

An Analysis of Collapse Mechanism of Thin-Walled Circular Tubes Subjected to Bending

Somya Poonaya, Chawalit Thinvongpituk, and Umphisak Teeboonma

Abstract—Circular tubes have been widely used as structural members in engineering application. Therefore, its collapse behavior has been studied for many decades, focusing on its energy absorption characteristics. In order to predict the collapse behavior of members, one could rely on the use of finite element codes or experiments. These tools are helpful and high accuracy but costly and require extensive running time. Therefore, an approximating model of tubes collapse mechanism is an alternative for early step of design. This paper is also aimed to develop a closed-form solution of thin-walled circular tube subjected to bending. It has extended the Elchalakani et al.'s model (Int. J. Mech. Sci.2002; 44:1117-1143) to include the rate of energy dissipation of rolling hinge in the circumferential direction. The 3-D geometrical collapse mechanism was analyzed by adding the oblique hinge lines along the longitudinal tube within the length of plastically deforming zone. The model was based on the principal of energy rate conservation. Therefore, the rates of internal energy dissipation were calculated for each hinge lines which are defined in term of velocity field. Inextensional deformation and perfect plastic material behavior was assumed in the derivation of deformation energy rate. The analytical result was compared with experimental result. The experiment was conducted with a number of tubes having various D/t ratios. Good agreement between analytical and experiment was achieved.

Keywords—Bending, Circular tube, Energy, Mechanism.

I. INTRODUCTION

IN the crashworthiness design of vehicle and other structures, it is important to prevent and reduce the frequency of death and the severity of injuries in the event of an accident. Many researchers have been investigating the crushability and absorption capacity of some structures. They have been interested in studying the thin-walled structures such as shell, tubes, stiffeners and stiffened sandwich panels. These structures have been identified as a very efficient impact energy absorbing system and called "energy absorber". The study of deformation in energy absorbers accounts for geometrical shape, mode of collapse as well as strain hardening and strain rate effect. In general, there are several approaches to determine the energy absorption of structural members; by using finite element analysis, experiments and theoretical analysis. The finite element analysis and experiments approaches are not desirable because of high cost and extensive running time.

Therefore, the theoretical analysis of collapse mechanism is an alternative for the early step of design. This is a simple method to predict the collapse behavior and energy absorption

of such members. It is relatively simple, fast and economics prediction.

Theoretical analysis of the collapse can be made by using hinge line method. When thin-walled members are crushed by any load, the collapse strength is reached. Then, plastic deformations are occurred over some folding lines and are called "hinge lines". When hinge lines are completed around the structure, global or local collapse will progress. The internal energy dissipation in deformed structure is determined by plastic deformation along each hinge line. The sum of internal energy dissipation of each hinge lines is equal to the external energy.

Many attempts have been made to understand and predict the crushing behavior and energy absorption characteristics of thin-walled tubes subjected to pure bending. D. Kecman., [1] studied the deep bending collapse of thin-walled rectangular columns and proposed a simple failure mechanism consisting of stationary and rolling plastic hinge line. The analytical solution was achieved using limit analysis techniques. L.C. Zhang and T.X. Yu., [2] studied the ovalisation of a tube with an arbitrary cross section and one symmetric plane to obtain a full moment-curvature response. Their analysis shows that the flattening increases nonlinearly as the longitudinal curvature increases up to a limiting maximum value. T. Wierzbicki and S.U. Bhat., [3] derived a closed-form solution for the pressure necessary to initiates and propagates a moving hinge on the tube. The calculations were performed using a rigid-plastic material and a simple moving hinge model was assumed to occur along hinge line. The deformation of a ring was modeled into a "dumbbell" shape. The analytical results agreed well with the experiments. T. Wierzbicki and M.S. Suh., [4] conducted a theoretical analysis of the large plastic deformations of tubes subjected to combined load in the form of lateral indentation, bending moment and axial force. The model is effectively decoupling the 2-D problem into a set of 1- D problems. The theoretical results gave good correlation with existing experimental data. A.G. Mamalis et. al., [5] studied the effect of shear on collapse characteristics during the bending of cylindrical tubes. Theoretical and experimental results were found to be in good agreement. S.J. Cimpoeu and N.W. Murray., [6] presented empirical equations of the moment-rotation relation of a square thin-walled tube subject to pure bending collapse where the width-to-thickness ratio less than 26. Results from the empirical model were compared with the analytical model of Kecman [1]. T. Wierzbicki et. al., [7] studied the bending collapse mechanism of thin-walled prismatic columns by the concept of the basic folding mode. They developed the collapse mechanism by adding the toroidal and rolling deformation in the compressive model. Close-form solutions were derived for the moment-rotation

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characteristic of square column in the post failure range. The stress profiles in the most general case of a floating neutral axis were also shown. The simplified analytical solution was shown to predict the moment-rotation relationship with an absolute error not greater than 7%. T. Wiezbriki and Simmao.,[8] studied the simplified model of circular tube in pure bending, which was valid for large and very large sectional distortion. Good agreement with numerical solution (ABAQUS) was obtained. T.H.Kim and S.R.Reid., [9] modified the mechanism model of Wiezbricki el al.1994.[7] ,which was suggested that the toroidal deformation and conical rolling should be defined differently from the case of axial compression to satisfy the bending kinematics condition. Good agreement was found between the mechanism model and the experiment. M.Elchalakani el al., [10] predicted the response of a circular steel tube under pure bending. They included the effect of ovalisation along the length of the tube into the model. Work dissipated through the toroidal and the rolling hinges was ignored. The hinge mechanism was assumed straight and inextension deformation. Good agreement between analytical result and experiment was achieved. M.Elchalakani el al., [11] presented a closed-form solution of the post-buckling collapse of the slender circular hollow section with $D/t > 85$ subjected to pure bending. The theoretical analysis closely matches with the experimental results.

The main objective of this paper is to develop a close-form solution of thin-walled circular tube subjected to bending using a rigid plastic mechanism analysis. It has extended Elchalakani's model [10] to include the rate of energy dissipation of rolling hinge in the circumferential direction. The model is based on the principal of energy rate conservation and is analyzed in the 3-D problem by adding the longitudinal hinges in plastic zone and has the half folding length H . The rolling radius r and the folding length H are determined by minimizing the mean crushing moment M_m with respect to those two parameters. The theoretical model is then presented and then resolved in the form of moment-rotation characteristics. The model is then compared with experiments.

II. ANALYTICAL MODEL

A. Phases of Collapse Mechanism

In general the collapse mechanism of a tube subjected to bending can be divided into three phases: i.e.; elastic behavior, ovalisation plateau and structural collapse. The analytical model of each phase is derived as explained;

1) Elastic behavior

In this phase, the moment increases linearly with constant slope up to a yield moment-rotation. The elementary theory of elasticity is used to predict the linear moment-rotation characteristic of a circular tube and is defined as;

$$M_y = \frac{2\sigma_y I}{D_0} \quad (1)$$

$$\theta_y = \frac{M_y L_0}{EI} \quad (2)$$

where M_y is the yield moment, L_0 is the pure moment length, E is the elastic modulus, I is the second moment of area, σ_y is the measured yield stress, θ_y is the yield rotation angle and D_0 is the outside diameter of the tube.

2) Ovalisation plateau

In phase 2, the ovalisation of the initially circular cross-section of tube subjected to bending is obvious and the material exhibits slight hardening. The bending moment is generally assumed fairly constant during the increment of bending rotation. The ultimate moment of an ovalised tube and the corresponding angle of rotation [10] can be written as

$$M_{ovalised} = S_{ovalised} \sigma_y = \frac{4}{3} (R_v^2 R_h - R_{vi}^2 R_{hi}) \sigma_y \quad (3)$$

where $S_{ovalised}$ is the plastic section modulus of an ovalised tube, σ_y is the measured yield stress of an ovalised tube.

$R_h = \frac{D_h}{2} = 0.55D_0$ and $R_v = \frac{D_v}{2} = 0.45D_0$ are the external horizontal and vertical radii of an ovalised tube, respectively. The internal horizontal and vertical radii are $R_{hi} = (R_h - t)$ and $R_{vi} = (R_v - t)$, respectively. The geometry of D_h , D_v , and t can be found in Fig. 1.

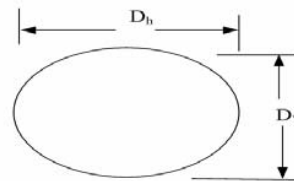


Fig. 1 Ovalisation of the cross-section of tube in phase 2

3) Structural collapse

In phase 3, the structure starts to collapse resulting load carrying capacity decrease rapidly. The plastic folding is deformed in the plastic zone, as shown in Fig. 2. The model involves the flattened hinge in the circumferential cross-section and the oblique hinge lines along the longitudinal of tube, as shown in Fig. 2 (a) and (b) respectively.

The traveling hinges of flattened region in the circumferential cross-section in Fig. 2 (a) were modeled as a line connecting points A and B. The rolling hinge has radius r and circular arc of current radius R_1 . Fig. 2 (b) and (c) show deformation of the four oblique hinge lines along the longitudinal tube within the length of plastic zone, H .

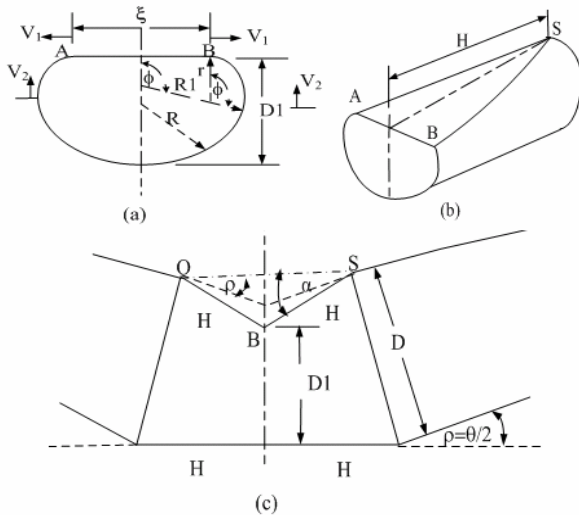


Fig. 2 Plastic collapse model of thin-walled circular tube subjected to pure bending

B. Assumptions

The assumptions made in the theoretical mechanism of thin-walled circular tube subjected to pure bending are:

1. The tube material is ductile, rigid-perfectly plastic, isotropic, homogeneous and material compatibility condition is maintained.
2. The tube circumference is inextensible.
3. Shear deformation and twist of the deformed tube are neglected.
4. The collapse hinge mechanism formed in the tube is cinematically admissible and the cross section of the tube deforms in a simplified manner as shown in Fig. 2.
5. All hinges are assumed straight. Work dissipation through the toroidal region is ignored, but the rolling hinges in the deformed cross-section is considered
6. The tube's cross section radius (R) at the beginning of the plastic hinge formation is the initial mean radius of the tube.
7. The tube does not elongate or contract in the axial direction.
8. The parameter H and r are constant during the collapse of section.

C. Geometrical Analysis

The kinematics of the collapse mechanism of the thin-walled circular tube subjected to bending is defined and the global geometry of collapse shown in Fig. 2.

The inextensible condition in circumferential direction of half circular cross-section of tube is:

$$\xi + r\phi + R_1(\pi - \phi) = \pi R \quad (4)$$

where R is the mean radius of circular tube, R_1 is the radius of deformed cross-section, ξ is the length of hinge line, AB in Fig. 2(a) and ϕ is the mechanism angle define in Fig. 2(a).

The vertical displacement D_I of circular cross-section, in Fig. 2 (a) is defined as;

$$D_I = R_1(1 - \cos \phi) + r(1 + \cos \phi) \quad (5)$$

And in Fig. 2 (c)

$$D_I = 2R \cos \rho - H \sin \alpha \quad (6)$$

where $\alpha = \frac{\pi}{2} - \arcsin\left(1 - \frac{2R}{H} \sin \rho\right)$, $\rho = \theta/2$, and θ is the bending rotation at the end of tube.

Equation (5) and equation (6) are rewritten in terms of mechanism angle ϕ and the bending rotation ρ ,

$$\frac{1 - \cos \phi}{\phi + \sin \phi} = \frac{2R \cos \rho - H \sin \alpha - 2r}{\pi(R - r)} \quad (7)$$

The velocity of hinge propagation, in fig. 2(a) is expressed as;

$$V_1 = \frac{d\xi}{dt} = \dot{\xi}, \quad V_2 = \frac{dR_1}{dt} = \dot{R}_1 \quad (8)$$

III. PLASTIC ENERGY DISSIPATION

The rate of internal energy dissipation resulting from continuous and discontinuous deformation rate fields is defined as:

$$\dot{W} = \int_S (M_{\alpha\beta} \dot{\kappa}_{\alpha\beta} + N_{\alpha\beta} \dot{\epsilon}_{\alpha\beta}) dS + \sum_{i=1}^n \int_{l^i} M_n^i [\dot{\psi}_i] dl^i \quad (9)$$

where S denotes the current shell mid surface, n is the total number of plastic hinge lines, l^i is the length of the i^{th} hinge, and $[\dot{\psi}_i]$ denotes a jump of the rate of rotation across the moving hinge line. The components of the rate of rotation and the rate of extension tensors are denoted as $\dot{K}_{\alpha\beta}$ and $\dot{\epsilon}_{\alpha\beta}$, respectively. The corresponding conjugate generalized stresses are the bending moment $\dot{M}_{\alpha\beta}$ and the membrane forces $\dot{N}_{\alpha\beta}$, $M_n = \left(\frac{2}{\sqrt{3}}\right) \frac{\sigma_0 t^2}{4}$ is the plane strain plastic

bending moment (per unit length) normal to the hinge line. All components are expressed in the cylindrical coordinate system on the surface of the shell.

There are 6 important components in the first integration term of (9) i.e. $\dot{\epsilon}_{xx}$, $\dot{\epsilon}_{x\theta}$, $\dot{\epsilon}_{\theta\theta}$, \dot{K}_{xx} , $\dot{K}_{x\theta}$, and $\dot{K}_{\theta\theta}$. Each of these components will be considered individually.

Regarding to the assumption of circular tube subjected to pure bending, for shear deformation and twist of the deforming shell element are neglected, resulting $\dot{\epsilon}_{\alpha\theta} = \dot{K}_{\alpha\beta} = 0$. The tube circumference is inextensible, hence $\dot{\epsilon}_{\theta\theta} = 0$. Later, the local change in the axial curvature is assumed to be small compared to the change in the circumferential curvature, resulting $\dot{K}_{xx} < \dot{K}_{\theta\theta}$ and thus \dot{K}_{xx} will be neglected. Finally, the axial strain rate is

zero, $\dot{\epsilon}_{xx} = 0$, due to assumption 7. Hence, the expression for the rate of internal energy dissipation of (9) can be reduced to:

$$\dot{W} = \int_s (M_{\theta\theta} \dot{\kappa}_{\theta\theta}) ds + \sum_{i=1}^n \int_{L^i} M_n^i [\dot{\psi}_i] dl^i \quad (10)$$

where $M_{\theta\theta} = \sigma_{\theta\theta} \left(\frac{t^2}{4} \right)$ is the plastic bending moment.

The remaining two terms in (10) are corresponding to the rate of energy of shape distortion or lateral crushing of the tube:

$$\dot{W}_{crush} = \int_s (M_{\theta\theta} \dot{\kappa}_{\theta\theta}) ds + \sum_{i=1}^n \int_{L^i} M_n^i [\dot{\psi}_i] dl^i \quad (12)$$

where the plastic bending moment is $M_{\theta\theta} = \frac{\sigma_{\theta\theta} t^2}{4}$, $\sigma_{\theta\theta}$ is the circumferential stress and t is the thickness of tube, The rate of curvature, $\dot{\kappa}_{\theta\theta}$ is defined by,

$$(\dot{\kappa}_{\theta\theta})_A = -\frac{\dot{R}_1}{R_1^2} \quad (\dot{\kappa}_{\theta\theta})_B = -\frac{\dot{r}}{r^2} \quad (13)$$

If the rolling hinge, r and R_1 are assumed unchanged along the rotational angle θ , the rate of curvature is equal to zero; $\dot{\kappa}_{\theta\theta} = 0$.

By using the assumptions and geometrical analysis above, the energy dissipation in the tube can be derived as followed.

A. Rate of Energy Dissipation in the Crushing of Rings

Since $\dot{\kappa}_{\theta\theta} = 0$; (12) can be reduced to;

$$\dot{W}_{crush} = \sum_{i=1}^n \int_{L^i} M_n^i [\dot{\psi}_i] dl^i \quad (14)$$

where M_n is the bending moment normal to the hinge line and the rate of rotation $\dot{\psi}$ at the moving hinge line is calculated from the condition of kinematic continuity:

$$\dot{\psi} = V[K] \quad (15)$$

where V is the velocity of the traveling hinge in the tangential direction, and K is the circumferential curvature. $[K] = K^+ - K^-$ is the jump in curvature from either side of the hinge line. Ahead of the hinge $K^+ = 1/r$ and behind the hinge $K^- = 0$ (flat sections), the rate of rotation at the hinge is

$$\dot{\psi}_1 = V_A \left[\frac{1}{r} \right]; \quad \dot{\psi}_2 = V_B \left[\frac{1}{r} - \frac{1}{R_1} \right] \quad (16)$$

where V_A and V_B are tangential velocities in the current deformed configuration of the ring in Fig. 2(a) and are given by

$$V_A = \frac{d\xi}{dt} = \dot{\xi}; \quad V_B = \frac{-dR_1(\pi - \phi)}{dt} \quad (17)$$

The bending moment normal to the hinge line M_n is assumed to be equal to the circumferential bending moment on the continuous deformation region, $M_n = M_{\theta\theta} = \frac{\sigma_{\theta\theta} t^2}{4}$

An approximate yield condition [7] for the present problem is given in (18)

$$\left(\frac{M_{\theta\theta}}{M_0} \right)^2 + \left(\frac{N_{xx}}{N_0} \right)^2 = 1 \quad (18)$$

Thus, the plastic compressive-tensile stress σ_{xx} in axial direction resulting from bending is coupled with bending-induced hoop stress $\sigma_{\theta\theta}$ through an inscribed yield condition, thus to one point with the coordinate,

$$\frac{\sigma_{\theta\theta}}{\sigma_0} = \frac{M_{\theta\theta}}{M_0} = \frac{1}{\sqrt{2}}, \quad \frac{\sigma_{xx}}{\sigma_0} = \frac{N_{xx}}{N_0} = \frac{1}{\sqrt{2}} \quad (19)$$

The rate of energy dissipation in the crushing mode can be rewritten by substituting (15), (16), (17), and (19) in (14):

$$\dot{W}_{crush} = \frac{M_0 t}{16\sqrt{2}R^2} H \left((R_1 - r) \cos\phi \left[\frac{1}{r} \right] + R_1 \left[\frac{1}{r} - \frac{1}{R_1} \right] \right) \dot{\phi} \quad (20)$$

where $M_0 = \int_A \sigma_0 z dA = 4\sigma_0 R^2 t$ is the fully plastic bending moment of the undeformed cross-section.

B. Rate of Energy Dissipation over the Central Hinge AB

Energy is dissipated in the central hinge line (AB). Its length varies along the angle α and the rotation angle $\rho = \theta/2$ (Fig. 2)

$$\dot{W}_{AB} = \sum_i M_n l_i \dot{\psi}_i = \frac{M_0 t}{8\sqrt{2}R^2} (\xi) \dot{\alpha} \quad (21)$$

Where i is the number of hinge lines, $l_i = \xi$ is the length of hinge line and $\dot{\psi} = 2\dot{\alpha}$ is the rate of rotation angle of hinge lines.

C. Rate of Energy Dissipation over the Oblique Hinge Lines SA, SB, OA, and OB

The plastic energy are dissipated over the four oblique hinge lines SA, SB, OA and OB, as shown in Fig. 2 (b). They are easily calculated as;

$$\dot{W}_{OB} = \sum_i M_n l_i \dot{\psi}_i = \frac{M_0 t}{4\sqrt{2}R^2} \dot{\xi} \quad (22)$$

where $i = 4$ is the number of hinge lines, $l_h = \sqrt{H^2 + \xi^2}$ is the length of each hinge line and $\dot{\psi} = \frac{V_A}{l_h} = \frac{\dot{\xi}}{l_h}$ is the rate of rotation angle of each hinge line, which $\dot{\psi}$ is assumed to vary along the tangential velocity V_A of the moving hinge AB.

D. Instantaneous Moment

The global energy balance of a single structural element can be established by equating the rate of energy dissipation and the rate of external work.

$$\dot{W}_{ext} = \dot{W}_{int} \quad (23)$$

where \dot{W}_{ext} is the rate of external work, and \dot{W}_{int} is the instantaneous rate of energy dissipation at the plastically deformed region.

The rate of total internal energy dissipated, \dot{W}_{int} is,

$$\dot{W}_{int} = \dot{W}_{crush} + \dot{W}_{AB} + \dot{W}_{OB} \quad (24)$$

$$\text{External rate of energy dissipation is } \dot{W}_{ext} = M\dot{\theta} \quad (25)$$

The instantaneous moment can be determined by substituting (24) and (25) in (23).

$$M = M(H, r, \theta) \quad (26)$$

The length of the plastic folding region H and the rolling radius r can be determined from the postulate of minimum total plastic work.

$$\frac{dM}{dr} = 0 \text{ and } \frac{dM}{dH} = 0 \quad (27)$$

The value of H and r are substituted into (26) to give the expression for the instantaneous moment in terms of the rotation angle θ :

$$M = M(\theta) \quad (28)$$

Details of each moment component are presented in Appendix.

IV. EXPERIMENTS

In order to verify the model, the experiment was conducted with 6 tubes (UB1 to UB6) of mild steel of different diameter to thickness ratios. The nominal diameter to thickness ratios ranges from 21.16 to 42.57. The length of each specimen is 1500 mm. The material properties are determined by using the tensile coupons tested according to the British Standard BSEN 10 002-1:1990 [12]. Results from tensile tests are shown in Table I.

TABLE I
DIMENSION AND MECHANICAL PROPERTIES OF TEST SPECIMENS

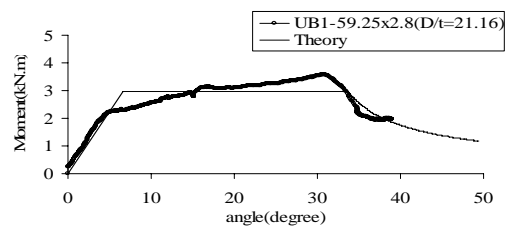
Specimen	Diameter (mm)	Thickness (mm)	D/t	Yield Stress σ_y (MPa)	Ultimate Stress σ_U (MPa)
UB1	59.25	2.80	21.16	353	383
UB2	59.00	2.30	25.65	289	344
UB3	46.85	1.80	26.03	342	451
UB4	59.35	1.80	32.97	374	389
UB5	58.55	1.60	36.59	270	350
UB6	74.50	1.75	42.57	321	428

The experimental setup was designed to obtain a pure bending moment over the middle span of the specimen. The

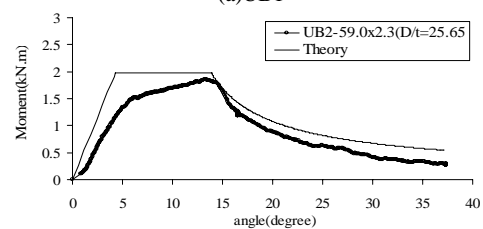
influence of shear and axial forces should be avoided or minimized as much as possible. To meet this requirement, S.J. Cimpoeu [6] introduced a machine that able to apply a pure bending moment without imposing shear and axial forces. A machine based on that similar concept has been built at Ubonratchathani University to apply a pure bending test on those 6 specimens.

V. RESULTS AND DISCUSSION

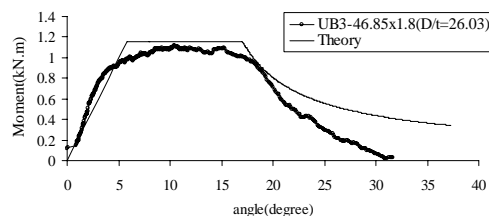
The comparison of analytical and experimental results is shown in Fig. 3(a) to (f) in the form of moment-rotation curve. In phase 1, the moment increases linearly with constant slope up to a point where the yield is. Then, phase 2 (Ovalisation) starts. In the ovalisation phase, the moment is constant, following equation (3) until a certain angle. It should be noted that the angle where to complete the ovalisation is still mysterious and will be investigated further. The present paper predicts this angle from experimental observation and shifts the curve of phase 3 to the ovalisation-end position. For phase 3, structural collapse, it starts with a curve that delay parabolically and following equation (28). In this phase, energy dissipations in the crushing rings, the central hinge and the oblique hinge lines are involved as explained in equation (14), (21) and (22) consequently. In general, the analytical model provides similar results with experiment. The discrepancy between experiment and analytical may be attributed to some assumptions and the imperfection of experimental specimens.



(a)UB1



(b)UB2



(c)UB3

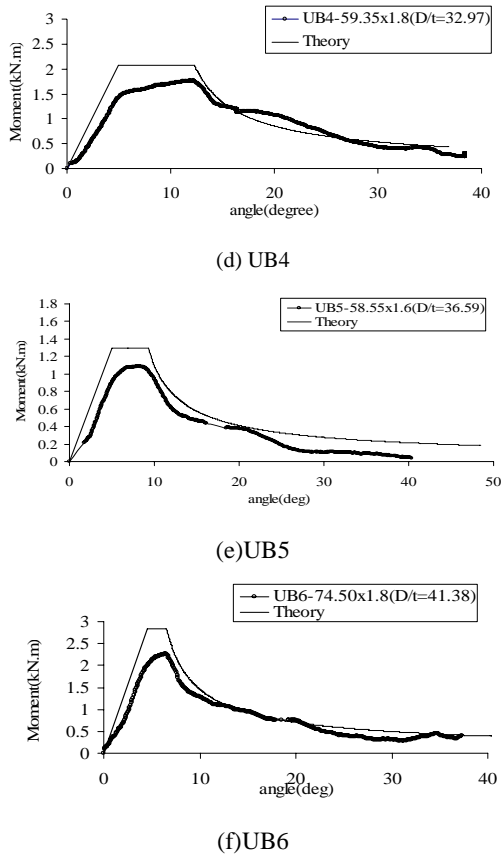


Fig. 3 Comparison between the experimental results and theoretical results

VI. CONCLUSION

This paper proposed an approximated method to predict the closed-form solution of the plastic collapse mechanism of thin-walled circular tube subjected to bending. The model is based on a rigid plastic mechanism analysis and the rate of energy dissipation in term of the velocity field. The collapse model includes the deformed cross-section, each consisting of four traveling hinges and four oblique hinge lines along the longitudinal tube. It is postulated that there are three phases for the collapse behavior of thin-walled circular tube subjected to bending, namely; elastic behavior, ovalisation plateau and structural collapse. This analytical model will be extended to calculate the critical angle of rotation at the formation of plastic hinge between the ovalisation plateau and the structural collapse phases in future publication. The prediction shows good agreement over a wide range of experimental data.

APPENDIX

The following are the details of the moment components given in (28). These formulations were calculated by using Mathcad, [13].

$$M_1 = \left(\frac{-1}{4} \cdot \frac{(-R+r)}{r} \cdot M_o \cdot \frac{\pi}{R} \cdot t \cdot \sqrt{2} \cdot \phi \right) \theta$$

is a moment component for the crushing of ring.

$$M_2 = \left(\frac{5.9 \cdot 10^{-2} \cdot \frac{(-R+r)^2}{R^2} \cdot H}{\theta} \cdot M_o \cdot t \cdot \phi^3 \right)$$

is a moment component for the central hinge.

$$M_3 = \left(\frac{1}{32} \cdot \frac{\sqrt{2}}{R^2} (R-r) \cdot M_o \cdot t \cdot \phi \right) \theta$$

is a moment component for the oblique hinge.

ACKNOWLEDGMENT

Authors wish to thank the Department of Industrial Engineering, Ubonratchathani University for support of the test rig.

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