

# An Approach to Correlate the Statistical-Based Lorenz Method, as a Way of Measuring Heterogeneity, with Kozeny-Carman Equation

H. Khanfari, M. Johari Fard

**Abstract**—Dealing with carbonate reservoirs can be mind-boggling for the reservoir engineers due to various diagenetic processes that cause a variety of properties through the reservoir. A good estimation of the reservoir heterogeneity which is defined as the quality of variation in rock properties with location in a reservoir or formation, can better help modeling the reservoir and thus can offer better understanding of the behavior of that reservoir. Most of reservoirs are heterogeneous formations whose mineralogy, organic content, natural fractures, and other properties vary from place to place. Over years, reservoir engineers have tried to establish methods to describe the heterogeneity, because heterogeneity is important in modeling the reservoir flow and in well testing. Geological methods are used to describe the variations in the rock properties because of the similarities of environments in which different beds have deposited in. To illustrate the heterogeneity of a reservoir vertically, two methods are generally used in petroleum work: Dykstra-Parsons permeability variations (V) and Lorenz coefficient (L) that are reviewed briefly in this paper. The concept of Lorenz is based on statistics and has been used in petroleum from that point of view. In this paper, we correlated the statistical-based Lorenz method to a petroleum concept, i.e. Kozeny-Carman equation and derived the straight line plot of Lorenz graph for a homogeneous system. Finally, we applied the two methods on a heterogeneous field in South Iran and discussed each, separately, with numbers and figures. As expected, these methods show great departure from homogeneity. Therefore, for future investment, the reservoir needs to be treated carefully.

**Keywords**—Carbonate reservoirs, heterogeneity, homogeneous system, Dykstra-Parsons permeability variations (V), Lorenz coefficient (L).

## I. INTRODUCTION

It is known that the bedding architecture of sedimentary rocks have been formed during long-term processes, i.e. deposition, compaction, and tectonic deformations. Therefore, there is a variation in the properties of different beds both laterally and vertically (Fig. 1). A reservoir as a sedimentary rock inherits this variation in properties [1]. In petroleum engineering when dealing with a hydrocarbon reservoir, it is required to measure the properties of that reservoir so as to understand the behavior of the reservoir and to control the flow through the porous media up to production pipes. These properties include permeability, porosity, wettability, water

saturation, and other rock characteristics [2]. To determine these properties, petroleum engineers classify the reservoir into some units that have similar properties and therefore similar flow behavior. In order to classify the reservoir, it is important to have a thorough understanding of the reservoir's properties variations.

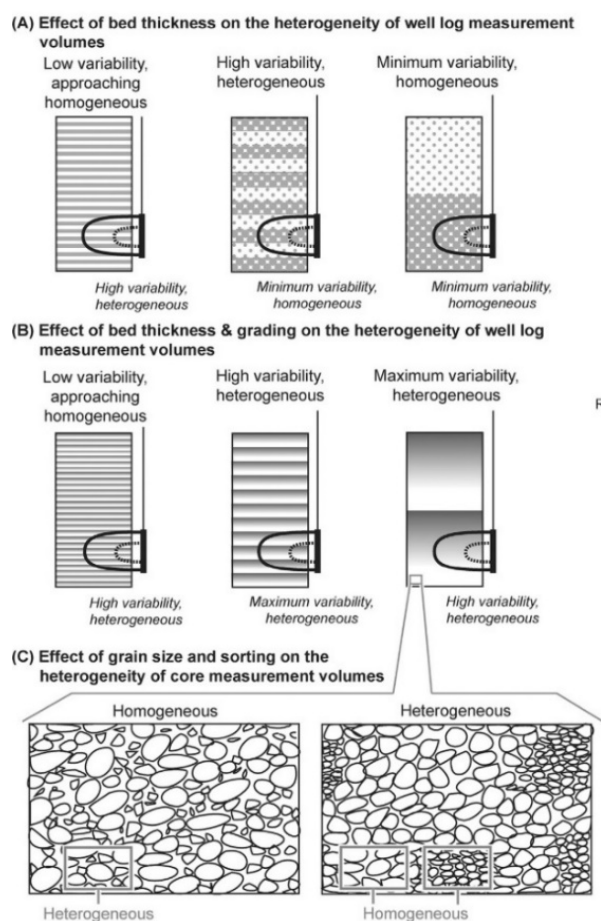


Fig. 1 Schematic illustration of the influence of thin beds (A, B), grading (B) and grain size and sorting (C) on petrophysical measurement volumes. (A, B) focus on deep and shallow well log measurements, and (B) focuses on core and thin section measurements [3]

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This variation which is known as heterogeneity is discussed and specified by different authors. There is a lot and different

approaches and methods which can be found in literature concerning heterogeneity. Therefore, according to its application in describing the complexity of the nature of the reservoir and its petrophysical characteristics, many petroleum authors over time have studied this variation [4]-[6].

Reservoir heterogeneity is then defined as the variation in these properties as a function of space. Heterogeneity needs to be specified vertically and aerially. Geological methods are used to describe the variations in the rock because of the similarities of depositional environments for different beds. In order to describe the heterogeneity of a reservoir vertically, two methods are mostly used in petroleum work:

- Dykstra-Parsons permeability variations V
- Lorenz coefficient L

They both determine the degree of heterogeneity by specifying a number between 0 and 1. A zero degree of heterogeneity means a homogeneous reservoir, and a coefficient of unity defines a completely heterogeneous reservoir [7], [8].

Here, in this paper, we present an evaluation on the vertical heterogeneity of an oil field in South Iran using Dykstra-Parsons coefficient (V) and Lorenz coefficient (L), and then, we compare the results with the previous findings.

## II. DEFINITION

### A. Lorenz Coefficient (L) Method to Evaluate the Vertical Heterogeneity

As a matter of fact, Lorenz developed a technic to measure the degree of inequality in the distribution of wealth across a population [9]. Schmalz and Rahme, 45 years later, suggested to use Lorenz curve to evaluate the vertical heterogeneity for a reservoir pay zone; They modified the Lorenz curve for use in petroleum engineering by plotting a curve of normalized cumulative permeability capacity versus the normalized cumulative volume capacity as a function of porosity on a Cartesian scale [10], [11], [7].

Dividing the area above the straight line of  $y=x$  by the area below it (see Fig. 1), a number is determined which is called Lorenz coefficient and it varies between 0 and 1 where 0 represents a completely homogeneous reservoir [8].

### B. Dykstra-Parsons Permeability Variation (v), to Evaluate the Vertical Heterogeneity

Dykstra – Parsons coefficient is commonly used in the quantification of permeability variation which is introduced by Dykstra and Parsons in 1950 and it is a statistical measure of non-uniformity of a set of data. It is generally used for permeability distribution but also can be extended for other properties of rock [12]. The geologic processes that create permeability in reservoir rocks appear to leave permeabilities distributed around the geometric mean [8]. The permeability values versus % of thickness are then plotted on a log-probability graph (see Fig. 2). The slope and intercept of a line of best fit, for all data, from this plot is then used to calculate the 50<sup>th</sup> and 84.1<sup>th</sup> probability percentile in order to calculate the expression below as the Dykstra-Parsons permeability variation number:

$$V = \frac{k_{50th} - k_{84.1th}}{k_{50th}} \quad (1)$$

## III. APPROACH

The relation between permeability (k) and porosity ( $\Phi$ ) can be shown as:

$$k = \left( \frac{1}{2\tau s^2 v} \right) \frac{\Phi^3}{(1-\Phi)^2} \quad (2)$$

Wyllie and Spangler proposed that it is more general to replace the factor 2 by a parameter called the pore shape factor  $k_{ps}$ . It is reported that the product  $k_{ps} \tau$  may be approximated by 5 for most cases [6]. Equation (2) for porous rocks can then be written as:

$$k = \left( \frac{1}{5s^2 v} \right) \frac{\Phi^3}{(1-\Phi)^2} \quad (3)$$

Equation (3) is the most popular form of the Kozeny equation, even though in actual porous rock  $k_{ps} \tau$  is variable and much greater than 5 [6].

According to Maclaurin series:

$$\frac{1}{(1-\Phi)^2} = 1 + 2\Phi + 3\Phi^2 + 4\Phi^3 + \dots \quad (4)$$

Because of the homogeneity and to avoid excess calculation, we consider  $m = \left( \frac{1}{5s^2 v} \right)$ . Therefore:

$$k = m \frac{\Phi^3}{(1-\Phi)^2} = m\Phi^3 + 2m\Phi^4 + 3m\Phi^5 + \dots \cong m\Phi^3 \quad (5)$$

Summing over some interval gives:

$$\sum kh = \sum mh\Phi^3 \quad (6)$$

Finally,  $\sum kh = m\Phi^2 \sum \Phi h$  is the straight line of Lorenz plot ( $y=x$ ).

As shown above, for a completely homogeneous unit, the Kozeny-Carman equation gives a straight line in the Lorenz curve. If it is a heterogeneous rock, then  $m$  and  $\Phi$  vary in different layers and we would expect deviation from straight line.

## IV. CASE STUDY

103 data are collected from a well in southern field in Iran. The data included permeability, porosity, and the depths at which they are measured. These data are extracted from cores that were taken from each depth. The reservoir of investigation is carbonate and due to the complexity and heterogeneity of carbonate reservoirs especially south Iran, before running the Lorenz and Dykstra-Parsons methods on this reservoir, one would expect the results to give high heterogeneity.

## V. RESULTS AND DISCUSSION

When Lorenz coefficient (L) is 0 in a purely homogeneous

formation, with rock properties that do not change with location in the reservoir; the cumulative property will increase by a constant value with depth which showed the line of perfect equality [3]. And also, the Dykstra-Parsons permeability variations ( $v$ ) show a straight diagonal line without any curvature. It means that a complete homogeneity, with regard to a reservoir rock, can be visualized in a formation that consists of 1- a single mineralogy 2- all grains of similar shapes and size. This complete homogeneity never actually occurs, but many formations are close enough to this situation that they can be considered as homogenous. Most of the models used for pressure-transient analysis assume this approximation [13].

By an elevation in the amount of these parameters to one, and increase in the heterogeneity of the property will cause a departure of the Lorenz curve away from the line of perfect equality, and the slope of the line of the best fit increases along with the difference between the 50<sup>th</sup> and 48.1<sup>th</sup> percentile. This means that the reservoir rock is composed of multiple mineralogies, and the components differ also in size and shape.

By current data from a well which is extracted from a carbonate reservoir, this variation is predicted not to be near zero because of different values of porosities and permeabilities. Based on our data, the Lorenz and Dykstra-Parsons curves can be constructed as below.

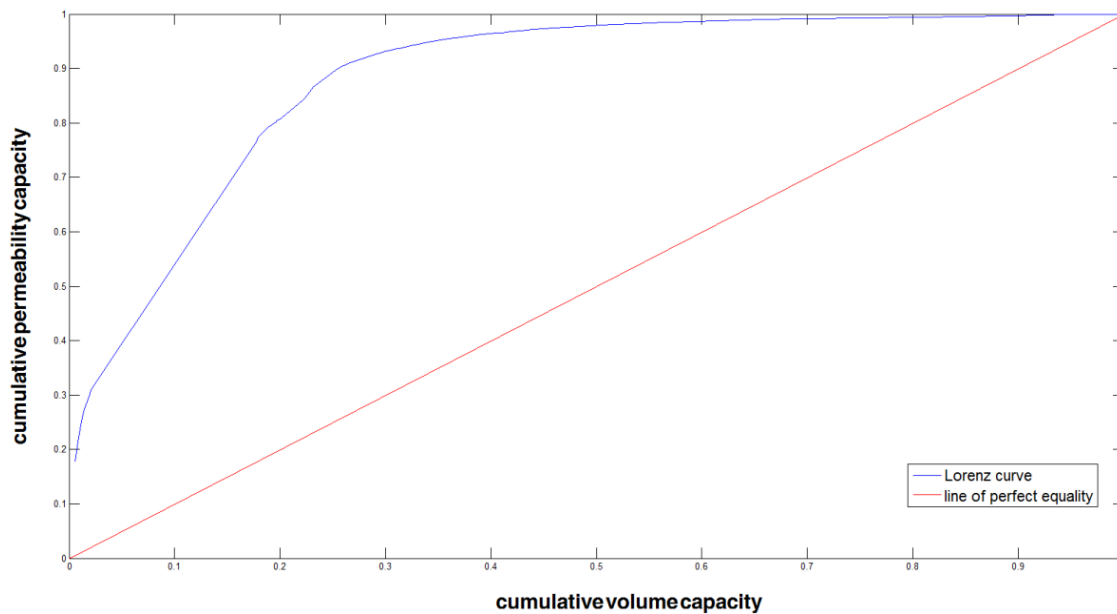


Fig. 2 Schematic illustration of a Lorenz plot: a cumulative of a property (e.g., flow capacity, permeability or porosity), sorted from in ascending order, is plotted on the y-axis, and a second cumulative property (e.g., storage capacity, porosity, or depth increment) is plotted on the x-axis

As can be seen Fig. 3, our extracted curve is far from a perfect straight line, and this is due to the scattering in the data. Therefore, the reservoir is heterogeneous, and a single property such as permeability or porosity cannot be assigned to it. To study this type of a reservoir, it is needed to be treated carefully, usually by portioning the reservoir bulk into some units. To accomplish that, it is recommended to use Lorenz curve and divide the reservoir into units with each unit showing straight line in Lorenz plot.

Based on the area between the Lorenz Curve and the line of perfect equality, the Lorenz coefficient ( $v$ ) will be equal to 0.7648. This number shows an approximately intermediate heterogeneous reservoir.

According to (1), the Dykstra-Parsons permeability variation number is equal to 0.500000, which again indicates an intermediate heterogeneous reservoir. This agrees with the recent interpretation on Lorenz curve.

## VI. CONCLUSION

- Lorenz coefficient ( $L$ ) and Dykstra-Parsons permeability variations ( $v$ ) are two common methods to evaluate the heterogeneity. These two methods indicate a number between 0 and 1. For the given reservoir, the following results are determined.
- Lorenz coefficient ( $L$ ) is equal to 0.7648
- Dykstra-Parsons permeability variations ( $v$ ) is equal to 0.500000
- Both of these methods show an intermediate to high heterogeneous reservoir.
- The statistical- based curve of Lorenz that has been used in petroleum engineering was correlated with a petroleum concept, i.e. Kozeny-Carman equation.

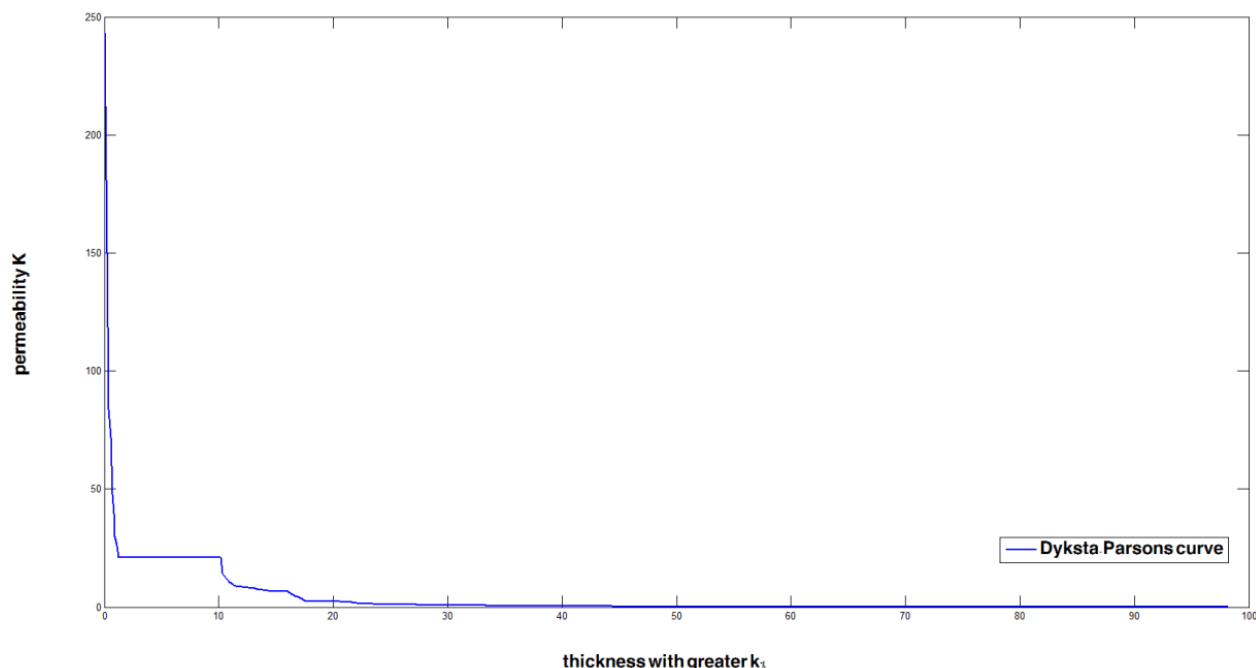


Fig. 3 Schematic illustration of a Dykstra-Parsons Plot based on current data

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