An Exact Algorithm for Location–Transportation Problems in Humanitarian Relief

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Abstract—This paper proposes a mathematical model and examines the performance of an exact algorithm for a location–transportation problems in humanitarian relief. The model determines the number and location of distribution centers in a relief network, the amount of relief supplies to be stocked at each distribution center and the vehicles to take the supplies to meet the needs of disaster victims under capacity restriction, transportation and budgetary constraints. The computational experiments are conducted on the various sizes of problems that are generated. Branch and bound algorithm is applied for these problems. The results show that this algorithm can solve problem sizes of up to three candidate locations with five demand points and one candidate location with up to twenty demand points without premature termination.

Keywords—Disaster response, facility location, humanitarian relief, transportation.

I. INTRODUCTION

THE number of natural disasters has increased and the severity has grown over recent years. Some people lose their lives. Many people have to lose their possessions or leave their residence. Efficient and effective disaster operations management has become a vital research topic. The life cycle of disaster operations management comprises four phases, which are the mitigation phase, the preparedness phase, the response phase, and the recovery phase [1]. The first two phases are pre-positioning phases that need to be performed prior to the onset of a disaster. The other two are post-disaster phases. The period of time in each phase depends on the type of disaster (a quick-onset or a slow-onset disaster). The disaster response is a crucial phase. The objective of disaster response in the humanitarian relief chain is to rapidly provide relief (emergency food, water, medicine, shelter, and supplies) to areas affected by large-scale emergencies, so as to minimize human suffering and death [2]. Most research topics have emphasized designing a disaster management framework, such as the study appearing in [3]. Few research papers have focused on constructing a disaster response operation framework and application. The research in the latter area aims to determine a solution by using a mathematical method. It starts with identifying the problems and formulating the mathematical model to represent the real problems. Next, an efficient method has to be found to give the best or at least a good quality solution for the mathematical model. Finally, the solution to answer the real problems is interpreted and validated. Relief logistics play an important role in this

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framework. The scope of relief logistics relates to ten subsystems, which are planning, inventory distribution, transportation, procurement, maintenance, control, human resources, information and communication, and administration subsystems [3]. The first three subsystems have been intensively studied under the following topics: facility location problems, inventory problems, transportation/routing problems and scheduling problems. Both individual analyses and the integration of these four problems have been researched.

The facility location-transportation problem for disaster response (FLTDR) is an emergency logistics aspect of the response phase. The framework for the main emergency logistics activities and their associated facilities and flows were proposed by [4] and are summarized in Fig. 1
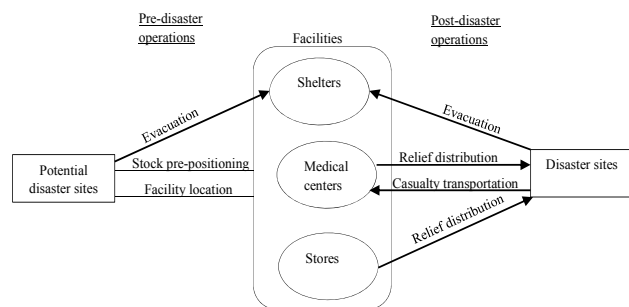

Fig. 1 Framework for disaster operations [4]

The arrows in Fig. 1 indicate the activities and the directions of the main flows of activity. Evacuation deals with the flow of people, relief distribution with resources, and casualty transportation of wounded people. Non-directional arrows do not indicate flows but rather express that a relationship exists between two components.

The FLTDR relates to solving location and transportation problems simultaneously. The location problem requires designing a network for distributing humanitarian aid (e.g., water, food, medical goods, and survival equipment). It mainly consists of determining the number, the position, and the mission of a humanitarian aid distribution center within the disaster region. The transportation problem deals with the distribution of humanitarian aid from the distribution center to demand points [5].

The purpose of this study is to propose a mathematical model for FLTDR problems and to examine the performance of an exact algorithm, which is a branch and bound algorithm for the problem under capacity restriction, transportation, and budgetary constraints. This paper focuses on calculating the

number of distribution centers to be constructed; determining the locations of distribution centers; identifying the quantity of relief items to be stored, and determining the assignment of vehicles to supply the humanitarian aid items so as to maximize the relief item coverage under the following assumptions. Each particular house or building within the affected area could require humanitarian aid and is thus a potential demand point. The demand quantities are estimated by a homeland security organization or experts. The demand quantities can only be satisfied by the distribution center, which is assumed to stock and distribute multiple types of relief item. The relief items are divided with respect to their response time criticalities and target response time intervals.

The amount of stock to be held at the distribution center depends on the number and location of distribution centers in the network as well as the assignment of demand locations to the distribution centers, while distribution center location and assignment decisions are affected by the quantity of relief items to be stocked at each distribution center. Each distribution candidate site has a global and a per product capacity that fixes the maximum quantity to be stored within the site. The location candidates and the capacity of distribution centers are considered in the pre-disaster phase based on the demand locations and quantities. Both location and stock decisions are limited by pre-disaster budgetary restrictions.

The vehicles available at candidate sites are of various types and there are different numbers of available vehicles. The different docking times of each vehicle type at each site and the time needed for loading and unloading one unit of each product for each vehicle type are considered. The traveling time from a distribution center to a demand location is determined corresponding to distance and vehicle type. There are also some restrictions on the total weight and the total volume of vehicles. A maximum daily work time for each vehicle type is imposed. A given vehicle can perform as many trips as needed during a day as long as the corresponding work time limit is respected. Each vehicle trip is assumed to visit only one demand point at a time. One demand point may be visited many times. However, because of the maximum daily work time, the number of trips to a specific delivery point by a particular vehicle will be limited to a maximum value, which is set at 2. Finally, shipping costs from distribution centers to demand points are restricted by post-disaster budgetary restrictions. Next, the mathematical model formulation of this problem is presented.

II. MODEL FORMULATION

The proposed mathematical model was modified from [5] by creating a new objective function, adding budgetary constraints and changing the transportation cost function to make the problem match the real situation. The parameters and decision variables are defined as follows.

A. Parameters Description

I Set of demand points; $I = \{1,\ldots,n\}$

J Set of items; $J = \{1,\ldots,p\}$

L Set of candidate sites; $L = \{1,\ldots,u\}$

H Set of vehicle types at site l; $H = \{1,\ldots,m_l\}$

K Set of number of vehicles for each vehicle type at site l; $K = \{1,\ldots,u_{hl}\}$

V Set of vehicle trip; $V = \{1,2\}$

d_{ij} Demand for item type j at demand point i

s_{jl} Capacity of site l for item type j

S_l Capacity of site l for all item

Q_h Weight capacity of a vehicle of type h

V_h Volume capacity of a vehicle of type h

τ_{lh} Docking time for a vehicle of type h at site l

t_{ilh} Travel time from site l to demand point i by vehicle h

α_{jh} Time of loading and unloading one unit of item type j into a vehicle of type h

D_h Maximum daily work time for a vehicle of type h

ω_j Criticality weight for item type j; $\sum_{j=1}^{p} \omega_j = 1$ and $\omega_j \geq 0$

w_j Weight of one unit of item type j

v_j Volume of one unit of item type j

F_l Fixed cost of establishing distribution center l

g_{jl} Unit cost of acquiring and storing item type j at distribution center l

c_{ilh} Unit cost of shipping items from distribution center l to demand point i by vehicle type h

B_0 Emergency relief budgets allocated for pre-positioning relief supplies

B_1 Emergency relief budgets allocated for post-disaster distribution

B. Decision Variables

$$y_l = \begin{cases} 1 & \text{if a distribution center is located at site } l \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ilhkv} = \begin{cases} 1 & \text{If demand point } i \text{ is visited from distribution center } l \\ & \text{with the } k^{th} \text{ vehicle of type } h \text{ on its } v^{th} \text{ trip} \\ 0 & \text{otherwise} \end{cases}$$

Q_{ijlhkv} = Quantity of item type j delivered to point i from distribution center l with the kth vehicle of type h on its vth trip

p_{jl} = Quantity of item type j provided at site l

C. Mathematical Model

$$Max \sum_{i \in I} \sum_{j \in J} \frac{\omega_j}{d_{ij}} \sum_{l \in L} \sum_{h \in H} \sum_{k \in K} \sum_{v \in V} Q_{ijlhkv} \qquad (1)$$

subject to

$$\sum_{l \in L} \sum_{h \in H} \sum_{k \in K} \sum_{v \in V} Q_{ijlhkv} \leq d_{ij} \quad \forall i \in I, j \in J \qquad (2)$$

$$\sum_{i \in I} \sum_{h \in H} \sum_{k \in K} \sum_{v \in V} Q_{ijlhkv} \leq p_{jl} \quad \forall j \in J, l \in L \qquad (3)$$

$$\sum_{i \in I} \sum_{k \in K} \sum_{v \in V} \left((2t_{ilh} + \tau_{lh}) X_{ilhkv} + \sum_{j \in J} \alpha_{jh} Q_{ijlhkv} \right) \leq D_h y_l \quad \forall h \in H, l \in L \qquad (4)$$

$$\sum_{j \in J} w_j Q_{ijlhkv} \leq Q_h X_{ilhkv} \quad \forall i \in I, l \in L, h \in H, k \in K, v \in V \qquad (5)$$

$$\sum_{j \in J} v_j Q_{ijlhkv} \leq V_h X_{ilhkv} \quad \forall i \in I, l \in L, h \in H, k \in K, v \in V \quad (6)$$

$$\sum_{j \in J} p_{jl} \leq S_l y_l \quad \forall l \in L \quad (7)$$

$$p_{jl} \leq s_{jl} \quad \forall j \in J, l \in L \quad (8)$$

$$\sum_{l \in L} F_l y_l + \sum_{j \in J}\sum_{l \in L} g_{jl} p_{jl} \leq B_0 \quad (9)$$

$$\sum_{i \in I}\sum_{l \in L}\sum_{h \in H}\sum_{k \in K}\sum_{v \in V} X_{ilhkv} c_{ilh} \leq B_1 \quad (10)$$

$$y_l \in \{0,1\} \quad (11)$$

$$X_{ilhkv} \in \{0,1\} \quad (12)$$

$$Q_{ijlhkv} \geq 0, integer \quad (13)$$

$$p_{jl} \geq 0, integer \quad (14)$$

The objective function (1) maximizes the total fraction of weight demand covered by the established distribution centers. Constraint set (2) guarantees that the quantity of item j delivered for each demand point i does not exceed its demand. Constraint set (3) ensures that the total quantity of a given item type j delivered from a distribution center l does not exceed the quantity of item type j available in this distribution center. Constraint set (4) requires that the maximum daily work time restriction related to each vehicle k of type h located at a distribution center l is not exceeded. These constraints also prohibit trips from unopened sites. Constraint sets (5) and (6) express the vehicle capacity constraints for each trip in terms of weight and volume. Constraint set (7) and (8), respectively, insure that the total and the per item capacity of the distribution center are satisfied. Constraint (9) requires that the pre-disaster expenditure related to establishing a distribution center and holding inventory does not exceed the pre-disaster budget. Constraint (10) ensures that the total transportation costs do not exceed the post-disaster budget. Finally, constraint sets (11)-(14) define the nature of decision variables used in the model.

III. NUMERICAL EXPERIMENTS

The exact algorithm to solve the integer programming problem (1) in this research is a branch and bound algorithm implemented in MATLAB. The numerical experiments were performed on a personal computer with CPU Intel Core 2 Duo E8400 at 3.0 GHz with 6 Gb. RAM. In order to analyze the impact of the number of demand points (n) and the number of candidate locations of distribution centers (u) on the performance of the algorithm, a 4^2 full factorial design with three replications is used to carry out the numerical experiment to test the two following hypotheses. The first

hypothesis is to test whether treatments (the parameter of the problem n and u) affect the response:

H_0: There is no treatment effect

H_1: There is at least one main treatment effect

The other hypothesis is to test whether a treatment interaction affects the response:

H_0: There is no treatment interaction

H_1: There is at least one treatment interaction

The four levels of treatment n are 5, 10, 15 and 20. The levels of treatment u are 1, 2, 3, and 4. The data sets of sixteen problem cases are generated associated with the number of demand points and the number of candidate locations. The number of item type (p) is fixed at 2. The number of vehicle type at each site (m_l) and the number of vehicles for each vehicle type at site l are randomized between 1 and 5. The data sets are obtained by randomly generating d_{ij}, p_{jl} and the parameters existing on the left of the constraint set (4) to (10). The others are randomly generated from the determined range to avoid the number of feasible solution being limited to a small number. The minimum and maximum values of range were fixed by respectively multiplying 0.5 and 300 with the value of the right hand side. Three instances are generated for each data set. The branch and bound algorithm used to solve these instances are set to be prematurely terminated at 7200 seconds or 2 hours in order to limit the computational time for large-size problems. The percentage of weight demand coverage, computational time and the solutions of decision variables are recorded. The results of the experiments are statistically analyzed by using analysis of variance at a level of significance $\alpha = 0.05$ in MINITAB

IV. RESULTS AND DISCUSSIONS

This part comprises two sections. The first section shows the experimental results and interpretation of the results. The other expresses the result of the analysis of variance (ANOVA) of the computing time.

A. The Experimental Results

The average computing time and percentage of weight demand coverage (the objective function value in terms of percentage) are shown in Table I.

From Table I, the computing time increases according to both n and u. To statistically explore the severity of the effect of these two parameters, they are analyzed in the next section using analysis of variance (ANOVA). The algorithm application is limited for small size problems. Only for problem sizes of $n = 5$, $u \leq 3$ and $5 < n \leq 20$, $u = 1$ can the algorithm give the solution within 2 hours. Therefore, solving problem sizes of $n=5$, $u=4$ and $n > 5$, $u > 1$ using this algorithm may not provide the optimal solution because of premature termination.

At the same u, the average percentage of demand coverage decrease when n increases. At the same n, the average percentage of demand coverage tends to be higher when u increases. Moreover, from the problem solutions of all cases, all candidate locations are used. These two results show that to

meet more demand from disaster victims, more distribution centers are needed.

TABLE I
THE RESULTS OF THE NUMERICAL EXPERIMENTS

Case	n	u	Computing Time (seconds)				Average Percentage of Weight Demand Coverage (%)
			Instance 1	Instance 2	Instance 3	Average	
1	5	1	8.38	198.68	77.00	94.69	17.94
2	5	2	102.24	5050.17	42.85	1731.75	30.80
3	5	3	78.06	7190.56	189.51	2486.04	43.81
4	5	4	7185.66	7178.18	7199.52	7187.79	57.05
5	10	1	1782.35	75.80	778.90	879.02	12.73
6	10	2	6885.70	7121.80	7160.12	7055.87	30.78
7	10	3	7196.97	7180.55	7185.74	7187.75	36.60
8	10	4	7170.69	7182.39	7126.47	7159.85	40.42
9	15	1	400.03	124.20	7189.16	2571.13	11.06
10	15	2	7108.99	7192.07	7192.61	7164.56	18.03
11	15	3	7191.52	7177.97	7196.82	7188.77	30.43
12	15	4	7163.30	7142.13	7180.88	7162.10	39.56
13	20	1	7196.58	704.70	706.48	2869.25	11.01
14	20	2	7124.73	7192.44	7133.71	7150.29	11.90
15	20	3	7157.33	7171.74	7129.28	7152.78	23.31
16	20	4	7058.51	7199.11	7124.95	7127.52	23.05

B. Analysis of Variance for Computing Time

Before drawing conclusions from the ANOVA table, the assumption of experimental or residual error, which is normal and independently distributed, should be examined by analyzing the residual plots illustrated in Fig. 2.
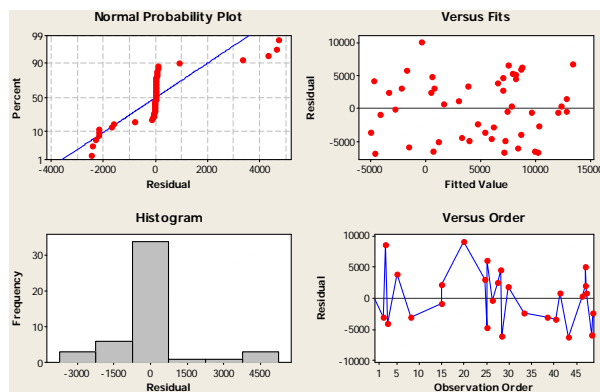

Fig. 2 Residual plots for computing time response

From Fig. 2, the Normal Probability Plot shows that the residuals are in linear form. It can be concluded that the data distribution is a normal distribution. Likewise, the upside down bell shape of the histogram also shows that the data distribution is normal. The other two graphs show that the residual is independently distributed because the plotted data are distributed randomly. Thus, it can be concluded that the residual is normal and independently distributed. After assumption verification, the ANOVA table for the experiment summarized in Table II is considered.

TABLE II
ANOVA FOR THE COMPUTING TIME RESPONSE

Source of Variation	Degrees of Freedom	Adjusted Sum of Squares	Adjusted Mean Square	F	P-Value
n	3	83600675	27866892	8.01	0.000
u	3	212813562	70937854	20.38	0.000
$n*u$	9	47358668	5262074	1.51	0.186
Error	32	111395661	3481114		
Total	47				

From Table II, the treatment n and u, significantly affect the response because p-values are less than the level of significance $\alpha = 0.05$. The parameter u has a stronger effect because it has a higher statistical value F. There is no significant interaction nu on the response.

V. CONCLUSION AND RECOMMENDATION

The branch and bound algorithm can solve the proposed FLTDR model, but it is unable to solve large size problems. The solutions can provide important information for logistics operations management for disaster relief. The computing performance depends on both the number of demand points and the number of candidate locations for distribution centers. To extend the performance of the algorithm, parallel computing, which decomposes a large size problem into many small size problems to be solved simultaneously, should be considered in future research. The number of candidate locations for distribution centers should be taken into consideration to divide sub-problems because of the strong effects. In addition, heuristic algorithms should be also taken into account as an efficient algorithm for a large size problem. The number of candidate locations reflects the percentage of demand coverage. The greater the percentage of weight demand coverage required, the greater is the number of distribution centers needed. However, it is limited by resource constraints. Therefore, the sensitivity analysis of the vital resource restrictions such as budgetary variables, the number of vehicles, etc. is an interesting area for future research.

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