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An Analysis of Global Stability of a Class of Neutral-Type Neural Systems with Time Delays

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Abstract—This paper derives some new sufficient conditions for the stability of a class of neutral-type neural networks with discrete time delays by employing a suitable Lyapunov functional. The obtained conditions can be easily verified as they can be expressed in terms of the network parameters only. It is shown that the results presented in this paper for neutral-type delayed neural networks establish a new set of stability criteria, and therefore can be considered as the alternative results to the previously published literature results. A numerical example is also given to demonstrate the applicability of our proposed stability criterion.

Keywords—Stability Analysis, Neutral-Type Neural Networks, Time Delay Systems, Lyapunov Functionals.

I. Introduction

In recent years, the analysis of dynamical behavior of Hopfield neural networks, Cohen-Grossberg neural networks, cellular neural networks, bidirectional associative memory neural networks have been paid much attention due to their potential applications in various engineering problems regarding image and signal. It is known that, in the analysis of dynamical behavior of neural networks, the class of the activation functions employed in the design and time delays are two key parameters. In the classical neural network models such as Hopfield neural networks, Cohen-Grossberg neural networks, cellular neural networks, bidirectional associative memory neural networks, the time delays are in the states of the neural system. However, since the time derivatives of the states are the functions of time, in order to completely determine the stability properties of equilibrium point, some delay parameters must be introduced into the time derivatives of states of the system. The neural network model having time delays in the time derivatives of states is called delayed neutral-type neural networks. In the recent literature, many researchers have studied the equilibrium and stability properties of standard neural networks and neural networks of neutral type with a single delay and many delays and presented various sufficient conditions for the global asymptotic stability of the equilibrium point [1]-[28]. The most of the previous literature results are basically expressed in the linear matrix inequality (LMI) forms. The LMI approach to the stability problem of neutral type neural networks involves some difficulties with determining the constraint conditions on the network parameters as it requires to test positive definiteness of high dimensional matrices. In the current paper, by employing a suitable Lyapunov functional, we will present new delay-independent sufficient conditions for global asymptotic stability of the equilibrium point for the class of neutral-type neural networks with many delays. Our results establish various relationships between the network parameters only. Therefore, the results of this paper can be easily verified when compared with the previously reported literature results in the LMI forms.

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II. Problem Statement

In this paper, we consider the following class of delayed neural network model described by a set of nonlinear neutral delay differential equations :

$$\begin{aligned}\dot{x}_i(t) &= d_i(x_i(t))\Big[-c_i(x_i(t))+\sum_{j=1}^{n}a_{ij}f_j(x_j(t)) \\ &\quad +\sum_{j=1}^{n}b_{ij}f_j(x_j(t-\tau_j))+u_i\Big] \\ &\quad +\sum_{j=1}^{n}e_{ij}\dot{x}_j(t-\tau_j) \qquad (1)\end{aligned}$$

for $i = 1,..,n$, where n is the number of the neurons in the network, x_i denotes the state of the ith neuron, $d_i(x_i)$ represents an amplification function, and $c_i(x_i)$ is a behaved function that keeps the solution of system (1) bounded. The constants a_{ij} denote the strengths of the neuron interconnections within the network, the constants b_{ij} denote the strengths of the neuron interconnections with time delay parameters $\tau_j(t)$. e_{ij} are coefficients of the time derivative of the delayed states. Finally, the functions $f_j(\cdot)$ denote the neuron activation functions, and the constants u_i are some external inputs. In system (1), $\tau_j \geq 0$ represents the delay parameter with $\tau = max(\tau_j)$, $1 \leq j \leq n$. Accompanying the neutral system (1) is an initial condition of the form : $x_i(t) = \phi_i(t) \in C([-\tau, 0], R)$, where $C([-\tau, 0], R)$ denotes the set of all continuous functions from $[-\tau, 0]$ to R.

In what follows, we give the usual assumptions on the functions d_i, c_i and f_i :

A_1 : The functions $d_i(x)$, are continuously bounded, and there exist positive constants m_i and M_i such that

$$0 < m_i \leq d_i(x) \leq M_i,\ i = 1, 2, ..., n,\ \forall x \in R$$

A_2 : The functions $c_i(x)$ are continuous and there exist positive constants γ_i and ψ_i such that

$$0 < \gamma_i \leq \frac{c_i(x) - c_i(y)}{x - y} = \frac{|c_i(x) - c_i(y)|}{|x - y|} \leq \psi_i,\ i = 1, 2, ..., n,$$

$\forall x, y \in R, x \neq y$.

A_3 : The activation functions are Lipschitz continuous, i.e., there exist positive constants $L_i > 0$ such that

$$|f_i(x) - f_i(y)| \leq L_i |x - y|,\ i = 1, 2, ..., n,\quad \forall x, y \in R, x \neq y$$

We note here that if $E = 0$, then system (1) describes a Cohen-Grossberg neural network. If $d_i(x_i) = 1$ and $c_i(x_i) = x_i$, $i = 1, 2, ..., n$, in a Cohen-Grossberg neural network, this Cohen-Grossberg neural network describes a Hopfield-type neural network. If a Hopfield-type neural network uses a piecewise-wise linear activation function, then this Hopfield-type neural network describes a cellular neural network. Therefore, stability analysis of system (1) can be easily specialized for standard neural network models.

III. Stability Analysis

In this section,we obtain sufficient conditions for global stability of the equilibrium point of neutral system defined by (1). To this end, we will first shift the equilibrium point $x^* = [x_1^*, x_2^*, ..., x_n^*]^T$ of system (1) to the origin. By using the transformation $z(t) = x(t) - x^*$, the equilibrium point x^* can be shifted to the origin. The neutral-type neural network model (1) can be rewritten as :

$$\begin{aligned} \dot{z}_i(t) &= \alpha_i(z_i(t))\Big[-\beta_i(z_i(t)) + \sum_{j=1}^{n} a_{ij} g_j(z_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij} g_j(z_j(t-\tau_j))\Big] + \sum_{j=1}^{n} e_{ij} \dot{z}_j(t-\tau_j) \end{aligned} \tag{2}$$

which can be written in the form :

$$\begin{aligned} \dot{z}(t) &= \alpha(z(t))\big[-\beta(z(t)) + Ag(z(t)) + Bg(z(t-\tau))\big] \\ &\quad + E\dot{z}(t-\tau) \end{aligned}$$

where

$$\begin{aligned} & z(t) = [z_1(t), z_2(t), ..., z_n(t)]^T \\ & A = (a_{ij})_{n\times n}, B = (b_{ij})_{n\times n}, E = (e_{ij})_{n\times n} \\ & g(z(t)) = [g_1(z_1(t)), g_2(z_2(t)), ..., g_n(z_n(t)]^T \\ & \alpha(z(t)) = diag(\alpha_1(z_1(t)), \alpha_2(z_2(t)), ..., \alpha_n(z_n(t))) \\ & \beta(z(t)) = [\beta_1(z_1(t)), \beta_2(z_2(t)), ..., \beta_n(z_n(t))]^T \\ & g(z(t-\tau)) \\ & = (g_1(z_1(t-\tau_1)), g_2(z_2(t-\tau_2)), ..., g_n(z_n(t-\tau_n)))^T \end{aligned}$$

For the transformed system (2), the functions α_i, β_i and g_i are of the form :

$$\begin{aligned} & \alpha_i(z_i(t)) = d_i(z_i(t) + x_i^*),\ i = 1, 2, ..., n \\ & \beta_i(z_i(t)) = c_i(z_i(t) + x_i^*) - c_i(x_i^*),\ i = 1, 2, ..., n \\ & g_i(z_i(t)) = f_i(z_i(t) + x_i^*) - f_i(x_i^*),\ i = 1, 2, ..., n \end{aligned}$$

Assumptions A_1, A_2, A_3 respectively imply that

$$\begin{aligned} & 0 < m_i \leq \alpha_i(z_i(t)) \leq M_i,\ i = 1, 2, ..., n \\ & \gamma_i z_i^2(t) \leq z_i(t)\beta_i(z_i(t)) \leq \psi_i z_i^2(t),\ i = 1, 2, ..., n \\ & |g_i(z_i(t))| \leq L_i |z_i(t)|,\ i = 1, 2, ..., n \end{aligned}$$

We also note the following facts :

Fact 1 : If a, b, c and d are real vectors of dimension n, then the following equality holds :

$$\begin{aligned} & [-a + b + c + d]^T [a + b + c + d] \\ & = -a^T a + b^T b + c^T c + d^T d + 2b^T c + 2b^T d + 2c^T d \end{aligned}$$

Fact 2 : If a and b are real vectors of dimension n, then the following inequality holds :

$$2a^T b \leq \frac{1}{\varepsilon} a^T a + \varepsilon b^T b$$

where ε is any positive real number.

We now present the main result of this paper :

Theorem 1 : For the neutral system defined by (2), let $A_1 - A_3$ hold. Then, the origin of system (2) is globally asymptotically stable if there exist a positive constants ε such that the following conditions hold :

$$\begin{aligned} \rho &= \frac{\gamma^2}{L^2} - 2(1 + \frac{1}{\varepsilon})(||A||_2^2 + ||B||_2^2) > 0 \\ \xi &= \frac{1}{M^2} - \frac{\varepsilon + 1}{m^2} ||E||_2^2 > 0 \end{aligned}$$

where $m = \min_{1\leq i\leq n}(m_i)$, $M = \max_{1\leq i\leq n}(M_i)$, $\gamma = \min_{1\leq i\leq n}(\gamma_i)$, $L = \max_{1\leq i\leq n}(L_i)$.

Proof : We construct the following positive definite Lyapunov functional :

$$\begin{aligned} V(z(t)) &= 2\sum_{i=1}^{n} \int_0^{z_i(t)} \frac{\beta_i(s)}{\alpha_i(s)} ds \\ &\quad + \sum_{i=1}^{n} \int_{t-\tau_i}^{t} \frac{1}{\alpha_i^2(z_i(s))} \dot{z}_i^2(s) ds \\ &\quad + k\sum_{i=1}^{n} \int_{t-\tau}^{t} g_i^2(z_i(s)) ds \end{aligned}$$

where k is a positive constant to be determined later. The time derivative of $V(z(t))$ along the trajectories of the system (2) is obtained as follows :

$$\begin{aligned} \dot{V}(z(t)) &= 2\sum_{i=1}^{n} \frac{\beta_i(z_i(t))}{\alpha_i(z_i(t))} \dot{z}_i(t) + \sum_{i=1}^{n} \frac{1}{\alpha_i^2(z_i(t))} \dot{z}_i^2(t) \\ &\quad - \sum_{i=1}^{n} \frac{1}{\alpha_i^2(z_i(t-\tau_i))} \dot{z}_i^2(t-\tau_i) \\ &\quad + k\sum_{i=1}^{n} g_i^2(z_i(t)) - k\sum_{i=1}^{n} g_i^2(z_i(t-\tau_i)) \end{aligned}$$

which can be written as

$$\begin{aligned} \dot{V}(z(t)) &= 2\beta^T(z(t))\alpha^{-1}(z(t))\dot{z}(t) \\ &\quad \times [\alpha^{-1}(z(t))\dot{z}(t)]^T [\alpha^{-1}(z(t))\dot{z}(t)] \\ &\quad - [\alpha^{-1}(z(t-\tau))\dot{z}(t-\tau)]^T \\ &\quad \times [\alpha^{-1}(z(t-\tau))\dot{z}(t-\tau)] \\ &\quad + kg^T(z(t))g(z(t)) - kg^T(z(t-\tau))g(z(t-\tau)) \end{aligned}$$

We can write the following :

$$\begin{aligned}&2\beta^T(z(t))\alpha^{-1}(z(t))\dot{z}(t)\\&=2\beta^T(z(t))[-\beta(z(t))+Ag(z(t))+Bg(z(t-\tau))]\\&+2\beta^T(z(t))\alpha^{-1}(z(t))E\dot{z}(t-\tau)\end{aligned}$$

$$\begin{aligned}&[\alpha^{-1}(z(t))\dot{z}(t)]^T[\alpha^{-1}(z(t))\dot{z}(t)]\\&=[-\beta(z(t))+Ag(z(t))+Bg(z(t-\tau))\\&+\alpha^{-1}(z(t))E\dot{z}(t-\tau)]^T\times\\&[-\beta(z(t))+Ag(z(t))+Bg(z(t-\tau))\\&+\alpha^{-1}(z(t))E\dot{z}(t-\tau)]\end{aligned}$$

Hence, it follows that

$$\begin{aligned}&2\beta^T(z(t))\alpha^{-1}(z(t))\dot{z}(t)+[\alpha^{-1}(z(t))\dot{z}(t)]^T[\alpha^{-1}(z(t))\dot{z}(t)]\\&=[\beta(z(t))+Ag(z(t))+Bg(z(t-\tau))+\alpha^{-1}(z(t))E\dot{z}(t-\tau)]^T\\&\times[-\beta(z(t))+Ag(z(t))+Bg(z(t-\tau))+\alpha^{-1}(z(t))E\dot{z}(t-\tau)]\end{aligned}$$

In the light of Fact 1, we obtain

$$\begin{aligned}&2\beta^T(z(t))\alpha^{-1}(z(t))\dot{z}(t)+[\alpha^{-1}(z(t))\dot{z}(t)]^T[\alpha^{-1}(z(t))\dot{z}(t)]\\&=-\beta^T(z(t))\beta(z(t))\\&+g^T(z(t))A^TAg(z(t))+g^T(z(t-\tau))B^TBg(z(t-\tau))\\&+\dot{z}^T(t-\tau)\alpha^{-2}(z(t))E^TE\dot{z}(t-\tau)\\&+2g^T(z(t))A^TBg(z(t-\tau))\\&+2g^T(z(t))A^T\alpha^{-1}(z(t))E\dot{z}(t-\tau)\\&+2g^T(z(t-\tau))B^T\alpha^{-1}(z(t))E\dot{z}(t-\tau)\end{aligned}$$

which, when used in the time derivative of $V(z(t))$, yields :

$$\begin{aligned}\dot{V}(z(t))\;&=-\beta^T(z(t))\beta(z(t))+g^T(z(t))A^TAg(z(t))\\&+g^T(z(t-\tau))B^TBg(z(t-\tau))\\&+\dot{z}^T(t-\tau)\alpha^{-2}(z(t))E^TE\dot{z}(t-\tau)\\&+2g^T(z(t))A^TBg(z(t-\tau))\\&+2g^T(z(t))A^T\alpha^{-1}(z(t))E\dot{z}(t-\tau)\\&+2g^T(z(t-\tau))B^T\alpha^{-1}(z(t))E\dot{z}(t-\tau)\\&-[\alpha^{-1}(z(t-\tau))\dot{z}(t-\tau)]^T\\&\times[\alpha^{-1}(z(t-\tau))\dot{z}(t-\tau)]\\&+kg^T(z(t))g(z(t))-kg^T(z(t-\tau))g(z(t-\tau))\end{aligned}$$

from which it follows that

$$\begin{aligned}\dot{V}(z(t))\;\leq\;&-||\beta(z(t))||_2^2+||A||_2^2||g(z(t))||_2^2\\&+||B||_2^2||g(z(t-\tau))||_2^2\\&+||\alpha^{-1}(z(t))||_2^2||E||_2^2||\dot{z}(t-\tau)||_2^2\\&+2||A||_2||B||_2||g(z(t))||_2||g(z(t-\tau))||_2\\&+2||A||_2||\alpha^{-1}(z(t))||_2||E||_2\\&\times||g(z(t))||_2||\dot{z}(t-\tau)||_2\\&+2||B||_2||\alpha^{-1}(z(t))||_2||E||_2\\&\times||g(z(t-\tau))||_2||\dot{z}(t-\tau)||_2\\&-||\alpha^{-1}(z(t-\tau))||_2^2||\dot{z}(t-\tau)||_2^2\\&+k||g(z(t))||_2^2-k||g(z(t-\tau))||_2^2\end{aligned}$$

Assumption A_1 implies that

$$\frac{1}{M}\leq||\alpha^{-1}(z(t-\tau))||_2\leq\frac{1}{m}$$

and

$$\frac{1}{M}\leq||\alpha^{-1}(z(t))||_2\leq\frac{1}{m}$$

where $m=\min_{1\leq i\leq n}(m_i)$, $M=\max_{1\leq i\leq n}(M_i)$.

Assumption A_2 implies that

$$||\beta(z(t))||_2\geq\gamma||z(t)||_2$$

where $\gamma=\min_{1\leq i\leq n}(\gamma_i)$.

Assumption A_3 implies that

$$\frac{1}{L}||g(z(t))||_2\leq||z(t)||_2$$

where $L=\max_{1\leq i\leq n}(L_i)$. Thus, we obtain

$$||\beta(z(t))||_2\geq\frac{\gamma}{L}||g(z(t))||_2$$

Therefore, we can now write

$$\begin{aligned}\dot{V}(z(t))\;\leq\;&-\frac{\gamma^2}{L^2}||g(z(t))||_2^2+||A||_2^2||g(z(t))||_2^2\\&+||B||_2^2||g(z(t-\tau))||_2^2\\&+\frac{1}{m^2}||E||_2^2||\dot{z}(t-\tau)||_2^2\\&+2||A||_2||B||_2||g(z(t))||_2||g(z(t-\tau))||_2\\&+\frac{2}{m}||A||_2||E||_2||g(z(t))||_2||\dot{z}(t-\tau)||_2\\&+\frac{2}{m}||B||_2||E||_2||g(z(t-\tau))||_2||\dot{z}(t-\tau)||_2\\&-\frac{1}{M^2}||\dot{z}(t-\tau)||_2^2+k||g(z(t))||_2^2\\&-k||g(z(t-\tau))||_2^2\end{aligned}\tag{3}$$

We note the following inequalities :

$$\begin{aligned}&2||A||_2||B||_2||g(z(t))||_2||g(z(t-\tau))||_2\leq\\&||A||_2^2||g(z(t))||_2^2+||B||_2^2||g(z(t-\tau))||_2^2\end{aligned}$$

$$\begin{aligned}&2\frac{1}{m}||A||_2||E||_2||g(z(t))||_2||\dot{z}(t-\tau)||_2\leq\\&\frac{2}{\varepsilon}||A||_2^2||g(z(t))||_2^2+\frac{\varepsilon}{2m^2}||E||_2^2||\dot{z}(t-\tau)||_2^2\end{aligned}$$

$$\begin{aligned}&2\frac{1}{m}||B||_2||E||_2||g(z(t-\tau))||_2||\dot{z}(t-\tau)||_2\leq\\&\frac{2}{\varepsilon}||B||_2^2||g(z-\tau(t))||_2^2+\frac{\varepsilon}{2m^2}||E||_2^2||\dot{z}(t-\tau)||_2^2\end{aligned}$$

where ε is a positive constant. Using the above inequalities in (3) results in :

$$\begin{aligned}\dot{V}(z(t)) \;\leq\; & -\frac{\gamma^2}{L^2}||g(z(t))||_2^2+||A||_2^2||g(z(t))||_2^2 \\ & +||B||_2^2||g(z(t-\tau))||_2^2 \\ & +\frac{1}{m^2}||E||_2^2||\dot{z}(t-\tau)||_2^2+||A||_2^2||g(z(t))||_2^2 \\ & +||B||_2^2||g(z(t-\tau))||_2^2 \\ & +\frac{2}{\varepsilon}||A||_2^2||g(z(t))||_2^2 \\ & +\frac{\varepsilon}{2m^2}||E||_2^2||\dot{z}(t-\tau)||_2^2 \\ & +\frac{2}{\varepsilon}||B||_2^2||g(z-\tau)||_2^2 \\ & +\frac{\varepsilon}{2m^2}||E||_2^2||\dot{z}(t-\tau)||_2^2 \\ & -\frac{1}{M^2}||\dot{z}(t-\tau)||_2^2+k||g(z(t))||_2^2 \\ & -k||g(z(t-\tau))||_2^2 \\ = \; & (-\frac{\gamma^2}{L^2}+2(1+\frac{1}{\varepsilon})||A||_2^2)||g(z(t))||_2^2 \\ & +2(1+\frac{1}{\varepsilon})||B||_2^2||g(z(t-\tau))||_2^2 \\ & +(\frac{\varepsilon+1}{m^2}||E||_2^2-\frac{1}{M^2})||\dot{z}(t-\tau)||_2^2 \\ & +k||g(z(t))||_2^2-k||g(z(t-\tau))||_2^2\end{aligned}$$

Let

$$k=2(1+\frac{1}{\varepsilon})||B||_2^2$$

Then

$$\begin{aligned}\dot{V}(z(t)) \;\leq\; & (-\frac{\gamma^2}{L^2}+2(1+\frac{1}{\varepsilon})(||A||_2^2+||B||_2^2))||g(z(t))||_2^2 \\ & (\frac{\varepsilon+1}{m^2}||E||_2^2-\frac{1}{M^2}))||\dot{z}(t-\tau)||_2^2 \\ = \; & -\rho||g(z(t))||_2^2-\xi||\dot{z}(t-\tau)||_2^2\end{aligned}$$

Clearly, $\rho>0$ and $\xi\geq 0$ implies that $\dot{V}(z(t))<0$ for all $g(z(t))\neq 0$ (note that if $g(z(t))\neq 0$ then $z(t)\neq 0$). Now let $g(z(t))=0$. In this case $\dot{V}(z(t))$ is of the form :

$$\begin{aligned}\dot{V}(z(t)) \;\leq\; & -\gamma^2||z(t)||_2^2-\xi||\dot{z}(t-\tau)||_2^2 \\ \leq\; & -\gamma^2||z(t)||_2^2\end{aligned}$$

from which it follows that $\dot{V}(z(t))<0$ for all $z(t)\neq 0$. Now let $g(z(t))=z(t)=0$. We have hence

$$\dot{V}(z(t)) \;\leq\; -\xi||\dot{z}(t-\tau)||_2^2$$

$\xi>0$ implies that $\dot{V}(z(t))<0$ for all $\dot{z}(t-\tau)\neq 0$. If $g(z(t))=z(t)=\dot{z}(t-\tau)=0$, then

$$\dot{V}(z(t)) \;\leq\; -||B||_2^2||g(z-\tau)||_2^2$$

$\dot{V}(z(t))<0$, for all $g(z-\tau)\neq 0$ as $B\neq 0$. Therefore, $z(t)$ converges asymptotically to zero [29] and [35], hence meaning that the equilibrium point of neutral system (1) is asymptotically stable. On the other hand, the Lyapunov function used for the stability analysis is radially unbounded, it can be concluded that the equilibrium point of neutral system (1) is globally asymptotically stable.

We will compare our results with a previous stability given in [28] which is restated in the following theorem :

Theorem 2 [28] : For the neutral system defined by (2), assume that $||E||_2<1$. Then, the origin of (2) is globally asymptotically stable if the following condition holds :

$$\begin{aligned}\delta \;=\; & m\gamma-LM||A||_2(1+||E||_2) \\ & -LM||B||_2(1+||E||_2)-M\psi||E||_2>0\end{aligned}$$

where $m=\min\limits_{1\leq i\leq n}(m_i)$, $M=\max\limits_{1\leq i\leq n}(M_i)$, $\gamma=\min\limits_{1\leq i\leq n}(\gamma_i)$, $\psi=\max\limits_{1\leq i\leq n}(\psi_i)$, $L=\max\limits_{1\leq i\leq n}(L_i)$.

Remark : Note that the condition of Theorem 2 in [28] depends on the constant ψ while our result in Theorem 1 is expressed independently of ψ. Therefore, the result of Theorem 1 can be considered to be less conservative than the result of Theorem 2.

In order to show applicability and advantages of our results, we consider the following example :

Example 1 : Assume that the network parameters of neural system (1) are given as follows :

$$A=B=\frac{1}{\sqrt{2}}\begin{bmatrix} c & 0 \\ 0 & c\end{bmatrix}, E=\sqrt{2}\begin{bmatrix} c & 0 \\ 0 & c\end{bmatrix}$$

with $m=M=1$, $L=1$, $\gamma=\psi=1$, where c is a positive constant. We can calculate

$$||A||_2=||B||_2=\frac{c}{\sqrt{2}}, ||E||_2=\sqrt{2}c$$

For $\varepsilon=1$, applying the result of Theorem 1 to this example, we obtain

$$\rho=\frac{\gamma^2}{L^2}-2(1+\frac{1}{\varepsilon})(||A||_2^2+||B||_2^2)=1-4c^2>0$$

$$\xi=\frac{1}{M^2}-\frac{\varepsilon+1}{m^2}||E||_2^2=1-4c^2>0$$

Clearly, $c<\frac{1}{2}$ implies that $\rho>0$ and $\xi>0$. When checking the applicability of the condition of Theorem 2, one can see that the following condition must be satisfied

$$\begin{aligned}\delta \;=\; & m\gamma-LM||A||_2(1+||E||_2) \\ & -LM||B||_2(1+||E||_2)-M\psi||E||_2 \\ = \; & 1-\frac{2c}{\sqrt{2}}(1+\sqrt{2}c)-\sqrt{2}c \\ = \; & 1-2\sqrt{2}c-2c^2>0\end{aligned}$$

Hence, according to Theorem 2, for the network parameters given in this example, the sufficient condition for the stability of system (2) is obtained follows :

$$c<1-\frac{1}{\sqrt{2}}$$

Therefore, if

$$1-\frac{1}{\sqrt{2}}\leq c<\frac{1}{2}$$

then the results of [28] does not hold whereas the result of Theorem 1 is still applicable to this example. Thus, the result we obtained in Theorem 1 can be considered an alternative result to the previous stability result given in Theorem 2 of [28].

IV. Conclusions

By employing a simple and suitable Lyapunov functional, we have derived a new delay-independent sufficient condition ensuring the global asymptotic stability of a class of neutral-type neural networks with discrete time delays. The proposed condition establishes a relationship between the network parameters of the neural systems. The obtained result can be applied to Cohen-Grossberg neural networks, Hopfield-type neural networks and cellular neural networks. A constructive example also has been presented to show the advantages of our results over the previous literature results.

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