

An Aggregate Production Planning Model for Brass Casting Industry in Fuzzy Environment

Ömer Faruk Baykoç, and Ümit Sami Sakalli

Abstract—In this paper, we propose a fuzzy aggregate production planning (APP) model for blending problem in a brass factory which is the problem of computing optimal amounts of raw materials for the total production of several types of brass in a period. The model has deterministic and imprecise parameters which follows triangular possibility distributions. The brass casting APP model can not always be solved by using common approaches used in the literature. Therefore a mathematical model is presented for solving this problem. In the proposed model, the Lai and Hwang's fuzzy ranking concept is relaxed by using one constraint instead of three constraints. An application of the brass casting APP model in a brass factory shows that the proposed model successfully solves the multi-blend problem in casting process and determines the optimal raw material purchasing policies.

Keywords—Aggregate production planning, Blending, brass casting, possibilistic programming.

I. INTRODUCTION

BRASS is an alloy of copper and zinc; it also includes small amounts of other metals such as tin, lead, nickel, iron, aluminum, antimony. There are several brass types corresponding to different ratios of the metals. A critical process in brass casting is blending of the raw materials in a furnace so that the specified metal ratios are satisfied; this is called the melting operation. The raw materials are mainly composed of low-cost scrap materials, each of which contains several metals; to meet the required specifications, pure metals are also used in blending. Therefore, the blending problem requires the determination of the cheapest blend of available raw materials to meet the specifications. Applications of LP blending models range from the oil industry, to the chemical industry, to the food industry; a literature overview was provided by Ashayeri, van Eijs and Nederstigt [1]. Also, there were applications to steel production; Kim and Lewis [2] developed a large scale linear programming model over a planning horizon.

This model was recently modified by Rong and Lahdelma [3] to model uncertainty in raw material compositions by fuzzy chance constraints. On the other hand, there has been no application except Sakalli and Birgoren's [4] study to

brass production in the scientific literature to our best knowledge. They proposed a multi-blend APP model for brass casting in the deterministic case.

Aggregate production planning (APP) is concerned with determining production requirements of products to meet forecast demand in the medium term, often from 2 to 18 months. A wide variety of APP techniques have been developed since the 1950. In real-world APP problems, input data or parameters frequently are imprecise owing to some information being incomplete or unobtainable. Zadeh [5] proposed the fuzzy set theory providing a highly effective means of handling with imprecise data. Zimmermann [6] first introduced fuzzy set theory into conventional LP problems. Since then, fuzzy mathematical programming has developed into several fuzzy optimization methods for solving APP problems [7]. The studies on fuzzy APP model are Lee [8], Tang, Wang, and Fung [9], Wang and Fang [10], Wang and Fang [11], Tang, Fung, and Yung [12].

Zadeh [13] presented the theory of possibility, which is related to the theory of fuzzy sets by defining the concept of a possibility distribution as a fuzzy restriction, which acts as an elastic constraint on the values that can be assigned to a variable. Buckley [14, 15] formulated and described a procedure for solving existing PLP problems. Lai and Hwang [16] developed an auxiliary multiple objective linear programming (MOLP) model for solving a PLP problem with imprecise objective and/or constraint coefficients [11]. The studies on PLP problems are Hsieh and Wu [17], Tang, Wang and Fung [18], Hsu and Wang [19], Wang and Liang [11] and Liang [20].

II. BLENDING PROBLEM IN BRASS MELTING PROCESS

Brass types are defined according to percent weights of metals in their compositions. A certain level of variability in the percentages is tolerable in terms of material properties, therefore the standards for a brass type is comprised of lower and/or upper specification limits. Brass is produced by melting pure materials such as pure copper and zinc and scraps materials such as scrap brass parts and scrap cables. Scrap materials are much cheaper than pure materials; using them allows a significant reduction in material costs, but also poses a challenge to meet specification limits, because scrap materials contain different metals with varying percentages. Brass melting process is presented in Fig. 1.

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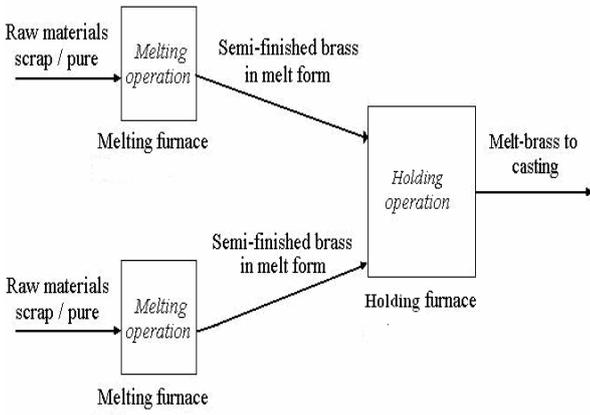


Fig. 1 Brass melting process

First, the foreman compute the amounts of scrap materials, and pure materials if necessary, to be melted together. This process is called “preparing charge”. Raw materials are blended and melted in a melting furnace; next a sample from the melt brass is tested in the lab to check violations of specification limits. If there is any violation, pure copper or zinc is added to the melt brass as a remedy for bringing in control of the percentages. When there is no violation, the melt brass is poured into the resting furnace.

III. SOLVING LINEAR PROGRAMMING MODELS WITH IMPRECISE COEFFICIENTS

Linear programming model with imprecise coefficients is given in Eq. 1:

$$\begin{aligned} \max \quad & \tilde{c}x \\ \text{s.t.} \quad & \tilde{A}x \leq \tilde{b}, \text{ and } x \geq 0 \end{aligned} \tag{1}$$

where \tilde{A} , \tilde{b} and \tilde{c} are imprecise and have possibility distributions. We present Lai and Hwang approaches to solve Eq. 1 in Section III. A.

A.. Lai and Hwang Approach

The fuzzy objective function is fully defined by three corner points $(C^p, 0)$, $(C^m, 1)$ and $(C^o, 0)$ geometrically. Lai and Hwang [21] suggested that maximizing the fuzzy objective can be obtained by pushing these three critical points in the direction of the right- hand side. Therefore the objective function in Eq. 1 is translated to the following form:

$$\begin{aligned} \max \quad & (C^m x, C^p x, C^o x) \\ \text{s.t.} \quad & x \in X \end{aligned} \tag{2}$$

Instead of maximizing these three objectives simultaneously, Lai and Hwang proposed to maximize $C^m x$, minimize $[C^m x - C^p x]$ and maximize $[C^o x - C^m x]$. This lead us the following auxiliary multi-objective linear programming model of Eq. 3:

$$\begin{aligned} \min \quad & z_1 = (C^m - C^p)x \\ \max \quad & z_2 = C^m x \\ \max \quad & z_3 = (C^o - C^m)x \\ \text{s.t.} \quad & x \in X \end{aligned} \tag{3}$$

Lai and Hwang suggested using Zimmermann’s fuzzy programming method to convert the auxiliary multi-objective linear programming model into an equivalent single-goal LP problem. First the positive ideal solutions (PIS) and negative ideal solutions (NIS) of the objective functions can be specified as follows [21, 22]:

$$\begin{aligned} z_1^{PIS} &= \min(C^m - C^p)x, & z_1^{NIS} &= \max(C^m - C^p)x \\ &x \in X & &x \in X \\ z_2^{PIS} &= \max(C^m)x, & z_2^{NIS} &= \min(C^m)x \\ &x \in X & &x \in X \\ z_3^{PIS} &= \max(C^o - C^m)x, & z_3^{NIS} &= \min(C^o - C^m)x \\ &x \in X & &x \in X \end{aligned} \tag{4}$$

The linear membership function of each objective function is defined as follows:

$$f_1(z_1) = \begin{cases} 1 & z_1 < z_1^{PIS} \\ \frac{z_1^{NIS} - z_1}{z_1^{NIS} - z_1^{PIS}} & z_1^{PIS} \leq z_1 \leq z_1^{NIS} \\ 0 & z_1 > z_1^{NIS} \end{cases}$$

$$f_2(z_2) = \begin{cases} 1 & z_2 > z_2^{PIS} \\ \frac{z_2 - z_2^{NIS}}{z_2^{PIS} - z_2^{NIS}} & z_2^{NIS} \leq z_2 \leq z_2^{PIS} \\ 0 & z_2 < z_2^{NIS} \end{cases} \tag{5}$$

$f_3(z_3)$ is similar to $f_2(z_2)$.

Lai and Hwang used fuzzy ranking concepts for the constraints and combined it with their strategy for imprecise objective function. For a minimal acceptable possibility, β , the constraints of the Eq. 1 can be modeled as follows:

$$A_\beta^m x \leq b_\beta^m, A_\beta^p x \leq b_\beta^p, A_\beta^o x \leq b_\beta^o, x \geq 0 \tag{6}$$

Finally, Zimmermann’s following equivalent single-objective linear programming model is used to solve the model [23].

$$\begin{aligned} \max \quad & \alpha \\ \text{s.t.} \quad & f_i(z_i) \geq \alpha, \quad i=1,2,3 \end{aligned} \tag{7}$$

$$A_\beta^m x \leq b_\beta^m, A_\beta^p x \leq b_\beta^p, A_\beta^o x \leq b_\beta^o, x \geq 0$$

The blending problem, in brass casting, has two different aspects. One is the problem of computing the optimal blends for a daily or weekly production: this is formulated as a single-blend model to be used on operational level at the foundry. The other aspect is the problem of computing optimal amounts of raw materials for the total production of several types of brass in a period: this is formulated as a multi-blend model to be used by the planning department as an APP tool for determining optimal raw material purchasing policies.

A. Problem Formulation

The brass casting APP model is translated to a crisp model by using Lai and Hwang's approach and given in Eq. 8-20. Objective functions of the model are minimizing the most possible value of the imprecise total costs, maximizing the possibility of obtaining lower total costs and minimizing the risk of obtaining higher total costs.

Model notation

There are n number of raw material types, m number of ingredients, t number of products (brass type).

- i raw material type, $i = 1, \dots, n$
- j ingredient type (metals), $j = 1, \dots, m$
- k product (brass) type, $k = 1, \dots, t$
- \tilde{C}_i cost of raw material i per kg in a planning period
- P_{ij} percentage of ingredient j in material i in a planning period, $i = 1, \dots, h$
- \tilde{R}_{ij} percentage of ingredient j in material i in a planning period, $i = h, \dots, n$
- V_i yield coefficient for material i
- W_j yield coefficient for ingredient j
- U_{kj} upper limit on the percentage of ingredient j in product k
- L_{kj} lower limit on the percentage of ingredient j in product k
- \tilde{MX}_i maximum quantity of material i to be procured in a planning period
- \tilde{MN}_i minimum quantity of material i to be procured in a planning period
- \tilde{D}_k amount of product k to be produced in a planning period

Decision variables

X_{ik} amount of material i to be used for production of product k (kg)

Mathematical model

$$\text{Max } z_1 = (z^m - z^p) = \sum_{i=1}^n \left(c_i^m - c_i^p \right) \sum_{k=1}^t X_{ik} \quad (8)$$

$$\text{Min } z_2 = z^m = \sum_{i=1}^n \left(c_i^m \sum_{k=1}^t X_{ik} \right) \quad (9)$$

$$\text{Min } z_3 = (z^o - z^m) = \sum_{i=1}^n \left(\left(c_i^o - c_i^m \right) \sum_{k=1}^t X_{ik} \right) \quad (10)$$

$$\sum_{i=1}^h (V_i W_j P_{ij} X_{ik}) + \sum_{i=h}^n (V_i W_j R_{ij\beta}^p X_{ik}) \leq (U_{kj} D_{k\beta}^p), \quad \forall k \quad \forall j \quad (11)$$

$$\sum_{i=1}^h (V_i W_j P_{ij} X_{ik}) + \sum_{i=h}^n (V_i W_j R_{ij\beta}^m X_{ik}) \leq (U_{kj} D_{k\beta}^m), \quad \forall k \quad \forall j \quad (12)$$

$$\sum_{i=1}^h (V_i W_j P_{ij} X_{ik}) + \sum_{i=h}^n (V_i W_j R_{ij\beta}^o X_{ik}) \leq (U_{kj} D_{k\beta}^o), \quad \forall k \quad \forall j \quad (13)$$

$$\sum_{i=1}^h (V_i W_j P_{ij} X_{ik}) + \sum_{i=h}^n (V_i W_j R_{ij\beta}^p X_{ik}) \geq (L_{kj} D_{k\beta}^p), \quad \forall k \quad \forall j \quad (14)$$

$$\sum_{i=1}^h (V_i W_j P_{ij} X_{ik}) + \sum_{i=h}^n (V_i W_j R_{ij\beta}^m X_{ik}) \geq (L_{kj} D_{k\beta}^m), \quad \forall k \quad \forall j \quad (15)$$

$$\sum_{i=1}^h (V_i W_j P_{ij} X_{ik}) + \sum_{i=h}^n (V_i W_j R_{ij\beta}^o X_{ik}) \geq (L_{kj} D_{k\beta}^o), \quad \forall k \quad \forall j \quad (16)$$

$$\sum_{i=1}^n X_{ik} \geq w_1 * D_{k\beta}^p + w_2 * D_{k\beta}^m + w_3 * D_{k\beta}^o \quad \forall k \quad (17)$$

$$\sum_{i=1}^n X_{ik} \leq w_1 * MX_{i\beta}^p + w_2 * MX_{i\beta}^m + w_3 * MX_{i\beta}^o \quad \forall k \quad (18)$$

$$\sum_{i=1}^n X_{ik} \geq w_1 * MN_{i\beta}^p + w_2 * MN_{i\beta}^m + w_3 * MN_{i\beta}^o \quad \forall k \quad (19)$$

$$X_{ik} \geq 0 \quad \forall i \quad \forall k \quad (20)$$

In brass casting APP model, the maximum procurement amounts of material i (\tilde{MX}_i), the minimum procurement amounts of material i (\tilde{MN}_i), the amount of product k to be produced (\tilde{D}_k), the cost of raw material i per kg (\tilde{C}_i) and the percentages of the ingredients in scrap materials (\tilde{R}_{ij}) are all imprecise and have triangular possibility distributions. Fig. 2 represents the triangular possibility distribution of imprecise number $\tilde{C}_i = (C_i^p, C_i^m, C_i^o)$

All charge materials may or may not be utilized to their maximum available percentage P_{ij} or \tilde{R}_{ij} for example, utilization depends on the form of the material (ingots, turnings, etc.). The percent utilization of P_{ij} or \tilde{R}_{ij} is represented by a material yield coefficient V_i . Also, a certain amount of some ingredients are lost since they become gaseous or they are trapped in the slag phase. The percent loss is modeled by an ingredient yield coefficient W_j . In brass production, the yield coefficients are close to 1, and sometimes can be approximated by 1.

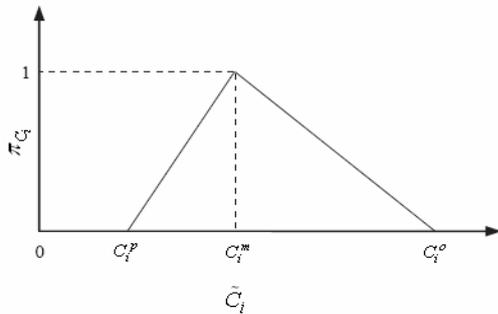


Fig. 2 Triangular possibility distribution

The raw materials are classified into two groups: first group includes pure materials and brass products return from other process to the casting process in the factory which is called work in process. The percentages of the ingredients in the pure materials are known and deterministic. However, works in process material's ingredients percentages are not deterministic. When the lab test results were analyzed, it had been seen that the brass products are following normal distributions.

For a single-blend model to be used on operational level, those probability distributions are very important to satisfy the metal ratios in the product. So the calculations of raw materials amounts to be added in charge must be done carefully considering the random variables.

On the other hand, the random variables are not very important for a medium or long term planning. The percentages of the ingredients are approximate to the mean parameter of the distributions in a long term planning period; this is "the law of large numbers" in the statistics. Therefore, the percentages of the ingredients in the work in process raw materials are known and deterministic for APP models. So the ingredients percentages in the first group raw materials are deterministic and represented with P_{ij} .

The second group raw materials include scrap materials. The percentages of the ingredients in scrap materials are not known precisely because of that, they are modeled as possibilistic distributions and represented with \tilde{R}_{ij} .

Eq. 11-16 ensure the percentage of ingredient j in product k between upper and lower specification limits.

Eq. 17 is constructed to meet the demand. Not all materials are always available in the market, or they can be procured in limited amounts. The predicted maximum amount that can be procured in a period is expressed as MX_i (Eq. 18). Also, there are contracts with raw material suppliers dictating procurement of certain raw materials; the minimum procurement amounts are MN_i (Eq. 19).

In Eq. 17, 18 and 19, only the right-hand sides are imprecise and have triangular possibility distributions. To obtain crisps right-hand side values, weighted average method is used which is proposed by Lai and Hwang [21] where $w_1 + w_2 + w_3 = 1$. w_1 , w_2 and w_3 represent the weights of the most pessimistic, most possible and most optimistic values of the imprecise right-hand side, respectively. This study applies the concept of the most likely values proposed by Lai and Hwang [21], assuming $w_2 = 4/6$ and $w_1 = w_3 = 1/6$.

The APP model of brass casting (Eq.8-20) does not produce feasible solutions every time based on possibility distributions of imprecise data. The reason of unfeasible solutions is absence of the decision variables satisfying Eq. 11-16.

Lai and Hwang convert the fuzzy inequalities to the crisp inequalities using the fuzzy ranking concepts. In this concept, a fuzzy number is greater than the other, if its pessimistic, possible and optimistic values are greater than the others pessimistic, possible and optimistic values, respectively.

In real life, objects are often compared by their attributes. The comparison of each attribute is done based on a certain measurement to indicate the difference between objects. In common sense, one object is better/greater than the others, if the best value of its attribute is better/greater than the best value of the same attribute of the other objects. In another way, the worst value of the attribute may also be used to compare two objects [24]. The number of attributes and the compared attributes can change based on desired realization degree. Therefore, the fuzzy inequalities can be converted to the crisp inequalities using one attribute from the three attributes of possibilistic distribution (pessimistic, possible and optimistic) instead of using three of them. The selection of using one or two or three attributes must be done based on the problem structure.

The APP model of the brass casting is suitable for using one attribute from the three attributes. Because the solutions of the APP model is not used for computing the optimal blends *to meet the required specifications* for a daily or weekly production on operational level; it is used for determining optimal raw material purchasing policies in a medium or long term planning. We can not prepare blend for a daily or weekly production using the APP solutions. Therefore, it is not absolutely necessary to satisfy Eq. 11, 12 and 13 or Eq. 14, 15 and 16 simultaneously in brass casting APP model.

In the proposed model, the fuzzy inequalities are converted to the crisp inequalities by using only Eq.13 and 16. The percentage of ingredient j in product k is ensured between the minimum value of the lower bound and maximum value of the upper bound by using these constraints. Therefore, the proposed model is obtained by using Eq.8, 9, 10, 13, 16, 17, 18, 19 and 20.

V. AN APPLICATION OF APP MODEL FOR BRASS CASTING

The application of APP model for brass casting is performed in MKEK Brass Factory in Kırıkkale, Turkey; it is the oldest brass factory with the largest production capacity in Turkey. The management wants to produce nineteen type of brass by using 40 different types of raw materials in a 10 months planning period. The percentages of metals in raw materials are 'effective percentages' which are $V_i * W_j * P_{ij}$. In this particular application, loss of the ingredients for most of the raw materials has been considered insignificant by the factory engineers, thus yield coefficients are approximated by 1.

The proposed brass casting APP model is performed for 3, 4, 5 and 6 minimum acceptable possibility ($\beta = 0.5$). The PIS and NIS values of the objective functions are found as (2660298, 1554423), (9964426, 16533540) and (2563058, 4315143) for z_1 , z_2 and z_3 respectively. When the equivalent LP model of the auxiliary MOLP problem is solved, the total cost is obtained as a triangular possibility distribution with (10108831, 12363180, 15562512) which is given in Fig. 5 and overall degree of DM satisfaction with multiple goal values is achieved at 0.633.

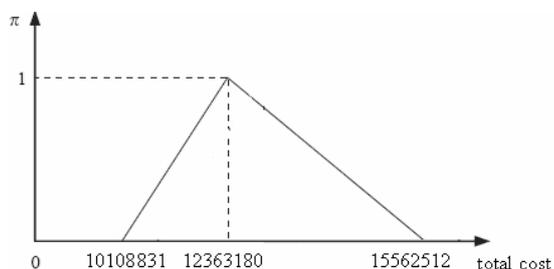


Fig. 5 The triangular possibility distribution of the total cost

The management of the factory is satisfied with those solutions and does not need to modify linear membership functions.

VI. CONCLUSION

In this paper a possibilistic APP model for blending problem has been discussed in a brass factory. Demand quantities, percentages of the ingredients in some of the raw materials, cost coefficients and minimum and maximum procurement amounts are all imprecise and have triangular possibility distributions.

The brass casting APP model can not be solved by using common approaches used in the literature every time. Therefore a mathematical model is proposed for solving this model. In the proposed model, the Lai and Hwang's fuzzy ranking concept is relaxed by using one constraint instead of three constraints.

An application of the proposed model is performed in brass factory in Turkey. The solution of the problem presents that the proposed model successfully solves the multi-blend problem in casting process and determines the optimal raw material purchasing policies for a medium or long term planning period.

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