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An Adaptive Dynamic Fracture for 3D Fatigue Crack Growth Using X-FEM

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***Abstract*—**In recent years, a new numerical method has been developed, the extended finite element method (X-FEM). The objective of this work is to exploit the (X-FEM) for the treatment of the fracture mechanics problems on 3D geometries, where we showed the ability of this method to simulate the fatigue crack growth into two cases: edge and central crack. In the results we compared the six first natural frequencies of mode shapes uncracking with the cracking initiation in the structure, and showed the stress intensity factor (SIF) evolution function as crack size propagation into structure, the analytical validation of (SIF) is presented. For to evidence the aspects of this method, all result is compared between FEA and X-FEM.

***Keywords*—**3D fatigue crack growth, FEA, natural frequencies, stress intensity factor (SIF), X-FEM.

I. Introduction

THE accurate prediction of the fatigue crack growth life of components is of great interest for fracture mechanic engineering. In this area, it is now increasingly necessary to accurately simulate three-dimensional growth of fatigue cracks in the complex structure. As a result, fracture mechanics calculation has been strongly developed in recent years. An important point is making itself when the slip must be explicitly described by numerical simulation. To mitigate these kind of problems, alternative methods have been proposed as the method of boundary elements condition, where only the surface of the crack must be mesh, mesh less methods where grid concept does not exist and that the crack is described by the functions ganglion weight values or methods based on the sharing of the unit (including generalized finite element method FEM and the extended finite element method X-FEM), which allow the use of meshes that do not comply with the crack from the discontinuity is taken into account by the functions of a special form. The finite element method is applicable digital specialist in the sector, in particular because of its availability in commercial codes. This is why it is the chosen method for numerical study. Even if, it is underlined that the proposed approach could have been applied to either the extended, generalized or similar finite element methods. The level set method is the Basic concept of X-FEM and it's device into two point:

– Enriched with the Heaviside: it's represent the cracked surface.
– Crack tip enrichment: it's represent the crack tip.

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II. Analytical Modelisation of Specimen Fracture

The fatigue mechanism in metallic materials should basically be associated with cyclic slip and the conversion into crack initiation and crack growth. The failure (fatigue life) comprises two periods, the crack initiation step and the crack growth step. The crack initiation step includes crack nucleation at the material surface and mico-crack growth structural. The crack propagation step usually is covered by Paris law [1]. In many cases the crack initiation step covers a relatively large percentage of the total fatigue life. The ultimate strength in fatigue failure is always precipitated by fatigue cracking at some level. This cracking, also known as fatigue crack growth (FCG), has become a foundational area of study as it pertains to damage design. Damage is a mechanistic philosophy and methodology whereby the remaining strength and/or life of a component are determined after measurable damage. According to this type of methodology, one deems a certain amount of damage to a component acceptable for use if it can be quantified at a non-critical stage. More specifically in regards to fracture mechanics and fatigue design, a certain known crack size a, is acceptable up to a certain critical crack size ac [2]. The starting point of this study has its pedigree fracture mechanics and a review of the work done in this field along with fatigue crack propagation is very significant to this research. This part will survey Linear Elastic Fracture Mechanics LEFM, 95% of blade design in the turbine engine industry is accomplished via 3D elastic anisotropic Finite Element Analysis, although crystal plasticity analysis is becoming a common tool for research applications, it is still not practical for blade design because of complex 3D geometry and loading involved [3], Nonlinear Fracture Mechanics EPFM, theoretical aspects only, and a part on how to model LEFM areas using FEA.

A. Linear Elastic Fracture Mechanics

Systematic analysis of the mechanical failure started in 15^{th} century by Leonardo da Vinci. He has studied mechanical strength of metal wires. However only in 18th century invention of steam machine and following industrialization brought the extensive studies of the failure behavior of metals. Brittle fracture testing was first performed by Inglis in 1913 and was finally quantified by Griffith [4] in 1920 using the First Law of thermodynamics and an energy release rate in looking at the stress analysis of an elliptical hole. Later, Irwin [5] and Orowan [6] introduced local domain of plasticity, there by representing the fatigue crack growth in metals more accurately. In 1958 Irwin proposed the stress-intensity factor K for Linear Elastic Fracture Mechanic analysis:

$$K = Y\sigma\sqrt{\pi a} \qquad (1)$$

K is universal parameter. It depends on crack size, load stress and geometrical configuration parameter. In spite of the significance of plastic strain in the crack tip zone, the linear elastic fracture mechanics (LEFM) became a foundation to the study of fracture and fatigue tolerant manufacturing. Westergaard, Dugdale, and all followed as modifications to Linear Elastic Fracture Mechanic continued in loads to more rigorously account for the mechanics of cracked components. Stress intensity factor is a parameter which characterizes the severity of the situation obtained by applying a load or fields stress on a zone cracked. This factor is obtained by consecration of stresses and strains in the vicinity of the crack-tip modes of cracking mode I, mode II, mode III. Factors KI, kII and KIII characterize both the geometry of the notch of the crack, and the nature of solicitation. They are unite in MPa $m^{0.5}$ [7]. The stress intensity factors (SIF) correspond to specific kinematic motion of cracks growths. SIF characterize the strength singularity of the stress field at the crack tip.

The finite element method (FEM) is now widely used in fracture mechanic engineering. Since its inception, it has been the subject of continuous development from the calculating community structures. Many works are dedicated to him [8], [9]. The extended finite element method proposes to enrich the approximation with discontinuous enrichment functions and singular the principle of partition of unity. Discontinuity is no longer necessarily carried by the mesh [10], [11]. The function of enrichment, as their name indicates, will be utilized to enrich the finite element model. They are given three main roles:

1) Represented the discontinuity.
2) Locate the crack-tip zone.
3) Capture: the solution at the crack-tip.

Meshing of three-dimensional solids is still one of the most burdensome tasks in finite element analysis. The difficulties of meshing became particularly serious with the advent of models. In treating such large-scale, unstructured finite element meshes, an inordinate amount of effort is devoted to:

1) Generating the mesh.
2) Coping with the unstructured character of the formulas during assembly and solution procedures.
3) post-processing.

Recently, it has become apparent that many of these difficulties can be circumvented by using structured meshes in conjunction with recently developed techniques for representing internal discontinuities [12], and internal details [13]. In fact, with these techniques, it becomes possible to model the detail associated with engineering situations with even greater fidelity than conventional finite element methods. For example, it is possible to model complex sliding or tearing surfaces within a body and to model cracks and small holes. One of the sources from which these capabilities have evolved is the seminal paper by Melenk and Babuska [14], in which the concept of partition of unity was first described. Belytschko and Black [15] employed the concept to model of cracks, in Moës [16] and Dolbow [17] step functions were introduced through the partition of unity to model arbitrary discontinuities. They called the method the extended finite element method (X-FEM). Babuska [14] and Strouboulis [18] illustrated the potential of the partition of unity concept in modeling small holes in a mesh and introducing so-called handbook solutions, they called it the generalized finite element method. The method was expanded in Strouboulis [13], where the focus was towards the extension of the classical finite element method to meshes that do not conform to boundaries of the problem. In Moës [16] and Belytschko [12] the quadrature issue was studied for meshes that do not conform to internal boundaries. Basic of XFEM Concepts is a:

1) Numerical technique for describing a fatigue crack growth and tracking their motion.
2) Couples naturally with XFEM and makes possible the modeling of 3D arbitrary crack propagation without remeshing level set method requires for two functions:
   – The first describes the crack surface, Φ (phi).
   – The second, Ψ (psi), is constructed so that the intersection of two level sets gives the crack-tip.
3) Uses signed distance functions to describe the crack growth geometry.

No explicit representation of the crack is needed and the crack is entirely described by nodal data. Enriched with the Heaviside: it's represent the cracked surface, crack-tip enrichment:

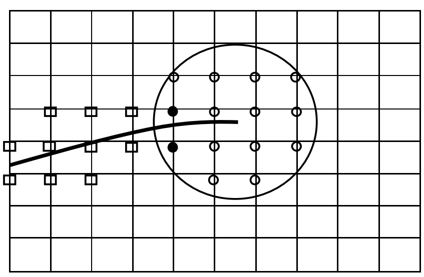

Fig. 1 Enriched with the Heaviside (Φ)

The level set method is the Basic concept of X-FEM and it's device into two point:

-The Heaviside function allows to model the discontinuity in displacement, and it's represented by following formula:

$$H(\underline{x}) = \begin{cases} +1 & si\ \emptyset > 0 \\ -1 & si\ \emptyset < 0 \end{cases} \qquad (2)$$

-Series of four functions is used to describe the singularity at the crack tip, and it's represented by following formula:

$$F_l(\underline{x}) = \sqrt{r}\begin{cases} sin(\theta/2) \\ sin(\theta/2)sin(\theta) \\ cos(\theta/2) \\ cos(\theta/2)sin(\theta) \end{cases} \quad (3)$$

where (r, θ) are polar coordinates related to the crack tip.

Modeling cracks using Level set method: Level set method offers an elegant way of modeling discontinuities. Level set method has been successfully applied for modeling cracks. In this section details regarding the crack modeling using level set functions and its coupling with XFEM will be discussed. Further, later in the part some of the key advantages of using level set functions in the framework of XFEM will be highlighted [19]. The extended finite element method allows one to shown cracks and weak (holes, material interfaces) discontinuities independent of the finite element mesh through the partition of unity. This allows one to avoid costly remeshing which occurs in the vicinity of the crack-tip in the traditional finite element framework when modeling crack growth. However, we have raised a point of discretization methods conventionally used in mechanics, focusing particularly on the basis of the X-FEM. The method level set and its coupling with X-FEM provides a satisfactory tool to solve free external problems and singular problem of example crack growth problems.

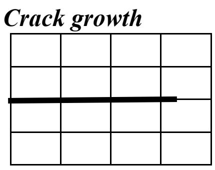

Fig. 2 FEM Crack coincides with the interface mesh

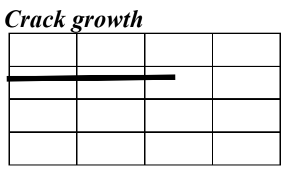

Fig. 3 X-FEM Crack is free

B. Lateral Edge Crack in 3D Model Under Tension

In following we illustrate the analytical method to calculate K. Theoretical analyses, Stress intensity factor K_I for this configuration is:

$$K_I = F(\lambda)\sigma\sqrt{\pi a} \quad (4)$$

We have these Dimension: Length: 6mm, width: 3mm, thickness: 1mm

The geometrical configuration factor is:

$$F(\lambda) = 1.12 - 0.231\lambda + 10.55\lambda^2 - 21.72\lambda^3 + 30.39\lambda^4 \quad (5)$$

C. Central Crack in 3D Model Under Tensile Load

In following we illustrate the analytical method to calculate K Theoretical analyses:

Stress intensity factor K_I for this configuration is:

$$K_I = F(\lambda)\sigma\sqrt{\pi a} \quad (6)$$

TABLE I
KI THEORETICAL ACCORDING TO STEPS (EDGE CRACK LENGTH)

a (mm)	KI Theoretical (MPa√mm)
0.5	1.4128
1	3.1655
1.5	5.3346
2	12.0904

Dimension: Length: 6mm, width: 3mm, thickness:1mm

The geometrical configuration factor is:

$$F(\lambda) = \sqrt{sec\left(\frac{\pi\lambda}{2}\right)(1 + 0.025\lambda^2 + 0.06\lambda^4)} \quad (7)$$

III. Numerical Simulation of 3D Fatigue Crack Growth

In this part, we will use a software simulation for comparison the results, and can give us an idea for the distribution of the stress, also we will see the strain of the model in using the two methods FEM and X-FEM. The early stages of design, numerical simulation are used to test the product before making prototypes. We can thus reduce the cost and time of a project but also virtually test the product on the effect of changes. The use of this tool in conjunction with the dynamic testing allows both to validate the model, but also to broaden the scope of investigations. These applications allow the validation of the mechanical behavior under static or dynamic. When we create the model we have to enter the material(ALUMINIUM) properties which are:

– Young's modulus: 70 GPa
– Poison's ratio: 0.33
– Density: 2.7 Kg/m^3

And we take the Boundary Conditions and Loads, loads consist of any of the loads fields, concentrated or distributed tensile. We take into account the boundary conditions which are:

– For the displacement we have:
  ENCASTRE ($U_1=U_2=U_3=UR_1=UR_2=UR_3= 0$) ;
  ZASYMM ($U_1=U_2=UR_3= 0$).
– For the first force we applied a mechanical pressure (tensile) (P=-10N).

TABLE II
KI THEORETICAL ACCORDING TO STEPS (EDGE CRACK LENGTH)

a (mm)	KI Theoretical (MPa√mm)
0.5	1.9007
1	2.5751
1.5	3.5475
2	5.5724

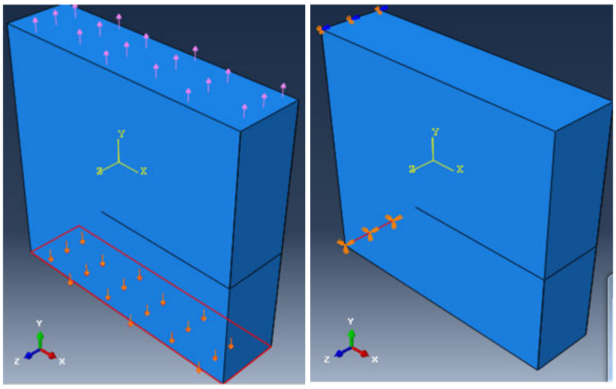

Fig. 4 Crack initiation, Boundary Conditions and Loads

Mesh and Creation of Cracked Domain, then we try to mesh our model with based to the crack:

- Element type: Tetrahedral
- Element number: 25488 elements.

A. Crack Initiation in Dynamical Part of Specimen

After the simulation we have the following three results:

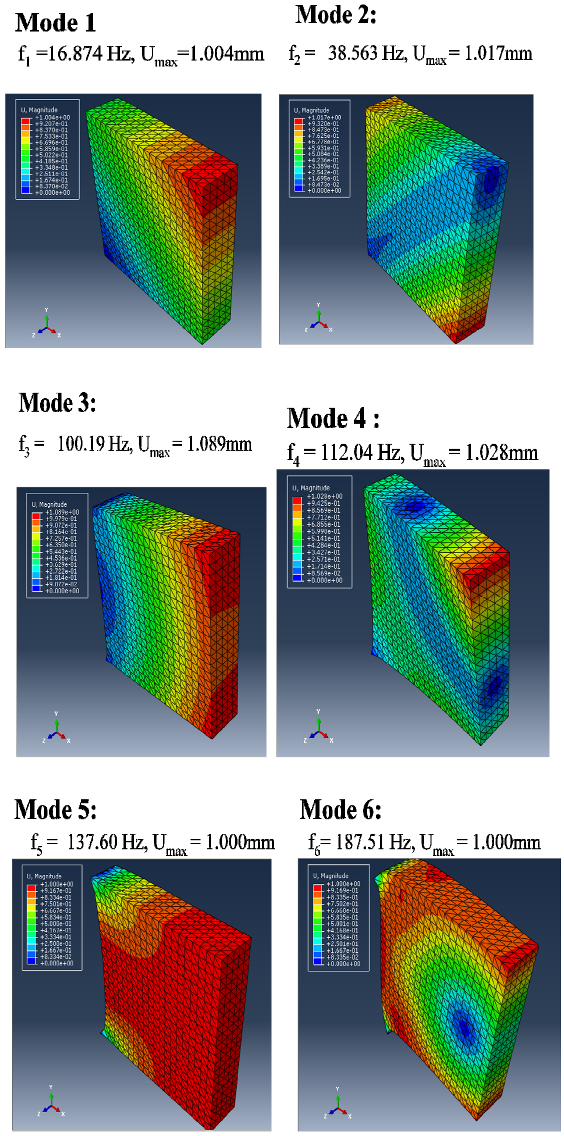

Fig. 5 First six modes shape of specimen uncracking

So, In this case we will create the same crack which is located in the medium of the model, the following fig show the cracked model and Creation of Cracked Domain:

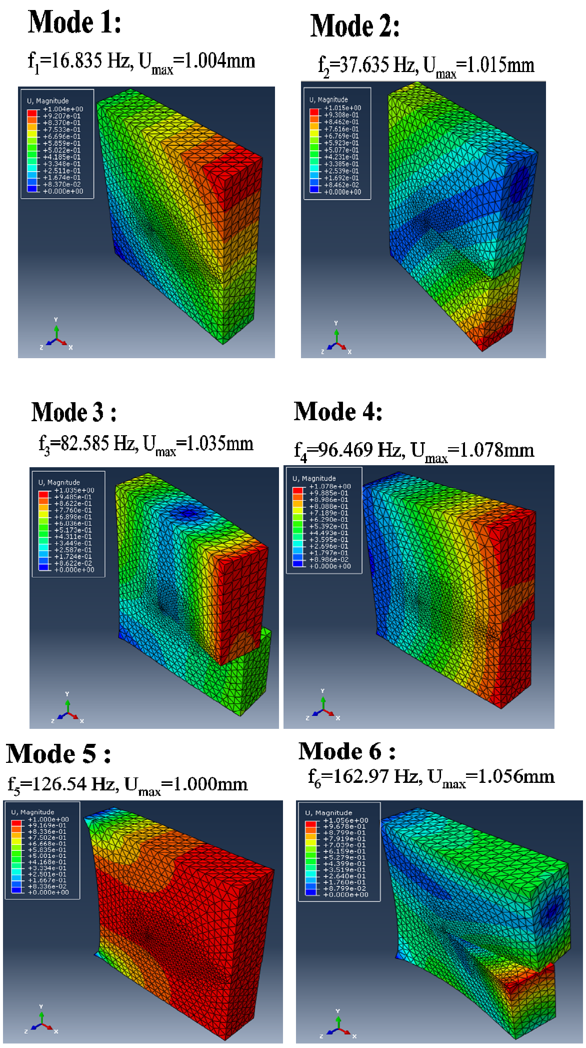

Fig. 6 First six modes shape of model cracking by FEM

TABLE III
COMPARISON OF THE NATURAL FREQUENCIES MODEL WITHOUT AND WITH CRACK USING FEM

Frequencies (HZ)	Without Crack	With Crack (FEM)	With Crack (X-FEM)
f_1	16.874	16.835	16.81
f_2	38.563	37.635	37.627
f_3	100.19	82.585	82.433
f_4	112.04	96.469	95.704
f_5	137.6	126.54	110.66
f_6	187.51	162.97	145.66

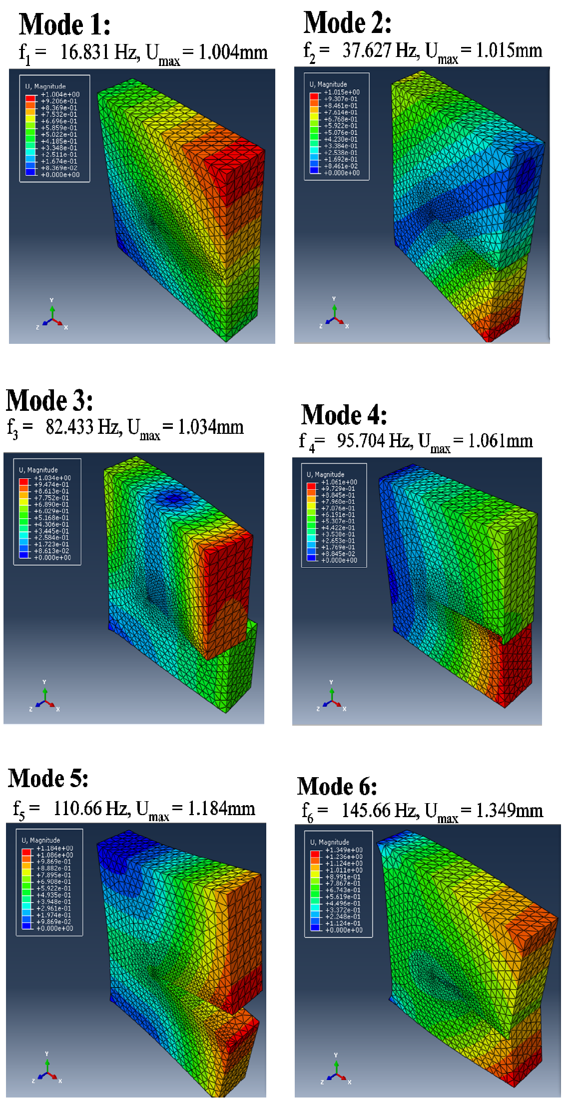

Fig. 7 First six modes shape of model cracking by X-FEM

The modal frequencies are decrease after the initiation crack. We note also when we use FEM the frequencies are lower than when we use X-FEM. The X-FEM is more reliable and precise in these results.

B. Crack Propagation in Lateral Edge Crack

After the simulation, we obtain these values for stress max as function crack size propagation:

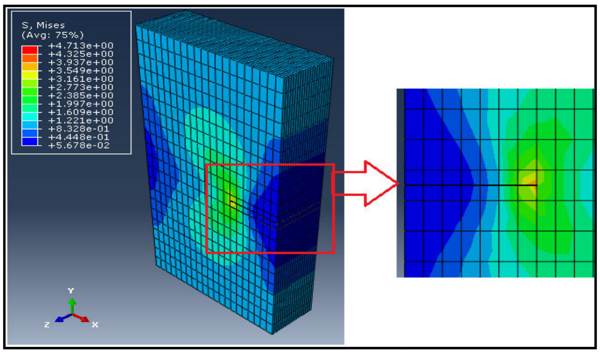

Fig. 8 Stress max Von Mises as function crack size (Edge crack)

We see when the crack lengths increase the S_{max} increase too. And we conclude that the Crack has a great effect on resistance of model.

TABLE IV
KI THEORETICAL ACCORDING TO STEPS (EDGE CRACK LENGTH)

a (mm)	Smax (MPa)
0.5	26.5
1.0	47.13
1.5	98.04
2.0	184.4

Propagation of the crack in the X-FEM for Lateral edge crack :

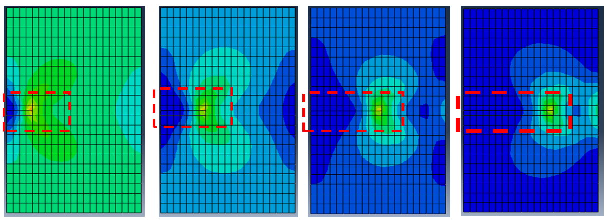

Fig. 9 Crack propagation of edge crack specimen

We see in these pictures the fatigue crack growth of the model ender tension, And the goal is to determine the SIF according the crack propagation.

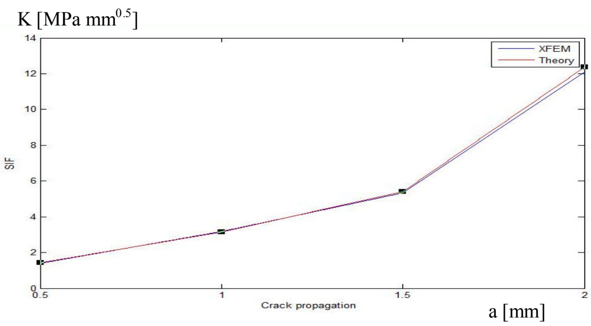

Fig. 10 Comparison between K_I theoretical and K_I numerical according to steps (crack length)

And we can see that both theoretical and numerical results are almost equal.

C. Crack Propagation in Central Crack

We obtain these stress max as function crack size propagation:

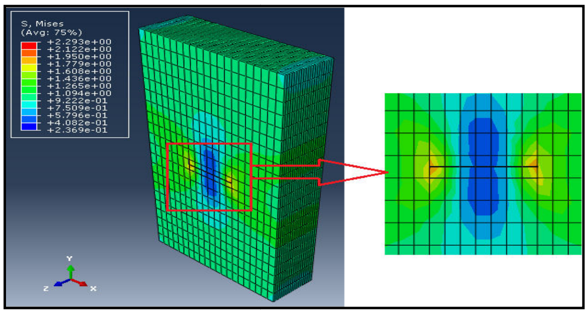

Fig. 11 Stress max Von Misess as function crack size (central crack)

We see when the crack lengths increase the S_{max} increase with it. And we affirmed that the Crack has a great effect on resistance of model.

Propagation of the crack in the X-FEM for this case (Central crack):

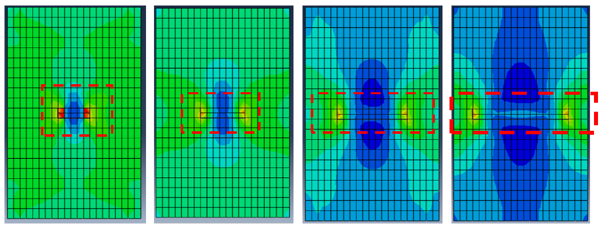

Fig. 12 Crack propagation of central crack specimen

And the Goal of this case is also to determine the SIF according to steps of crack propagations.

This graph interprets comparison between K_I theoretical and K_I numerical according to steps (crack length):

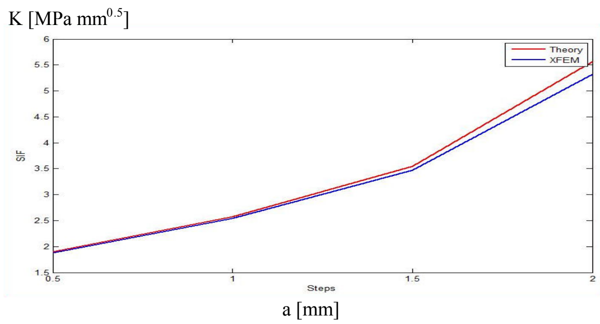

Fig. 13 Stress intensity factor K function as crack size propagation

We get the same results of the Lateral edge crack, when we compare the results of both theoretical and numerical methods, we can see that it's almost equal, so we confirmed that the extended finite element method is accurate and efficacy.

IV. Conclusion

In this study were treated different examples of application of the method X-FEM in 3D with several cases (crack central, lateral Edge crack). The numerical simulation results of each sample application were compared with theoretical results and also with the work of researchers in this field and have given a good approximation. The X-FEM has shown great flexibility and very good results without any need to refine the mesh at the crack. So the validation of XFEM numerical simulation of crack growth and stress intensity factor by theoretical model is confirmed.

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