equations is five diagonal and can be solved by using Thomas algorithm method. Finally the temperature distribution in each layer is determined by applying the inverse transformation (22) to the (33);

$$T^{(i)}(r, z) = \sum_{n=1}^{\infty} \left(U^{(i)}(r, n) \times \phi_n(z) \right) = \sum_{n=1}^{\infty} \left(a_n^{(i)} I_0 \left(\frac{\lambda_n}{\mu_i} r \right) + b_n^{(i)} K_0 \left(\frac{\lambda_n}{\mu_i} r \right) + w^{(i)}(r, n) \right) \times (a_1 \sin(\lambda_n z) - b_1 \lambda_n \cos(\lambda_n z)). \quad (36)$$

IV. RESULTS AND DISCUSSION

In this section, with an example that consists of all obtained coefficients; two-dimensional analytical conductive heat transfer in a cylindrical composite material is studied. The effect of derivation of fiber's angle on temperature distribution in one-layer and multi-layer laminates with various arrangements of layers has been studied. To show effects of variation of specifications in direction of fibers and perpendicular direction of fibers, graphite-epoxy has been used as composite laminate material. Conductive coefficient of graphite-epoxy in direction of fibers is 12.76 times larger than perpendicular direction of fibers, because Graphite is a conductive material and epoxy is heat insulator. Table I shows physical and thermal properties of fibers and matrix. Table II

shows properties of composite laminate that was made of epoxy and graphite:

TABLE I
PROPERTIES OF GRAPHITE FIBERS AND EPOXY MATRIX, [28]

| | |
|--|----------|
| Matrix material | Epoxy |
| Fibers material | Graphite |
| Conductive coefficient of matrix (W/m k) | 0.19 |
| Conductive coefficient of fibers (W/m k) | 14.74 |
| Heat capacity of matrix (J/kg k) | 1613 |
| Heat capacity of fibers (J/kg k) | 709 |

TABLE II
PROPERTIES OF GRAPHITE/EPOXY COMPOSITE MATERIAL, [28]

| | |
|--|------|
| k in parallel direction of fibers (W/m k) | 11.1 |
| K in perpendicular direction of fibers (W/m k) | 0.87 |
| Volumetric percentage of fibers | 75 |
| Melting point (k) | 450 |
| Heat capacity (J/kg k) | 935 |
| Density (kg/m^3) | 1400 |

TABLE III
GEOMETRY AND BOUNDARY CONDITIONS

| | |
|--|-----|
| Inner diameters of cylinder (m) | 0.5 |
| Outer diameter of cylinder (m) | 1 |
| Length (m) | 1 |
| Inner heat flux (W/m^2) | 400 |
| Linear heat convective coefficient ($\text{W/m}^2 \text{ k}$) | 150 |
| Lateral heat convective coefficient ($\text{W/m}^2 \text{ k}$) | 100 |
| Inner heat convective coefficient ($\text{W/m}^2 \text{ k}$) | 50 |
| Ambient temperature (k) | 300 |
| Inner temperature of cylinder (k) | 320 |
| Angle of fibers (Degree) | 90 |

To study effect of angle variation on temperature distribution, a one-layer composite laminate (or multi-layer laminate that its fibers' angle is equal) has been used. Table III presents geometry and boundary conditions of this laminate.

Fig.3 shows maximum temperature variations for numbers of Fourier series term in (34) (for a lamina with 90° fiber's angle). According to this figure, when the numbers of sentences of series are less than 200; this series will be converged speedily and variations are less than 0.01 that is a good approximation for engineering calculations. Consequently we calculating until 200th terms of Fourier series.

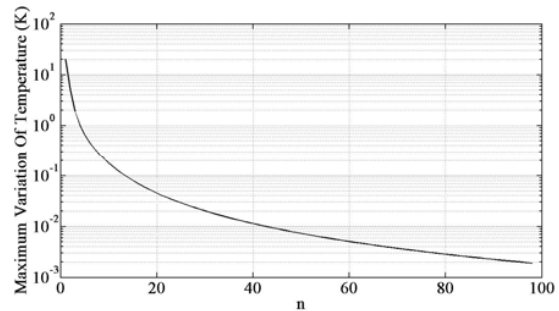
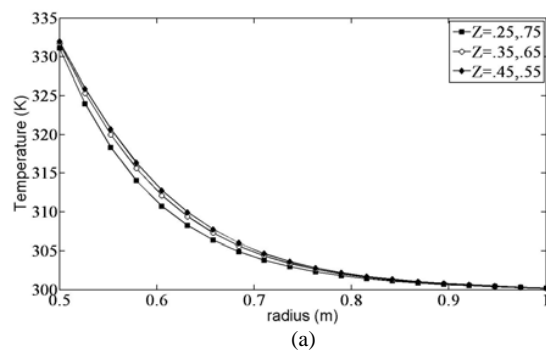


Fig.3 Maximum Temperature variations in terms of different Fourier series terms in a single layer laminate ($\theta = 90^\circ$)

Fig.4 shows the amount of temperature distribution in different z cross section of a one-layer laminate when fibers' angle is 0° and 90° , this figures present for two different amount of heat fluxes (Q). According to symmetry of boundary conditions in direction of z , temperature distribution is symmetric respect to midpoint of length in z direction. When the angle of fibers' is 0° the fibers are situated in ϕ direction and consequently heat transfer is comparable with a isotropic cylinder with conductive coefficient k_{22} , in other words $k_{rr} = k_{zz} = k_{22}$. As it seems from fig.4, for this specific boundary conditions that are considered, maximum temperature occurs in the inner wall of cylinder and when fibers' angle is 90° , temperature distribution is higher than the condition that fibers' angle is equal to zero. Also it is obvious that when fibers' angle is 90° , the pattern of temperature distribution is more monotonous than the state that fibers' angle is 0° , although the maximum and minimum of temperature in the first state is higher. So related to your design factor you must selected the fibers' angle. For example if you want to decrease the maximum temperature of composite cylinder you must situated the fibers in ϕ direction that fibers' angle is 0° . In Fig.5, variation of μ compare to fibers' angle is shown. The coefficient μ defined as (20).the figure is symmetric toward angle 90° , its value is maximum in this angle. it also has a



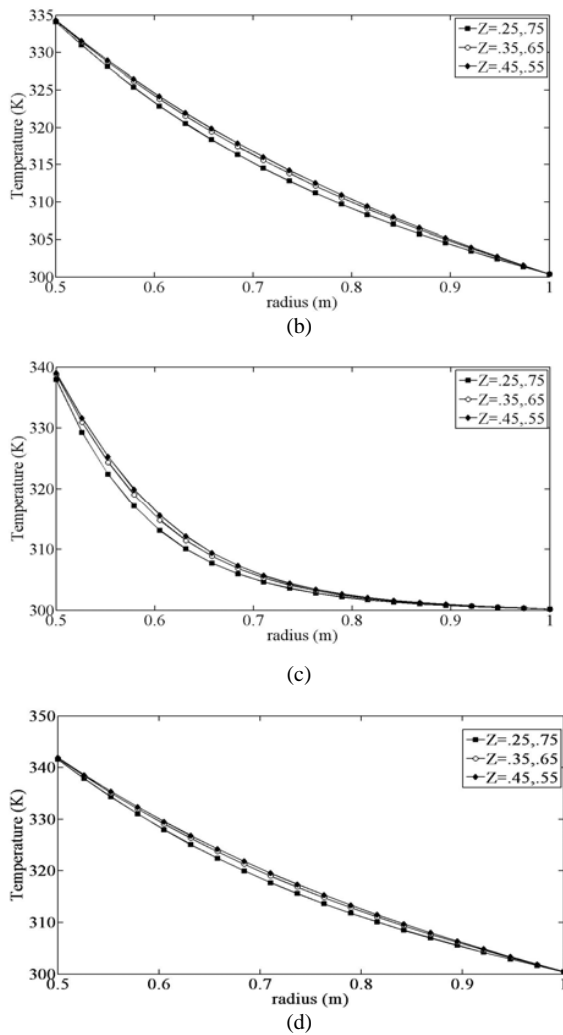


Fig.4 Temperature distribution in a single layer laminate under different heat fluxes and in different Fibers' angle, (a) $\theta = 0^\circ$ & $Q = 800(w/m^2)$, (b) $\theta = 90^\circ$ & $Q = 800(w/m^2)$, (c) $\theta = 0^\circ$ & $Q = 1200(w/m^2)$, (d) $\theta = 90^\circ$ & $Q = 1200(w/m^2)$

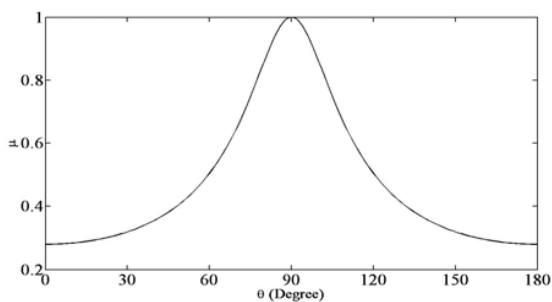


Fig.5 Diagram of coefficient μ in terms of fibers' angle (θ)

period equal to 180° . Considering (18) we can find that increasing in amount of μ causes decreasing effect of temperature gradient in z direction. In current research, to investigate effect of angle variations on temperature distribution; dimensionless temperature parameter is defined as below: $[(T - T_{\infty, in}) / (T_{\infty, out} - T_{\infty, in})]$, that $T_{\infty, in}$, $T_{\infty, out}$ are inside and outside ambient temperature, respectively.

Fig.6 presents variations of maximum of dimensionless temperature in terms of fibers' angle. Since Inner temperature of cylinder is higher than ambient temperature ($T_{\infty, in} - T_{\infty, out} < 0$), so; dimensionless temperature is negative. increasing fibers' angle from zero to 90 causes that conductive heat transfer coefficient in r direction decrease, but in the other hand regarding to Fig. 5 when fiber's angle is neared to 90 the amount of μ is increased that it caused less effect of temperature gradient in z direction (18); consequently as it shown in Fig.6 by growing fibers' angle from zero to 90 absolute amount of maximum of dimensionless temperature will be raised. For heat fluxes 400, 800 and 1200 raising fibers' angle from zero to 90 causes that the maximum temperature in laminates growth 1.7891, 2.3002 and 2.8114 respectively.

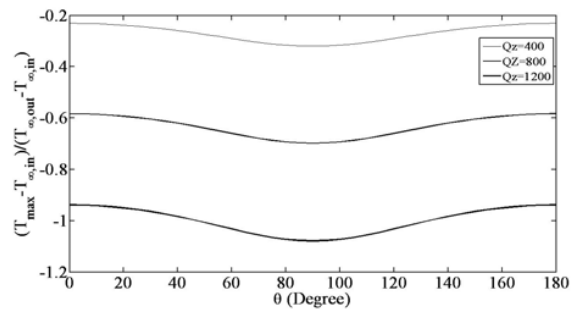
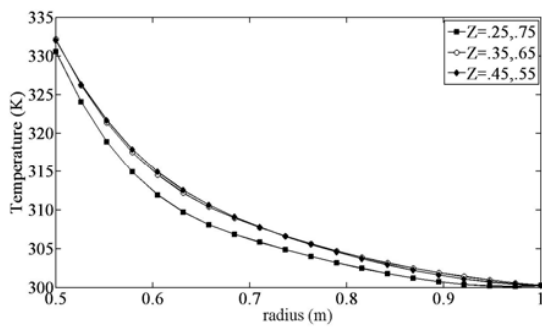
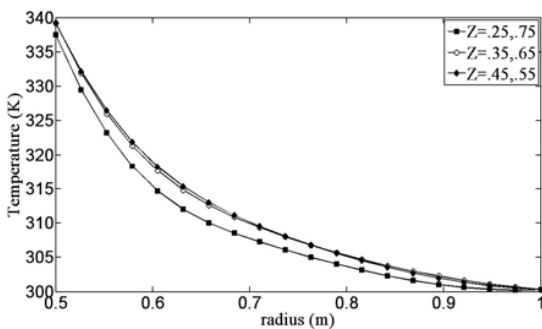


Fig.6 Maximum of relative temperature distribution in terms of fibers' angle (θ) under different inner heat fluxes

For other state of fibers' angle in a composite laminate, temperature distribution is between temperature distribution in a single layer laminate that fibers' angle is zero and single layer laminate that fibers' angle is zero 90. Fig.7 represents temperature distribution in a five-layers composite cylindrical laminate that is quasi-isotropic under various amount of heat fluxes. The boundary conditions and the material of composite are similar to single layer laminate. In this case thickness of each layer is 0.1 m. In quasi-isotropic laminates the arrangement of fibers in each lamina is $[0, 45, 90, 135, 180]$. By comparison between Fig.4 and Fig.10 it is clear that temperature distribution is in a state between zero and 90 of fibers angle.



(a)



(b)

Fig.7 Temperature distribution in quasi-isotropic laminate in different heat fluxes, (a) $Q = 800(\text{W} / \text{m}^2)$, (b) $Q = 1200(\text{W} / \text{m}^2)$

Fig.8 and Fig.9 present amounts of coefficients of temperature distribution series for a quasi-isotropic laminate; these coefficients are achieved with Thomas algorithm method. Because of the odd terms of this series a_n and b_n are zero, so its diagram has been shown in terms of $n/2$.

According to this figure (8), a_n are very small numbers that will be decreased sharply by increasing the amount of n . Since, regard to (34); a_n is coefficient of $I_0\left(\frac{\lambda_n}{\mu_i} r_i\right)$ and the amount of $I_0\left(\frac{\lambda_n}{\mu_i} r_i\right)$ will be raised remarkably by increasing of n ; Therefore, to converge of the series; the amounts of a_n must be very small. also there is a same deduction for b_n ; b_n is coefficient of $k_0\left(\frac{\lambda_n}{\mu_i} r_i\right)$ and the amount of $k_0\left(\frac{\lambda_n}{\mu_i} r_i\right)$ decrease rapidly respect to increasing of n , so the amount of b_n must be very big to keep convergence of series.

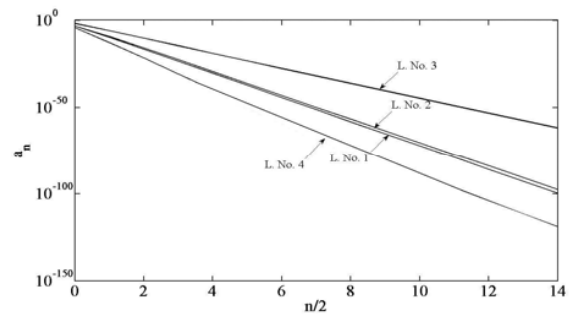


Fig.8 Fourier series terms (a_n) distribution in terms of $n/2$ in a quasi-isotropic laminate

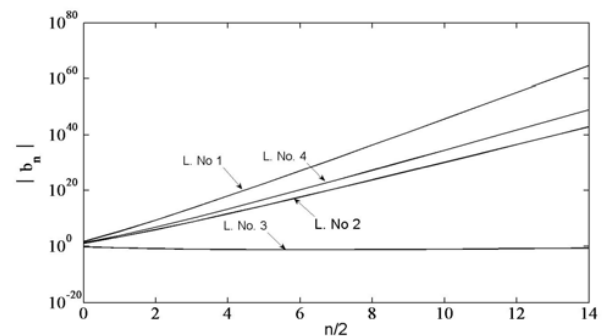


Fig.9 Fourier series terms (b_n) distribution in terms of $n/2$ in a quasi-isotropic laminate

V. CONCLUSION

In the present investigation, an exact analytical solution for two-dimensional steady-state temperature distribution (r, z) for the case of the general boundary condition is presented. The main results of this research are summarized as follows:

- Due to considering the general boundary condition, the present analytical solution can be generalized to the vast cases of thermal conditions for circular pipes, reservoirs and fins.
- For composite laminates with larger conductivity in fibers direction in comparison of perpendicular direction (such as graphite/epoxy), when the fibers' angle is 90° temperature distribution is more monotonous than the other direction but when the fibers' angle is 0° maximum and minimum of temperature in laminate is less than other directions.
- For other arrangement of fibers, the temperature distribution is in a state between two cases of $\theta = 0^\circ$ and $\theta = 90^\circ$.

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