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An Augmented Automatic Choosing Control Designed by Extremizing a Combination of Hamiltonian and Lyapunov Functions for Nonlinear Systems with Constrained Input

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Abstract— In this paper we consider a nonlinear feedback control called augmented automatic choosing control (AACC) for nonlinear systems with constrained input. Constant terms which arise from sectionwise linearization of a given nonlinear system are treated as coefficients of a stable zero dynamics. Parameters included in the control are suboptimally selected by extremizing a combination of Hamiltonian and Lyapunov functions with the aid of the genetic algorithm. This approach is applied to a field excitation control problem of power system to demonstrate the splendidness of the AACC. Simulation results show that the new controller can improve performance remarkably well.

Keywords— Augmented Automatic Choosing Control, Nonlinear Control, Genetic Algorithm, Hamiltonian, Lyapunov function

I. INTRODUCTION

IT is generally easy to design the optimal control laws for linear systems, but it is not so for nonlinear systems, though they have been studied for many years[1]∼[8]. One of the most popular and practical nonlinear control laws is synthesized by applying the linearization method by Taylor expansion and the linear optimal control method to a given nonlinear system. This is only effective in a small region around the steady state point or in almost linear systems[1]∼[3].

As one of approaches to overcome these drawbacks, an augmented automatic choosing control(AACC) is proposed for nonlinear systems[8]. Moreover, in many practical systems, there are physical constraints such as limitation and saturations of inputs. In this paper we consider a design method of the AACC for nonlinear systems with constrained inputs. Its process is as follows.

Assume that a system is given by a nonlinear differential equation. Choose a separative variable, which makes up nonlinearity of the given system. The domain of the variable is divided into some subdomains. On each subdomain, the system equation is linearized by Taylor expansion around a suitable point so that a constant term is included in it. This constant term is treated as a coefficient of a stable zero dynamics. The given nonlinear system approximately makes up a set of augmented linear systems, to which the optimal linear control theory is applied to get the linear quadratic (LQ) controls[2]. These LQ controls are smoothly united by sigmoid type automatic choosing functions to synthesize a single nonlinear feedback controller, which is limited to be satisfied with the constrained condition.

This controller is of a structure-specified type which has some parameters, such as the number of division of the domain, regions of the subdomains, points of Taylor expansion, and gradients of the automatic choosing function. These parameters must be selected optimally so as to be just the controller's fit. Since they lead to a nonlinear optimization problem, we are able to solve it by using the genetic algorithm (GA)[9] suboptimally. In this paper the suboptimal values of these parameters are obtained by acquiring both minimization of Hamiltonian and maximization of a stable region in the sense of Lyapunov.

This approach is applied to a field excitation control problem of power system, which is Ozeki-Power-Plant of Kyushu Electric Power Company in Japan, to demonstrate the splendidness of the AACC. Simulation results show that the new controller using the GA is able to improve performance remarkably well.

II. AUGMENTED AUTOMATIC CHOOSING CONTROL

Assume that a nonlinear system is given by

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathrm{D} \tag{1}$$

subject to

$$u_{j,min} \le u[j] \le u_{j,max} \quad (j = 1, \cdots, r) \tag{2}$$

where $\cdot = d/dt$, $x = [x[1], \cdots, x[n]]^T$ is an n-dimensional state vector, $u = [u[1], \cdots, u[r]]^T$ is an r-dimensional control input vector, $f(x) : \mathrm{D} \to R^n$ is a nonlinear vector-valued function with $f(0) = 0$ and is continuously differentiable, $g(x) : \mathrm{D} \to R^{n\times r}$ is a driving matrix with $g(0) \neq 0$ and is continuously differentiable, $u_{j,min}$: the minimum value of $u[j]$, $u_{j,max}$: the maximum value of $u[j]$, $\mathrm{D} \subset R^n$ is a domain , and T denotes transpose.

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Considering the nonlinearity of the system (1), introduce a vector-valued function $C : \mathrm{D} \to R^L$ which defines the separative variables $\{C_j(x)\}$, where $C = [C_1 \cdots C_j \cdots C_L]^T$ is continuously differentiable. Let D be a domain of C^{-1}. For example, if $x[2]$ is the element which has the highest nonlinearity of (1), then

$$C(x) = x[2] \in D \subset R \quad (L = 1)$$

(see Section 4). The domain D is divided into some subdomains: $D = \cup_{i=0}^{M} D_i$, where $D_M = D - \cup_{i=0}^{M-1} D_i$ and $C^{-1}(D_0) \ni 0$. $D_i (0 \le i \le M)$ endowed with a lexicographic order is the Cartesian product $D_i = \Pi_{j=1}^{L}[a_{ij}, b_{ij}]$, where $a_{ij} < b_{ij}$.

Introduce a stable zero dynamics :

$$\dot{x}[n+1] = -\sigma_i x[n+1] \tag{3}$$
$$(x[n+1](0) \simeq 1, \quad 0 < \sigma_i < 1),$$

where the value of σ_i shall be selected so that $\sigma_i = -\dot{x}[n+1]/x[n+1] \le -\dot{x}[k]/x[k]$ holds for all $k (k = 1, \cdots, n)$. This tries to keep $x[n+1] \simeq 1$ for a good while when the system (1) is not on $C^{-1}(D_0)$ (see Appendix).

Combine (1) with (3) to form an augmented system

$$\dot{\mathrm{X}} = \bar{f}(\mathrm{X}) + \bar{g}(\mathrm{X})u \tag{4}$$

where

$$\mathrm{X} = \begin{bmatrix} x \\ x[n+1] \end{bmatrix} \in \mathrm{D} \times R$$

$$\bar{f}(\mathrm{X}) = \begin{bmatrix} f(x) \\ -\sigma_i x[n+1] \end{bmatrix}, \bar{g}(\mathrm{X}) = \begin{bmatrix} g(x) \\ 0 \end{bmatrix}.$$

Let a cost function be

$$J = \frac{1}{2}\int_0^{\infty} \left(\mathrm{X}^T \mathrm{Q} \mathrm{X} + u^T \mathrm{R} u\right) dt \tag{5}$$

where

$$\mathrm{Q} \quad = \quad \begin{bmatrix} Q & 0 \\ 0 & q \end{bmatrix}, \quad R \ni q > 0,$$

$Q = Q^T > 0$ and $\mathrm{R} = \mathrm{R}^T > 0$ which denote positive symmetrix matrices. Values of Q and R are properly determined based on engineering experience.

On each D_i, the nonlinear system is linearized by the Taylor expansion truncated at the first order about a point $\hat{X}_i \in C^{-1}(D_i)$ and $\hat{X}_0 = 0$ (see Fig. 1):

$$\begin{aligned} f(x) + g(x)u &\simeq A_i x + w_i + B_i u \\ &\simeq A_i x + w_i x[n+1] + B_i u \end{aligned} \tag{6}$$

where

$$\begin{aligned} A_i &= \partial f(x)/\partial x^T|_{x=\hat{X}_i}, \; B_i = g(\hat{X}_i), \\ w_0 &= 0, \; w_i = f(\hat{X}_i) - A_i \hat{X}_i. \end{aligned}$$

That is, an approximation of (4) is

$$\dot{\mathrm{X}} = \bar{A}_i \mathrm{X} + \bar{B}_i u \qquad \text{on} \quad C^{-1}(D_i) \times R \tag{7}$$

where

$$\bar{A}_i \quad = \quad \begin{bmatrix} A_i & w_i \\ 0 & -\sigma_i \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}.$$

An application of the linear optimal control theory[2] to (5) and (7) yields

$$u_i(\mathrm{X}) = \mathrm{F}_i \mathrm{X} \tag{8}$$
$$\mathrm{F}_i = -\mathrm{R}^{-1} \bar{B}_i^T \mathrm{P}_i \tag{9}$$

where the $(n+1) \times (n+1)$ matrix P_i satisfies the Riccati equation :

$$\mathrm{P}_i \bar{A}_i + \bar{A}_i^T \mathrm{P}_i + \mathrm{Q} - \mathrm{P}_i \bar{B}_i \mathrm{R}^{-1} \bar{B}_i^T \mathrm{P}_i = 0. \tag{10}$$

Introduce an automatic choosing function of sigmoid type:

$$\begin{aligned} I_i(x) \quad = \quad \prod_{j=1}^{L} \Bigg\{ 1 - &\frac{1}{1 + \exp\left(2N_1 \left(C_j(x) - a_{ij}\right)\right)} \\ - &\frac{1}{1 + \exp\left(-2N_1 \left(C_j(x) - b_{ij}\right)\right)} \Bigg\} \end{aligned} \tag{11}$$

where N_1 is positive real value, $-\infty \le a_{ij}$ and $b_{ij} \le \infty$. $I_i(x)$ is analytic and almost unity on $C^{-1}(D_i)$, otherwise almost zero(see Fig. 2).

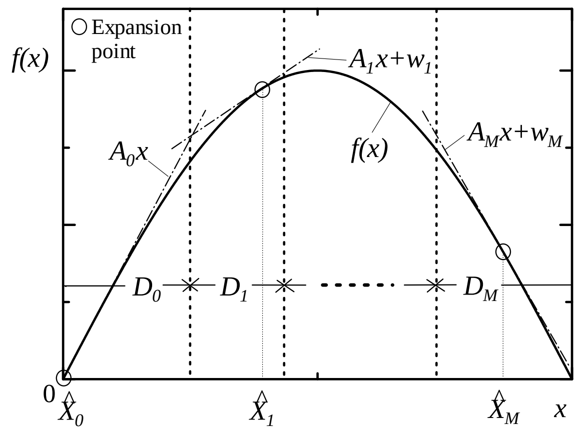

Fig. 1 Sectionwize linearization

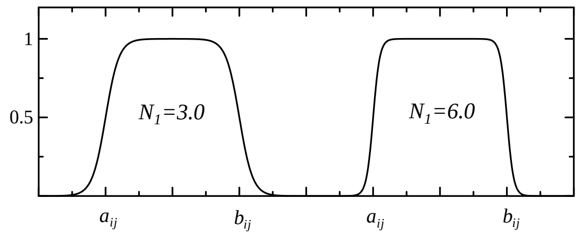

Fig. 2 Automatic Choosing Function(N_1=3.0, 6.0)

Uniting $\{u_i(\mathrm{X})\}$ of (8) with $\{I_i(x)\}$ of (11) yields

$$\begin{aligned} \hat{u}(\mathrm{X}) \quad &= \quad [\hat{u}(\mathrm{X})[1], \cdots, \hat{u}(\mathrm{X})[j], \cdots, \hat{u}(\mathrm{X})[r]]^T \\ &= \quad \sum_{i=0}^{M} u_i(\mathrm{X}) I_i(x). \end{aligned}$$

We finally have an augmented automatic choosing control which is satisfied with the constraint condition (2) by

$$u(\mathrm{X}) = [u(\mathrm{X})[1], \cdots, u(\mathrm{X})[j], \cdots, u(\mathrm{X})[r]]^T \tag{12}$$

where

$$u(\mathrm{X})[j] = \begin{cases} u_{j,max} & \text{if} \quad \hat{u}(\mathrm{X})[j] \geq u_{j,max} \\ u_{j,min} & \text{if} \quad \hat{u}(\mathrm{X})[j] \leq u_{j,min} \\ \hat{u}(\mathrm{X})[j] & \text{otherwise} \end{cases}$$

$$(1 \leq j \leq r).$$

III. PARAMETER SELECTION

The Hamiltonian for Eqs.(4) and (5) is given by

$$H(\mathrm{X},u,\lambda) = \frac{1}{2}\left(\mathrm{X}^T\mathrm{QX}+u^T\mathrm{R}u\right) + \lambda^T\left(\bar{f}(\mathrm{X})+\bar{g}(\mathrm{X})u\right). \quad (13)$$

Assume that the adjoint vector $\lambda(\mathrm{X}) \in R^{n+1}$ is defined by

$$\lambda(\mathrm{X}) = [\lambda^I(\mathrm{X})^T, \lambda^{II}(\mathrm{X})^T]^T \quad (14)$$

where $\lambda^I(\mathrm{X}) = [\lambda[1],\cdots,\lambda[r]]^T = -(G^T(x))^{-1}\mathrm{R}u(\mathrm{X})$, $\lambda^{II}(\mathrm{X}) = [\lambda[r+1],\cdots,\lambda[n+1]]^T = [0,E]\hat{\lambda}$,

$$\hat{\lambda} = \sum_{i=0}^{M}\{(\bar{B}_i - \bar{g}(\mathrm{X}))\bar{g}(\mathrm{X})^{\dagger} + E\}^T\mathrm{P}_i\mathrm{X}I_i(x) \quad \in R^{n+1},$$

$\bar{g}(\mathrm{X})^{\dagger}\bar{g}(\mathrm{X}) = E$, E is an appropriate-dimentional unit matrix, and † denotes pseudo inverse.

There are two necessary conditions of the optimality. One of them is $\partial H/\partial u = 0$ or $u = -\mathrm{R}^{-1}\bar{g}(\mathrm{X})^T\lambda = -\mathrm{R}^{-1}G^T(x)\lambda^I(\mathrm{X})$, which is satisfied with Eq.(12) from Eq.(14). By it, Eq.(13) becomes

$$H(\mathrm{X},u,\lambda) = \frac{1}{2}\mathrm{X}^T\mathrm{QX} - \frac{1}{2}u^T\mathrm{R}u + \bar{f}^T(\mathrm{X})\lambda. \quad (15)$$

The other one is $\dot{\lambda} = -\partial H/\partial \mathrm{X}$.

Next, introduce a Lyapunov function candidate:

$$V(\mathrm{X}) = \mathrm{X}^T\Pi(\mathrm{X})\mathrm{X} \quad (16)$$

where

$$\Pi(\mathrm{X}) = \sum_{i=0}^{M}\mathrm{P}_i\Pi_i(x) \ ,$$

$$\Pi_i(x) = \eta_i\prod_{j=1}^{L}\left\{1 - \frac{1}{1+\exp\left(2N_2\left(C_j(x)-a_{ij}\right)\right)} - \frac{1}{1+\exp\left(-2N_2\left(C_j(x)-b_{ij}\right)\right)}\right\}, \quad (17)$$

N_2 and η_i are positive real values.

By the Lyapunov's direct method[3], the equilibrium point 0 is uniformly stable on a connected set:

$$\mathrm{D}_V = \left\{x \in \mathrm{D} : V(\mathrm{X}) < \gamma,\ \dot{V}(\mathrm{X}) < 0\right\}$$

where

$$\gamma = \inf\left\{V(\mathrm{X}) : \mathrm{X} \neq 0,\ \dot{V}(\mathrm{X}) = 0\right\}. \quad (18)$$

In order to design the optimal control by Hamiltonian and expand the stable region in the sense of Lyapunov as wide as possible, we define a performance

$$PI = \omega_1\int_{\mathrm{D}}|H(\mathrm{X},u,\lambda)|/\mathrm{X}^T\mathrm{X}d\mathrm{X} + \omega_2\int_{\mathrm{D}}\left\|\dot{\lambda}+\partial H(\mathrm{X},u,\lambda)/\partial\mathrm{X}\right\|/\mathrm{X}^T\mathrm{X}d\mathrm{X} - \omega_3\cdot\gamma, \quad (19)$$

where $\omega_i(\omega_i \geq 0; i = 1,2,3)$ are weights.

A set of parameters included in the control (12):

$$\bar{\Omega} = \left\{M, N_1, N_2, a_{ij}, b_{ij}, \hat{X}_i, \eta_i\right\}$$

is suboptimally selected by minimizing PI with the aid of GA[9] as follows.

<ALGORITHM>

step1:Apriori: Set values $\bar{\Omega}_{apriori}$ appropriately.

step2:Parameter: Choose a subset $\Omega \subset \bar{\Omega}$ to be improved and rewrite it by $\Omega = \{M, N_1, \cdot\cdot\} = \{\alpha_k : k = 1,\cdot\cdot,K\}$.

step3:Coding: Represent each α_k with a binary bit string of $\widetilde{L}$ bits and then arrange them into one string of $\widetilde{L}K$ bits.

step4:Initialization: Randomly generate an initial population of $\widetilde{q}$ strings $\{\Omega_p : p = 1,\cdot\cdot,\widetilde{q}\}$.

step5:Decoding: Decode each element α_k of Ω_p by $\alpha_k = (\alpha_{k,max} - \alpha_{k,min})A_k/\left(2^{\widetilde{L}}-1\right) + \alpha_{k,min}$, where $\alpha_{k,max}$:maximum, $\alpha_{k,min}$:minimum, and A_k: decimal values of α_k.

step6:Adjoint: Make $\lambda = \lambda(\mathrm{X})_p \quad (p = 1,\cdot\cdot,\widetilde{q})$ for Ω_p by using Eq.(14).

step7:Fitness value calculation: Calculate PI_p by Eqs.(15) and (19), or fitness $F_p = -PI_p$. Integration of PI_p is approximated by a finite sum.

step8:Reproduction: Reproduce each of individual strings with the probability of $F_p/\sum_{j=1}^{\widetilde{q}}F_j$.

step9:Crossover: Pick up two strings and exchange them at a crossing position by a crossover probability P_c.

step10:Mutation: Alter a bit of string (0 or 1) by a mutation probability P_m.

step11:Repetition: Repeat step5∼step10 until prespecified $\widetilde{G}$-th generation. If unsatisfied, go to step2.

As a result, we have a suboptimal control $u(\mathrm{X})$ for the string with the best performance over all the past generations.

IV. NUMERICAL EXAMPLE

Consider a field excitation control problem of power system. Fig. 3 is a diagram of Ozeki-Power-Plant of

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Kyushu Electric Power Company in Japan. This system is assumed to be described[5][6][8] by

$$\widetilde{M}\frac{d^2\delta}{dt^2}+\widetilde{D}(\delta)\frac{d\delta}{dt}+P_e(\delta)=P_{in}$$
$$P_e(\delta)=E_I^2 Y_{11}\cos\theta_{11}+E_I\widetilde{V}Y_{12}\cos(\theta_{12}-\delta)$$
$$E_I+T'_{d0}\frac{dE'_q}{dt}=E_{fd}$$
$$E_I=E'_q+(X_d-X'_d)I_d(\delta)$$
$$I_d(\delta)=-E_I Y_{11}\sin\theta_{11}-\widetilde{V}Y_{12}\sin(\theta_{12}-\delta)$$
$$\widetilde{D}(\delta)=\widetilde{V}^2\left\{\frac{T''_{d0}(X'_d-X''_d)}{(X'_d+X_e)^2}\sin^2\delta+\frac{T''_{q0}(X_q-X''_q)}{(X_q+X_e)^2}\cos^2\delta\right\},$$

where δ: phase angle, $\dot{\delta}$: rotor speed, $\widetilde{M}$: inertia coefficient, $\widetilde{D}(\delta)$: damping coefficient, P_{in}: mechanical input power, $P_e(\delta)$: generator output power, $\widetilde{V}$: reference bus voltage, E_I: open circuit voltage, E_{fd}: field excitation voltage, X_d: direct axis synchronous reactance, X'_d: direct axis transient reactance, X_e: external impedance, $Y_{11}\angle\theta_{11}$: self-admittance of the network, $Y_{12}\angle\theta_{12}$: mutual admittance of the network, and $I_d(\delta)$: direct axis current of the machine. Put $x=[x[1],x[2],x[3]]^T=[E_I-\hat{E}_I,\delta-\hat{\delta}_0,\dot{\delta}]^T$ and $u=E_{fd}-\hat{E}_{fd}$, so that

$$\begin{bmatrix}\dot{x}[1]\\ \dot{x}[2]\\ \dot{x}[3]\end{bmatrix}=\begin{bmatrix}f_1(x)\\ f_2(x)\\ f_3(x)\end{bmatrix}+\begin{bmatrix}g_1(x)\\ 0\\ 0\end{bmatrix}u \qquad (20)$$

where

$$f_1(x) = -\frac{1}{kT'_{d0}}\left(x[1]+\hat{E}_I\right)+\frac{(X_d-X'_d)\widetilde{V}Y_{12}}{k}x[3]\cos\left(\theta_{12}-x[2]-\hat{\delta}_0\right)$$
$$f_2(x) = x[3]$$
$$f_3(x) = -\frac{\widetilde{V}Y_{12}}{\widetilde{M}}\left(x[1]+\hat{E}_I\right)\cos\left(\theta_{12}-x[2]-\hat{\delta}_0\right)-\frac{Y_{11}\cos\theta_{11}}{\widetilde{M}}\left(x[1]+\hat{E}_I\right)^2-\frac{\widetilde{D}(x)}{\widetilde{M}}x[3]+\frac{P_{in}}{\widetilde{M}}$$
$$\widetilde{D}(x) = \widetilde{V}^2\left\{\frac{T''_{d0}(X'_d-X''_d)}{(X'_d+X_e)^2}\sin^2\left(x[2]+\hat{\delta}_0\right)+\frac{T''_{q0}(X_q-X''_q)}{(X_q+X_e)^2}\cos^2\left(x[2]+\hat{\delta}_0\right)\right\}$$
$$g_1(x) = \frac{1}{kT'_{d0}}, \quad k=1+(X_d-X'_d)Y_{11}\sin\theta_{11}.$$

Assume that the constrained input is subject to

$$u_{min}+\hat{E}_{fd}\le E_{fd}\le u_{max}+\hat{E}_{fd}.$$

Parameters are

$\widetilde{M}=$	$0.016095[pu]$	$T'_{d0}=$	$5.09907[sec]$
$\widetilde{V}=$	$1.0[pu]$	$P_{in}=$	$1.2[pu]$
$X_d=$	$0.875[pu]$	$X'_d=$	$0.422[pu]$
$Y_{11}=$	$1.04276[pu]$	$Y_{12}=$	$1.03084[pu]$
$\theta_{11}=$	$-1.56495[pu]$	$\theta_{12}=$	$1.56189[pu]$
$X_e=$	$1.15[pu]$	$X''_d=$	$0.238[pu]$
$X_q=$	$0.6[pu]$	$X''_q=$	$0.3[pu]$
$T''_{d0}=$	$0.0299[pu]$	$T''_{q0}=$	$0.02616[pu]$.

Steady state values are

$\hat{E}_I=$	$1.52243[pu]$	$\hat{\delta}_0=$	$48.57°$
$\hat{\dot{\delta}}_0=$	$0.0[deg/sec]$	$\hat{E}_{fd}=$	$1.52243[pu]$.

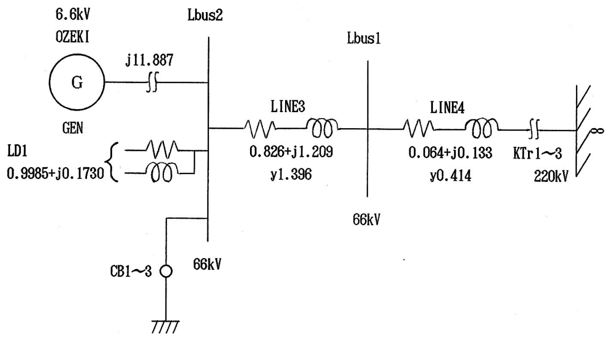

Fig. 3 Diagram of Ozeki-Power-Plant

Set X $=[x^T,x[4]]^T=[x[1],x[2],x[3],x[4]]^T$, $n=3$, $\hat{X}_0=\hat{\delta}_0=48.57°$, $C(x)=x[2]$, $L=1$, Q=diag(1,1,1,1), R=1, $\omega_1=\omega_2=1$, $\widetilde{P}=$ I and $x[4](0)=1$, where I is $(n+1)\times(n+1)$ unit matrix. Experiments are carried out for the new control(AACC), the automatic choosing control(ACC)[6], and the ordinary linear optimal control(LOC)[1][2].

1) AACC($\omega_3=10$):
We experiment a case of $\omega_3=10$ and the unknown parameter subset $\Omega=\{M,N_1,N_2,a_{ij},b_{ij},\hat{X}_i,\eta_i\}$. To reduce the overwork of computers, we select the Taylor expansion points $\{\hat{X}_i\}$ from among candidates $\{\hat{X}_k : k=1,\cdots,26\}$ which are prepared from 55° to 180° at intervals of 5°. Put $\hat{X}_0=48.57°$, $a_0=-\infty$ and $b_M=\infty$. Set $\sigma_0=\sigma_1=\cdots=\sigma_{26}=0.3262$ at (3) because $\min\{\sigma_{im}:0\le i\le 26\}=0.3262$ using Appendix. The parameters are suboptimally selected along the algorithm of section 3, where $\widetilde{G}$=100, $\widetilde{q}$=100, $\widetilde{L}$=8, P_c=0.8, P_m=0.03. D=[-0.5,0.5]×[-0.2,0.5]×[-2,2]×[0,1.0]. The constrained input value is $u_{max}=-u_{min}=0.5$. As a result, we have that $M=3$, $N_1=4.91$, $N_2=1.21$, $a_1=b_0=54.1°$, $a_2=b_1=113.0°$, $a_3=b_2=171.7°$, $\hat{X}_1=55°$, $\hat{X}_2=145°$, $\hat{X}_3=180°$ and $\eta_1=\eta_2=\eta_3=1.11$.

2) AACC($\omega_3=100$):
The parameters are suboptimally selected by using a similar way of the AACC($\omega_3=10$) under $\omega_3=100$. As a result, we have that $M=2$, $N_1=4.76$, $N_2=2.94$, $a_1=b_0=53.3°$, $a_2=b_1=90.3°$, $\hat{X}_1=70°$, $\hat{X}_2=115°$ and $\eta_1=\eta_2=1.44$.

3) AACC($\omega_2=0$):

The parameters are suboptimally selected by using a similar way of the AACC($\omega_3 = 10$) under $\omega_2 = 0$, which does not include the differential coefficient of the adjoint vector in Eq.(14). As a result, we have that $M = 3$, $N_1 = 7.18$, $N_2 = 1.56$, $a_1 = b_0 = 53.5°$, $a_2 = b_1 = 147.5°$, $a_3 = b_2 = 177.8°$, $\hat{X}_1 = 60°$, $\hat{X}_2 = 175°$, $\hat{X}_3 = 180°$ and $\eta_1 = \eta_2 = \eta_3 = 2.82$

4) ACC:

The parameters are suboptimally selected by using a similar way of the AACC($\omega_3 = 0$) under the same condition as it when $\Omega = \{M, N, a_{ij}, b_{ij}, \hat{X}_i\}$. As a result, we have that $M = 1$, $N = 7.0$, $a_1 = b_0 = 64.8°$ and $\hat{X}_1 = 75°$.

Table1 shows performances by the AACC, the ACC and the LOC. The cost function of Table1 is

$$\widetilde{J} = \frac{1}{2} \int_0^{20} \left(\mathrm{X}^T \mathrm{Q} \mathrm{X} + u^T \mathrm{R} u \right) dt.$$

Figs. 4 and 5 show the responses in case of $x^T(0) = [0, 1.4, 0]$. Figs. 6 and 7 show the responses in case of $x^T(0) = [0, 1.446, 0]$. These results indicate that the AACC($\omega_3 = 10, 100$) with constraint input is better than the AACC($\omega_2 = 0$), ACC and LOC.

V. CONCLUSIONS

We have studied an augmented automatic choosing control using zero dynamics for nonlinear systems with constrained input. This approach have been applied to a field excitation control problem of power system. Simulation results have shown that the new controller is able to improve performance remarkably well. The followings are left for the future works: problem of optimum selection of σ_i and ω_i, application to more complicated systems such as multi-machines power systems[7].

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Appendix

In the AACC, we make a linear approximation system in (6) : $f(x) \simeq A_i x + w_i x[n+1]$ by using an approximation of Taylor expansion: $f(x) \simeq A_i x + w_i$.
Thus we would like to keep $x[n+1] \simeq 1$, namely $x[n+1] \in R \;\; (x[n+1](0) \simeq 1)$ changes slower than the state vector $x \in R^n$ on $C^{-1}(D_i) \subset \mathrm{D}(i \neq 0)$. Whenever $x \in R^n$ enters into $C^{-1}(D_0)$ which has the steady state point, this AACC almost becomes the ordinary LQ control. That is, stay $x[n+1] \simeq 1$ for a good while except on $C^{-1}(D_0)$. We shall show how to do it.
Substituting (8) into (7) yields

$$\dot{\mathrm{X}} = \left(\bar{A}_i - \bar{B}_i \mathrm{R}^{-1} \bar{B}_i^T \mathrm{P}_i \right) \mathrm{X}. \qquad (21)$$

Assume the controllability of the linear feedback system described by (21). We define $\lambda_{ik} (0 \leq i \leq M, 1 \leq k \leq n+1)$ being eigenvalues of $\left(\bar{A}_i - \bar{B}_i \mathrm{R}^{-1} \bar{B}_i^T \mathrm{P}_i \right)$:

$$\begin{aligned} & \left| \left(\bar{A}_i - \bar{B}_i \mathrm{R}^{-1} \bar{B}_i^T \mathrm{P}_i \right) - \lambda_{ik} \mathrm{I} \right| \\ = & \begin{vmatrix} \left(A_i - B_i \mathrm{R}^{-1} B_i^T P_i \right) - \lambda_{ik} I & w_i \\ 0 & -\sigma_i - \lambda_{ik} \end{vmatrix} \\ = & \left(-\sigma_i - \lambda_{ik} \right) \left| \left(A_i - B_i \mathrm{R}^{-1} B_i^T P_i \right) - \lambda_{ik} I \right| \\ = & \; 0 \end{aligned} \qquad (22)$$

where $|\cdot|$ denotes determinant, and $P_i = P_i^T > 0$ is a solution of the Riccati equation: $P_i A_i + A_i^T P_i + Q - P_i B_i \mathrm{R}^{-1} B_i^T P_i = 0$.

The asymptotically stable condition of (21) is

$$Re(\lambda_{ik}) < 0 \quad \text{or} \quad Re(-\lambda_{ik}) > 0 \quad \text{for all} \quad \lambda_{ik}$$

where $Re(\cdot)$ denotes the real part.

Define σ_{im} as the minimal value of $\{Re(-\lambda_{ik})\}$ for $(A_i - B_i \mathrm{R}^{-1} B_i^T P_i)$ by $\sigma_{im} = \min Re\{-\lambda_{ik} : 1 \leq k \leq n\}$. That is, these $\{\lambda_{ik}\}$ are the solusion of $\left| (A_i - B_i \mathrm{R}^{-1} B_i^T P_i) - \lambda_{ik} I \right| = 0$ from (22).
We should be able to select σ_i of (3) from $\sigma_i \in (0, \sigma_{im}] \subset R$ properly.

TABLE I
PERFORMANCES

Method	$x^{\mathrm{T}}(0)$: initial point				
	$[0, 0.6, 0]$	$[0, 0.65, 0]$	$[0, 1.0, 0]$	$[0, 1.4, 0]$	$[0, 1.446, 0]$
LOC	2.587	×	×	×	×
ACC	2.096	2.388	×	×	×
AACC($\omega_2 = 0$)	1.991	2.172	2.703	×	×
AACC($\omega_3 = 10$)	1.995	2.178	2.663	3.096	×
AACC($\omega_3 = 100$)	1.997	2.182	2.705	3.050	4.988

× : very large value

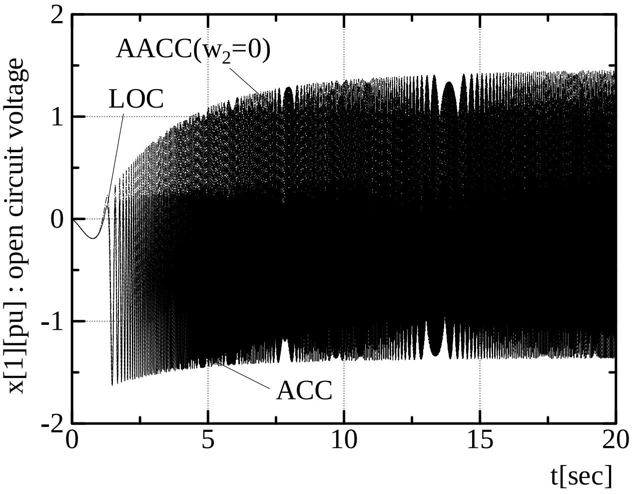

Fig. 4 Responses of LOC,ACC,AACC($\omega_2 = 0$)
$\left(x^T(0) = [0, 1.4, 0]\right)$

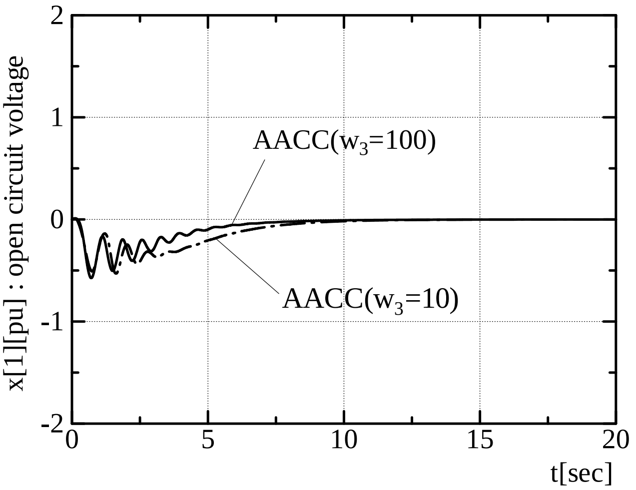

Fig. 5 Responses of AACC($\omega_3 = 10, 100$)
$\left(x^T(0) = [0, 1.4, 0]\right)$

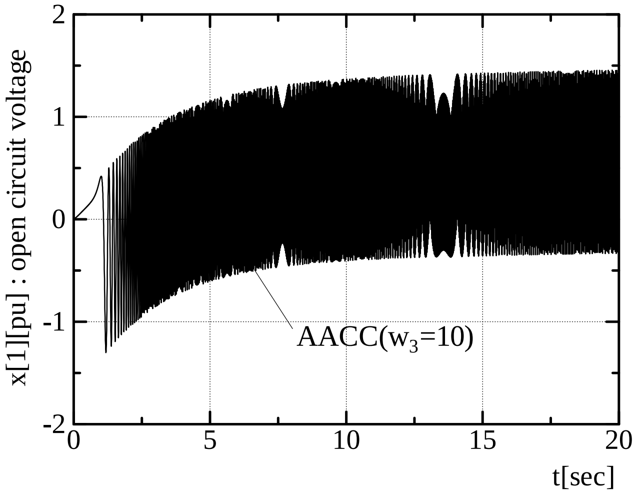

Fig. 6 Response of AACC($\omega_3 = 10$)
$\left(x^T(0) = [0, 1.446, 0]\right)$

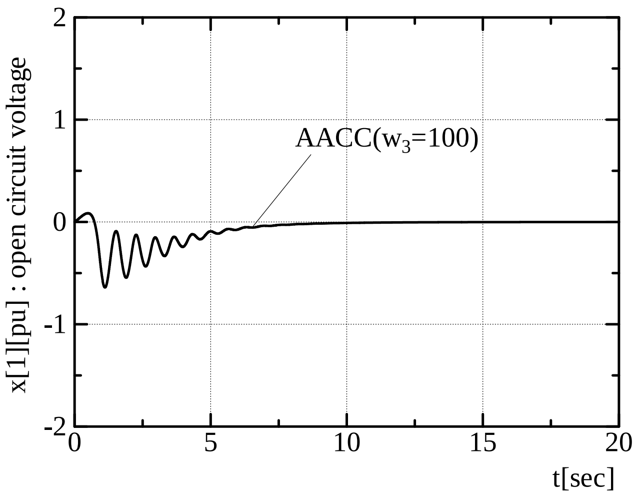

Fig. 7 Response of AACC($\omega_3 = 100$)
$\left(x^T(0) = [0, 1.446, 0]\right)$