An Algorithm for an Optimal Staffing Problem in Open Shop Environment

Daniela I. Borissova, and Ivan C. Mustakerov

Abstract—The paper addresses a problem of optimal staffing in open shop environment. The problem is to determine the optimal number of operators serving a given number of machines to fulfill the number of independent operations while minimizing staff idle. Using a Gantt chart presentation of the problem it is modeled as two-dimensional cutting stock problem. A mixed-integer programming model is used to get minimal job processing time (makespan) for fixed number of machines' operators. An algorithm for optimal open-shop staffing is developed based on iterative solving of the formulated optimization task. The execution of the developed algorithm provides optimal number of machines' operators in the sense of minimum staff idle and optimal makespan for that number of operators. The proposed algorithm is tested numerically for a real life staffing problem. The testing results show the practical applicability for similar open shop staffing problems.

Keywords—Integer programming, open shop problem, optimal staffing.

I. INTRODUCTION

THERE are many examples in which logistics systems modeling plays a critical role in determining improved methods for organization management. Management comprises planning, organizing, staffing, leading or directing, and controlling an organization. Staffing function is one of the most important managerial acts along with planning, organizing, directing and controlling. In many situations, it is needed to consider optimization of resources, whether they come in the form of materials and inventory, or in terms of people and staff. Efficient and effective management of staff (stuffing) is a prerequisite for competitiveness and overall organizational performance. Inefficient or ineffective staff management, on the other hand, will almost inevitably lead to lower levels of quality and productivity [1]. Human resources can be efficiently managed by a proper procedure for recruitment and selecting the candidates as per the job requirements. Very often, determining staffing needs depends on the subjective and rule of thumb judgment of an experienced manager or with the aid of spreadsheets [2]. An optimal staffing allows the organization to work at its efficient best. The staffing concerns problems of maximizing the utilization of manpower to perform a set of activities in order to achieve a certain goal [3]. There exist countless industry specific variations on staffing problems and among them

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staffing in open-shop environment is quite common in industry and manufacturing.

Open shop problems are at the core of many scheduling problems involving unary resources (machines or jobs), which have received an important amount of attention because of their wide range of applications [4]-[6]. In a classical open shop, a set of n jobs has to be processed on M machines. Every job consists of operations, each of which must be processed on a different machine for a given processing time [7]. The operations of each job can be processed in any order and the jobs are independent. At any time, at most one operation can be processed on each machine, and at most one operation of each job can be processed.

In this paper, a problem of open shop optimal staffing is investigated. The objective is to minimize the staff idle time in open shop environment subject to job completion time. This is investigated as a combinatorial optimization problem. The innovative approach proposed in the paper is to interpret that problem as a two-dimensional cutting and packing problem [8]-[13]. In general, two-dimensional cutting and packing problems consist of placing (without overlap) a given set of items into one object so as to minimize a given objective function. The paper proposes an algorithm for optimal staffing in open shop environment based on cutting stock modeling optimization.

The remainder of the paper is organized as follows. The problem specific is described in Section II. The mathematical modeling is given in Section III. In Section IV an optimal staffing algorithm in open shop environment is described. The computational results are presented in Section V and the paper is closed by the conclusions in Section VI.

II. PROBLEM STATEMENT

Investigated open shop staffing problem addresses a single job consisting of given number of operations that have to be processed on a number of machines. Each operation is assigned to a particular machine with known processing time. Each machine is served by a known number of operators. The operations of the job can be processed in any order in sequence or in parallel. The processing times are independent of the processing sequence. There is only one of each type of machine and the machines operators have competence to serve all of the machines. The corresponding to this problem input data (operations, machines, processing times and operators) are shown on Table I.

TABLE I
PROBLEM INPUT DATA

Input data	Operation 1 on Machine 1	Operation 2 on Machine 2	…	Operation m on Machine m
Task processing time	h_1	h_2	…	h_m
Number of operators serving the machine	w_1	w_2	…	w_m

The problem is to determine the optimal number of operators needed to complete all of the job operations in regard to minimal staff idle. The described problem specific can be visualized as a Gantt type chart [14] shown on Fig. 1.
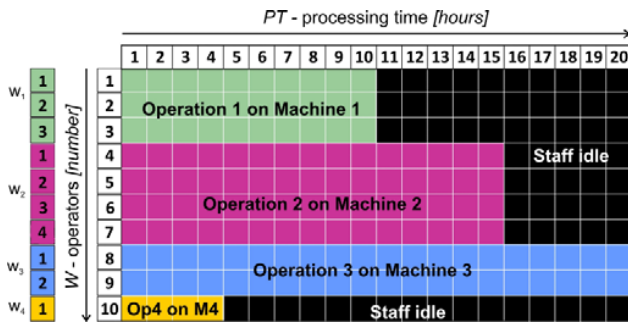

Fig. 1 Gantt type chart of investigated open shop staffing problem

Each operation assigned to a particular machine is illustrated in Fig. 1 as a rectangle with length – processing time and width – number of operators serving that machine. It can be seen from Fig. 1 that the graphical representation of the investigated problem resemble on two-dimensional cutting stock problem with fixed orientation of rectangles where the waste (staff idle) and overall length (processing time) are to be minimized. Along with this there exists a difference from the classical cutting stock problem that consists in variable width of the strip (operators number). The number of all operators serving machines should be optimized to get minimal staff idle. The innovative contribution of the paper is that it models the open shop staffing problem by means of cutting stock problems approach. Based on this model mixed-integer linear programming task is formulated and used in an algorithm for optimal staffing in open shop environment.

III. MATHEMATICAL MODEL FORMULATION

Using a mathematical model make it possible to evaluate various staffing policies subject to personnel and machines constraints. A mixed-integer programming model to address this aspect of the problem is developed based on cutting stock modeling. The corresponding notations are as follows:

PT = overall job processing time (makespan),
PT_{min} = minimal job processing time,
PT_{max} = maximal job processing time,
M = number of machines,
W = number of operators to serve the machines,
SA = $W*PT$ = strip area in Gantt diagram,
J = {1, 2, …, M} = set of operations' indexes,
h_j, $j \in$ J, = operation's processing time on j-th machine,

w_j, $j \in$ J = number of operators serving j-th machine,
$W_{min} = max\ \{w_j\}$ = minimal number of operators to serve the machines (for sequential processing scenario),
$W_{max} = \Sigma\ w_j$ = maximal number of operators to serve the machines (for parallel processing scenario),
$OA = \Sigma\ w_j\ h_j$ = all operations area in Fig.1,
$SI = SA - OA$ = staff idle,
l_{jm} = 0/1-variable; equal to 1 if j-th rectangle is located to the left of the m-th rectangle or 0 otherwise,
b_{jm} = 0/1-variable; equal to 1 if j-th rectangle is located below the m-th rectangle or 0 otherwise,
x_j, y_j = coordinates of left upper corner of the j-th rectangle representing j-th operation on j-th machine in Fig. 1.

The formulation of the mixed integer optimization task is stated as:

$$\min PT \tag{1}$$

subject to

$$PT \geq PT_{min} \tag{2}$$

$$PT \leq PT_{max} \tag{3}$$

$$x_j \geq 0,\ integer,\ j \in J \tag{4}$$

$$y_j \geq 0,\ integer,\ j \in J \tag{5}$$

$$x_j + h_j \leq PT \tag{6}$$

$$y_j + w_j \leq W \tag{7}$$

$$l_{jm} + l_{mj} + b_{jm} + b_{mj} = 1 \tag{8}$$

$$x_j + h_j \leq x_m + (1 - l_{jm})PT_{max} \tag{9}$$

$$y_j + w_j \leq y_m + (1 - b_{jm})W \tag{10}$$

$$SA = W*PT \tag{11}$$

$$SI = SA - OA \tag{12}$$

where

$$PT_{min} = \max\{h_j\} \tag{13}$$

$$PT_{max} = \max \sum_{j=1}^{J} h_j \tag{14}$$

$$OA = \sum_{j=1}^{J} w_j h_j \tag{15}$$

In the current modeling approach each operation is represented as a rectangle in a Gantt type chart (Fig. 1) with processing time in hours as x axis and machines' operators number as y axis. The objective function (1) minimizes the overall job processing time PT (makespan). The variable PT is limited by its minimal and maximal values in (2) and (3), that can be defined as PT_{min} equal to the largest operation's processing time (for parallel processing scenario) and PT_{max} equal to the sum of all operations processing times (for sequential processing scenario). The coordinate x_j, y_j of left upper corner of the j-th rectangle represents start of j-th operation on j-th machine. Equations (6) and (7) ensure that all rectangles are in the strip area while (8) expresses the fact that a particular rectangle can only be in one position to another rectangle – left, right, top or bottom. Equations (9) and (10) mean that no rectangles overlap each other. The solution of the task (1) – (12) defines minimal job processing time PT for the

given W. Since the total operations area OA (12) is constant and the total strip area SA can be calculated by (11), the staff idle SI for that number of operators W can be evaluated by (12).

This model definition assumes that operators number W has a preliminary fixed constant value. To define optimal number of operators W in regard to minimal staff idle, the defined optimization task (1) – (12) should be solved for all feasible operators' number combinations. Then the combination corresponding to minimal staff idle can be dtermined. For the goal an iterative algorithm based on multiple solving of optimization task is proposed.

IV. Algorithm for Optimal Staffing in Open Shop Environment

The flowchart of the optimal staffing algorithm in open shop environment is presented in Fig. 2.
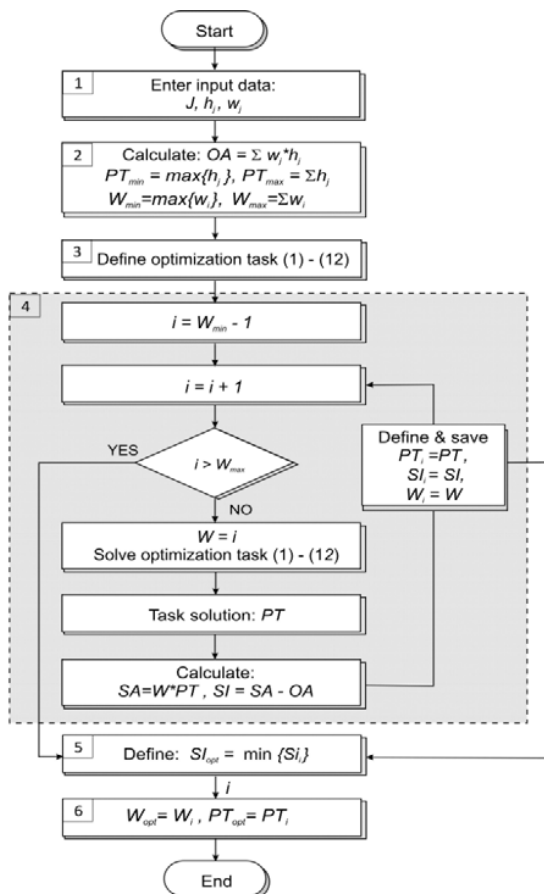

Fig. 2 Flowchart of optimal staffing algorithm

Applying this algorithm the optimal number of operators in regard to minimal staff idle is defined as follows:

Step 1: Define
- set of indexes J of operations and corresponding machines,
- processing times h_j of each operation j on j-th machine,
- number of needed operators w_{j-} to process operation j on j-th machine.

Step 2: Calculate
- summary area of all operations in Gantt type chart (Fig. 1) as $OA = \Sigma\ w_j h_j$,
- minimal feasible job processing time $PT_{min} = \max\{PT_j\}$ as the largest processing time of the various operations (for parallel processing),
- maximal feasible job processing time $PT_{max} = \Sigma PT_j$ as sum of all operations processing times (for sequential processing),
- minimal operators number $W_{min} = \max\{w_j\}$, as largest operators' number of the various operations (for sequential processing),
- maximal operators number $W_{max} = \Sigma w_j$ as sum of operators for all operations (for parallel processing).

The calculations of minimal and maximal values of PT and W on Step 2 are based on two boundary scenarios for job completion – execution of all operations in parallel or sequentially.

Step 3: Mathematical modeling of two-dimensional cutting stock problem and definition of optimization task formulation (1) – (15).

Step 4: The formulated optimization task is solved repeatedly with different numbers of operators W. The process starts with W_{min} then W increments by 1 and repeats until W_{max} is reached. After each task solution SA and SI are calculated and together with the optimal PT value are saved as SA_i, SI_i, PT_i, where $i \in \{ W_{min} , W_{min} + 1, ..., W_{max} \}$ is index of each particular optimization task.

Step 5: Define
- $SI_{opt} = \min \{ SI_i \}$,
- index i corresponding to SI_{opt}

Step 6: Define
- $W_{opt} = W_i$
- $PT_{opt} = PT_i$

The final result of algorithm execution is optimal number of operators W_{opt} for the investigated problem and optimal job processing time PT_{opt} for this number of operators.

V. Computational Results and Discussion

The proposed optimal staffing approach is numerically tested for a real manufacturing problem with job processing data shown in Table II.

TABLE II
REAL MANUFACTURING PROBLEM DATA

	Operation 1 on Machine 1	Operation 2 on Machine 2	Operation 2 on Machine 2	Operation 4 on Machine 4
Task processing time	10	15	20	4
Number of operators serving the machine	3	4	2	1
Number of machines	1	1	1	1

In accordance with Step 4 of the described iterative algorithm, the optimization task (1) – (15) must be solved sequentially for all feasible numbers of machines' operators i.e. for $W = W_{min}$, $W = W_{min} + 1$, $W = W_{min} + 2$, etc., to $W = W_{max}$, where $W_{min} = 4$ and $W_{max} = 10$.

The corresponding numerical results of task solutions are shown in Table III and the graphical representation of the results is illustrated on Fig. 3.

TABLE III
NUMERICAL RESULTS AFTER ALGORITHM COMPLETION

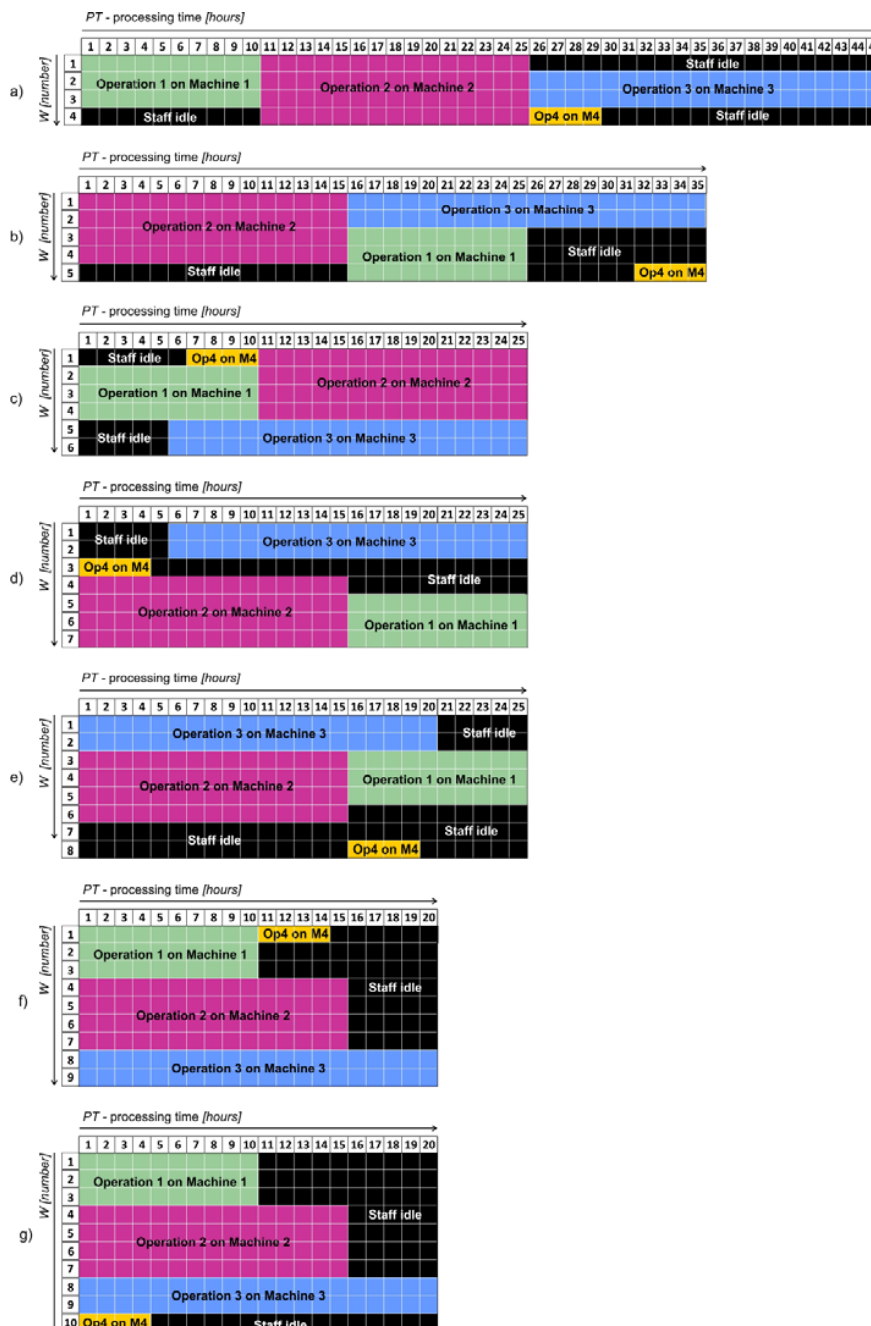| Task #	Operators W, number	Job processing time PT, hours	Staff idle SI, man-hours
4	4	45	46
5	5	35	41
6	6	25	16
7	7	25	41
8	8	25	66
9	9	20	46
10	10	20	66

Fig. 3 Graphica representation of results after algorithm completion

In Fig. 3 the case (a) represents scenario of sequential operations execution with minimal number of operators $W = W_{min}$ and maximal job processing time $PT = PT_{max}$. The case (g) conforms to scenario of parallel operations processing with maximal number of operators $W = W_{max}$ and minimal job processing time $PT = PT_{min}$. These are the boundary cases of the investigated optimal staffing problem.

As it is seen from Table III and Fig. 3 (c) the optimal number of operators is $W = 6$ corresponds to minimal staff idle $SI = 16$. Although the job processing time PT is not minimal compared to case (g), it is optimal for this particular number of operators W.

The computational difficulty of most open shop problems is known as being NP-hard. The computational difficulty of special cases like this is to be investigated for any particular real life problem. However, the powerful computational capacity of modern computers and availability of efficient mathematical programming solvers, makes attractive using of the proposed algorithm. The numerical example used to illustrate the proposed algorithm was provoked by a real industry staffing problem. For this problem it is easy to determine the optimal staff number just by looking at the results in Table III or in Fig. 3. For more complex problems the steps 5 and 6 have to be automated by proper software modules.

The optimization task solutions on Step 4 of algorithm have solution times in order of seconds by means of Lingo solver [15] on desktop PC with Intel® Celeron® 2.80 GHz CPU and 1.96 GB of RAM under MS© Windows XP operating system. Other real life problems would be more complex and will need many more iteration on Step 4. The proposed algorithm can be implemented in a decision support system, by coding all of the algorithm steps in software modules.

The used modeling approach leads to degenerate integer programs in the sense that several combinations of placement of rectangles have identical objective value. In Fig. 3 the rectangle corresponding to Operation 4 on Machine 4 can have other coordinates (in staff idle areas) without changing the objective function value. In some practical cases this could be advantage giving managers the freedom to choose other schedule parameters for such operations on the basis of other subjective criteria.

Further investigations of other large scale real life open shop staffing problems will be a goal for future work.

VI. Conclusion

The paper proposes an algorithm for optimal staffing in open shop environment. The original optimal staffing problem is transformed and investigated as two-dimensional cutting stock problem. The used cutting stock modeling approach leads to mixed integer optimization task formulation. The proposed algorithm is based on iterative solution of formulated optimization task to simulate different staff planning scenarios and to define the alternatives to choose from. The optimization task solutions for different feasible staffing policies are compared in regard of minimum staff idle to come up with the best number of machines' operators. A real life industry staffing problem is used to test numerically the described algorithm. The numerical testing proves its practical applicability. The proposed optimal staffing algorithm can be coded in proper software module to implement it in a decision support system.

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