

# Airliner-UAV Flight Formation in Climb Regime

Pavel Zikmund, Robert Popela

**Abstract**—Extreme formation is a theoretical concept of self-sustain flight when a big airliner is followed by a small UAV glider flying in the airliner wake vortex. The paper presents results of a climb analysis with the goal to lift the gliding UAV to airliners cruise altitude. Wake vortex models, the UAV drag polar and basic parameters and airliner's climb profile are introduced at first. Afterwards, flight performance of the UAV in a wake vortex is evaluated by analytical methods. Time history of optimal distance between an airliner and the UAV during a climb is determined. The results are encouraging. Therefore available UAV drag margin for electricity generation is figured out for different vortex models.

**Keywords**—Flight in formation, self-sustained flight, UAV, wake vortex.

## I. INTRODUCTION

THE paper deals with an idea of energy extraction from an airliner wake vortex by a small UAV. Maximal thrust of all engines exceeds enormous 1 MN in the case of B747 and A340 airliners. Energy of engines is consumed by a drag and potential and kinetic energy accumulation during climb. Induced drag is a component of total airliner drag which is important for flight formation analysis. The energy consumed by induced drag is transformed to a strong wake vortex pair. Let's look at wake vortex as a potential source of energy. The extreme formation used in the abstract of the paper means the formation of a big airliner followed by a significantly smaller UAV. The idea of the extreme formation is to extract enough energy from an airliner wake vortex by the UAV to follow the airliner without any own propulsion. The UAV glider will be called the wake vortex glider "WVG UAV" later in the text. The goal of the paper is a study of the extreme formation feasibility in a climb regime. A motivation to study extreme formation in a climb regime is to lift up the WVG UAV up to an airliner cruise altitude without own propulsion.

The formation flight is not a new concept of flying. Bird flocks use formation for long distance flights to save energy. The estimation of energy savings is about 14% [1]. The same savings were achieved also by fighter aircrafts in a cruise regime [2]. The extreme formation flight in a cruise regime was published with encouraging results in 2015 [3]. This paper extends the results in a cruise by a climb regime and improves the WVG UAV drag polar estimation with respect to the ailerons deflection.

The paper is organized as follows: The Section II deals with a climb regime definition. A typical speed and a climb speed

P. Zikmund is with the Institute of Aerospace Engineering, Brno University of Technology, Brno, 616 69 Czech Republic (phone: 00420-54114-3687; e-mail: zikmund@fme.vutbr.cz).

R. Popela is with the Institute of Aerospace Engineering, Brno University of Technology, Brno, 616 69 Czech Republic (e-mail: popela.r@fme.vutbr.cz).

of B747 are defined within the section. The Section III describes wake vortex models determination. Three different models are considered to cover different atmospheric conditions. The Section IV brings the WVG UAV sketch and drag polar estimation. The Section V brings essential paper content, a feasibility study of the extreme formation in climb. The WVG UAV climb analysis is performed. Optimal distance between an airliner and the WVG UAV is figured out and drag and required thrust are estimated. Conclusions of the paper are summarized in the Section VI.

TABLE I  
LIST OF SYMBOLS

Symbol	Quantity	Unit
$a_l$	Squire's parameter	-
$b$	span	m
$c_D$	drag coefficient	-
$c_L$	lift coefficient	-
$c_l$	roll moment coefficient	-
$r$	radial dimension	m
$r_c$	vortex core radius	m
$t$	time	s
$V_i$	induced speed	$\text{ms}^{-1}$
$\alpha$	angle of attack	rad,
$\delta_{\text{aileron}}$	aileron deflection	rad
$\Gamma$	vortex circulation	$\text{m}^2\text{s}^{-1}$
$\nu$	kinematic viscosity	$\text{m}^2\text{s}^{-1}$

## II. CLIMB PROFILE

The estimation of an airliner climb profile impacts significantly the extreme formation feasibility study. Powerful engines give to airliners high excess power. Airliners with the high excess power reach vertical speed around 15 m/s in case of maximum take-off weight. Not fully loaded airliners climb with even higher climb speed. A climb profile is influenced by the weight of the airliner and also by flight control requirements. Only intercontinental long-distance flights were considered in the climb profile estimation. Airliners heading to very distant place usually take-off with maximal take-off weight and climb slower than partially loaded airliners. The high weight of airliner causes strong wake vertices and that is important for the extreme formation. The climb profile was determined on the basis of data observed from flight radar [4]. The web page flight radar gives online monitored data of all airplanes, equipped with the ADS-B transponder, all around the world. Four random flights were used to interpolate and define a case model.

Fig. 1 shows history of a climb speed dependent on flight altitude. The case data solid line approximate observed data scattered lines. Time integration of the case data climb speed corresponds to a typical climb time which is approximately

20 min. The steps in the climb speed history are forced by flight control requirements. The two regions with low climb speed around 2000 and 4000 m correspond to a rapid acceleration of airliners which can be seen in Fig. 2.

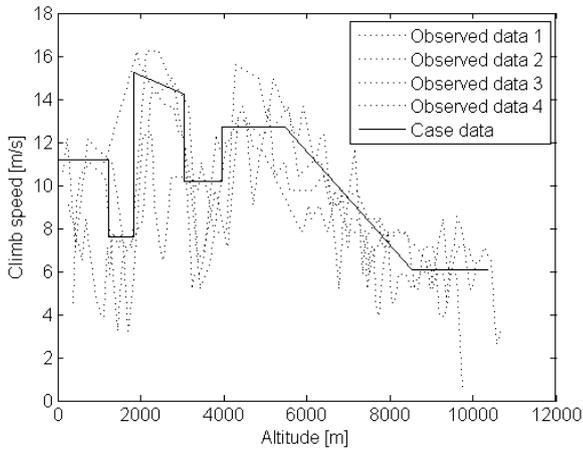


Fig. 1 Airliners climb profile – observed data and case model

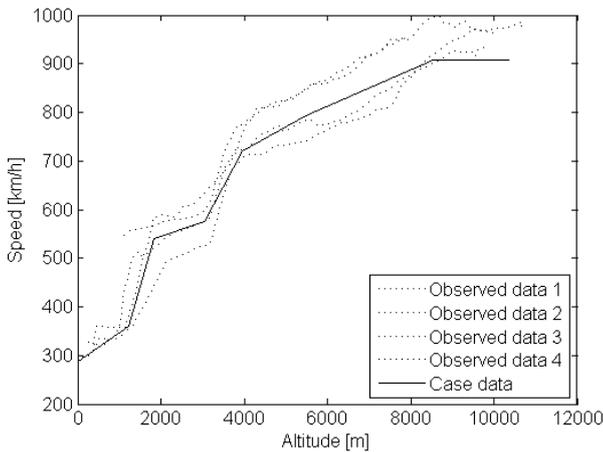


Fig. 2 Speed profile during climb – observed data and case model

Flight speed history during the climb is shown in Fig. 2. Unfortunately, the flight radar [4] gives only a ground speed which differs from an air speed by a wind component. Wind speed in the cruise level is not negligible. Therefore, the case data cruise speed was set up to a value given by Mach number 0.85 which is the typical cruise speed of B747. The climb profile given by the speed and the climb speed history was used as the case model for the extreme formation feasibility analysis.

### III. VORTEX CHARACTERISTICS AND MODEL

#### A. Vortex Models

Wake vortices have been studied intensively because of dangerous consequences when another airplane encounters a wake vortex [5], [6]. Therefore, many experiments were done and many papers were published within this field of interest.

Three vortex models were chosen for the extreme formation analysis. All of them were experimentally validated: Iversen [7], Lamb-Oseen and Hallock-Burnham [8]. Iversen’s model belongs to the first wake vortex descriptions and gives a good matching to flight experiments. Induced velocity distribution and vortex decay are given by dimensionless graphs in Figs. 3 and 4. The solid line for variable eddy viscosity was used in this analysis.

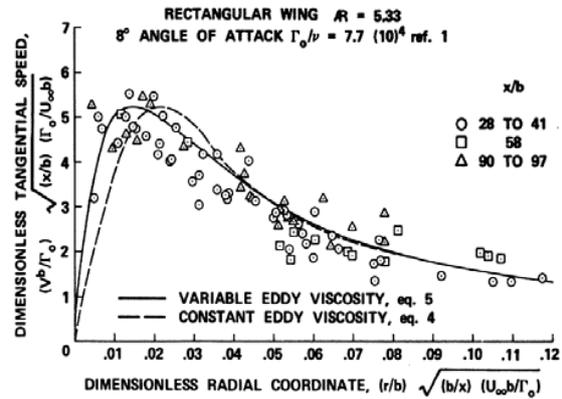


Fig. 3 Iversen model – induced velocity distribution [7]

Maximal tangential velocity in Iversen model is modelled by an interpolated line in Fig. 4. The line is broken into two straight lines by dimensionless downstream distance value 30. The value corresponds to varying time distance 3–5 s between an airliner and the WVG UAV during climb. 3 s was set up as the minimal downstream distance for Lamb-Oseen and Hallock-Burnham models. These models do not describe the earliest stage of a vortex life where the vortex is formed and keeps approximately constant the maximal tangential velocity.

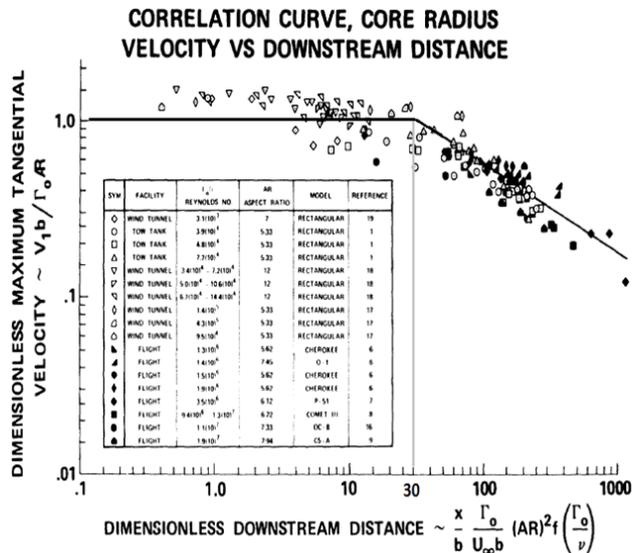


Fig. 4 Iversen model – vortex decay [7]

Lamb-Oseen and Hallock-Burnham induced velocity models [8] depend strongly on a vortex core size. The vortex

Models are defined by the following equations for Lamb-Oseen (1) and Hallock-Burnham (2).

$$V_{t(t)} = \frac{\Gamma_0}{2\pi r} [1 - e^{-\alpha(\frac{r}{r_c})^2}] \quad (1)$$

$$V_{t(t)} = \frac{\Gamma_0}{2\pi r} \frac{r^2}{r^2 + r_c^2} \quad (2)$$

### B. Vortex Initial Size and Decay

Iversen vortex model decay was obtained from the dimensionless maximum tangential velocity in Fig. 4. Lamb-Oseen and Hallock-Burnham models decay are given by a vortex core radius growth. The initial stage of a vortex life is characterized by a very small circulation decrease. This stage is defined by normalized time interval from 0 to 1. The value 1 corresponds to a constant  $t_0$  which represents the constant used for normalisation time for vortex pairs (3). Therefore, vortex circulation is considered as a constant in the range of  $t < t_0$  [9], [10]. The value of the constant  $t_0$  varies from 24 to 39 s during the climb of the case model.

$$t_0 = \frac{2\pi b_0^2}{\Gamma_0} \quad (3)$$

A tangential induced velocity decreases in spite of the constant value of circulation. That is given by growing of a vortex core radius. A vortex core radius size differs in literature within the range of 1 to 7 % of an airplane span. The initial core size and growth rate was obtained from the literature [8], [11], [12]. The initial value of 1 % of airliner span was used in the extreme formation analysis. The core grow is described by (4):

$$r_c = \sqrt{r_{c0}^2 + 4\alpha\delta vt} \quad (4)$$

where eddy viscosity coefficient  $\delta$  is given by (5).

$$\delta = 1 + a_1 \text{Re} = 1 + a_1 \frac{\Gamma}{v} \quad (5)$$

Value  $5 \times 10^{-5}$  of Squire's parameter  $a_1$  was estimated with respect to the literature [8]. The time history of the maximal upwards induced velocities is showed in Fig. 5.

Fig. 6 shows a comparison of induced velocity distribution for the three vortices models in a distance 7 s from the vortex generating airplane.

### C. Vertical Speeds in Vortex

The vortex models describe the induced speed distribution for a single vortex. Wake vortices behind a fixed-wing airplane are always generated as a vortex pair. Both vortices influence each other mutually. The result of the influence is a vortices descent. Therefore, resultant vertical speeds are higher in the downwards direction than in the upwards direction. Similar influence is applied because of a climb. The induced velocities are decreased by a climb speed of an airliner in the

vertical direction. Then the resultant upwards induced speed can be used for the VWG UAV as a propulsion force in gliding.

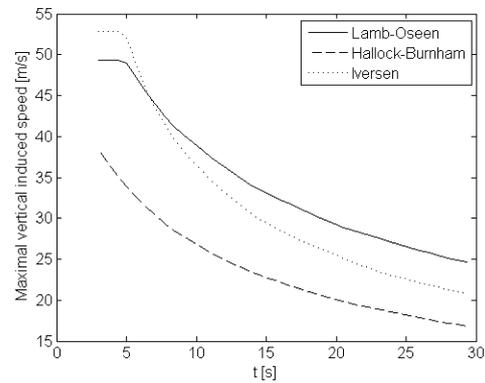


Fig. 5 Vortex models decay comparison (altitude 2500 m)

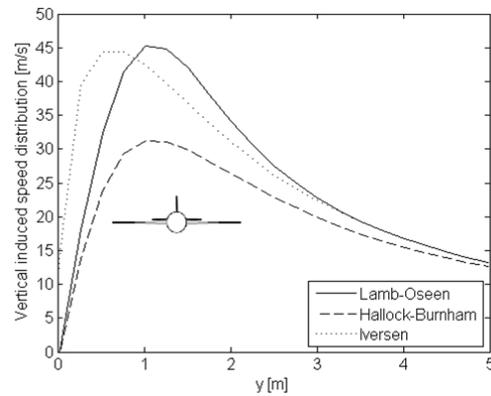


Fig. 6 Induced velocity distribution (vortices age 7 s, altitude 2500 m)

## IV. WVG UAV

The detailed description of the WVG UAV preliminary design is out of the paper focus. The master thesis [13] gives a deeper insight into the design than this section which brings only a brief process characterization and results. The goals of the design were WVG UAV geometry and a drag polar estimation. The inputs to the design process were the vortex characteristics and airliner cruise regime parameters. These pre-requisites are defined in the previous section regarding the wake vortex models. The geometrical design and the drag polar estimation were carried out iteratively to achieve satisfactory results. Roskam method [14] was used for the drag polar estimation.

Resultant value of wing loading is a surprising parameter which was obtained by the preliminary design. The weight and the wing area ratio of the WVG UAV reaches values from 350 to 500 kgm<sup>-2</sup> which are comparable to the airliners wing loading. The highest value corresponds to the WVG UAV weight 160 kg. This weight is a result for the best cruise glide performance for designed VWG UAV [3]. The climb regime analysis of the extreme formation leads to lighter WVG UAV than in the cruise case. Optimal weight for the case model

climb regime with the same WVG UAV geometry is 115 kg. The fuselage is shaped due to Whitcomb's area rule [15] to eliminate a wave drag in a transonic flight. Fig. 7 shows the preliminary sketch of the WVG UAV.

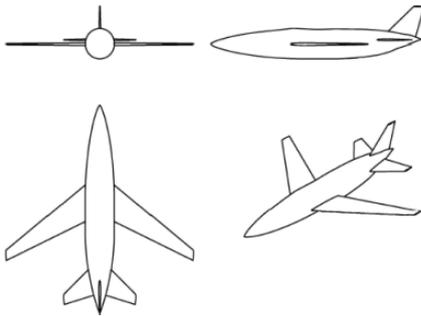


Fig. 7 WVG UAV concept layout [11]

Control surfaces are designed in the following configuration. The ailerons are going to be supported by spoilers [16]. The tail surfaces are all moving. Main characteristics of the UAV are presented in the following Table II.

TABLE II  
WVG UAV CHARACTERISTICS

Weight	115 kg	Wing span	1.45 m
Wing area	0.32 m <sup>2</sup>	Length	1.6 m

Small dimensions of the wing mean low Reynolds number. The initial velocity of an airliner is approximately 80 m/s. This speed denotes Reynolds numbers  $4.6 \times 10^5$  for a wing tip air foil and  $1.3 \times 10^6$  for a mean aerodynamic chord. The speed rises up quickly with a climb of an airliner. The airliners reach velocity 150 m/s in the altitude of 1800 m and Reynolds numbers achieve satisfying values of  $7.5 \times 10^5$  and  $2.1 \times 10^6$ . The drag polar is estimated in the following form (6):

$$c_D = c_{D0w} + c_{D0fus} + c_{D0emp} + c_{DL} c_{L^2} + c_{Dtrans} + c_{\delta aileron} \delta_{aileron} \quad (6)$$

The first three coefficients represent wing, fuselage and empennage drag in a zero lift condition. The next multiplied variables mean an induced drag. The coefficient  $c_{Dtrans}$  represents wave drag in transonic speed. The last coefficient covers a drag due to the ailerons deflection. This value was estimated according to the literature [13], [16].

#### V. CLIMB FEASIBILITY STUDY

The feasibility study is based on a flight performance analysis with taking into account controllability limitation. The analysis is performed by an analytical solution of flight in the extreme formation. The computation domain is defined by following variables: altitude, downstream distance between an airliner and the WVG UAV and span-wise dimension. The analysis begins by an evaluation of upwards induced speeds in the whole domain. An incoming flow angle is determined for

left and right position of the wing mean aerodynamic chord then. A roll moment coefficient is evaluated by (7):

$$c_l = \frac{y_{MAC}}{2b} c_{L\alpha w} \Delta \alpha \quad (7)$$

where  $y_{MAC}$  is a span-wise direction dimension of the mean aerodynamic chord,  $b$  is a span,  $c_{L\alpha w}$  is a wing lift slope and  $\Delta \alpha$  is a difference between left and right mean aerodynamic chord angle of attack. The aileron deflection is evaluated from the roll moment. The drag is estimated with knowledge of the aileron deflection. The estimated drag is compared to a drag in a glide regime then.

#### A. Optimal Distance between Airliner and WVG UAV

The optimal downstream distance between an airliner and the WVG UAV is evaluated from two criterions with the equal weights. The first criterion represents a width of the region where the WVG UAV can follow an airliner. The second criterion is a drag reserve which is obtained as a difference between two drag values. These drag values are defined for two theoretical regimes. The first regime corresponds to a forward speed component  $v_x$  of an airliner and a descent speed  $v_z$  from WVG UAV drag polar. The second regime corresponds to an available upwards speed in a wake vortex. The difference in the evaluated drag in both regimes gives the drag reserve which may be used for electricity generation in the WVG UAV. The optimal downstream distance between an airliner and the WVG UAV was evaluated by maximizing the drag reserve for all altitudes. Resultant distance histories are shown in Fig. 8. The optimal time distance reached boundary of the computational domain. The boundary is given by time 3 s. Vertices are not developed in shorter distance and used vortex models do not simulate the very first stage of vortex growth.

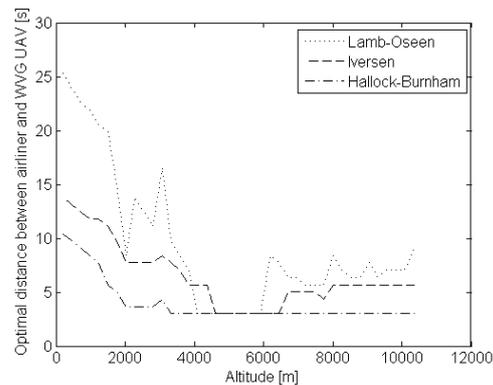


Fig. 8 Comparison of optimal downstream distances

#### B. Drag Reserve in Extreme Formation During Climb

The altitude where the optimal distance is the shortest is critical for the extreme formation in a climb. The altitude between 4000 and 6000 m is characterised by a combination of an airliner high climb speed with rising flight speed near the cruise speed. Fig. 9 shows the drag reserve during a climb for the three vortex models.

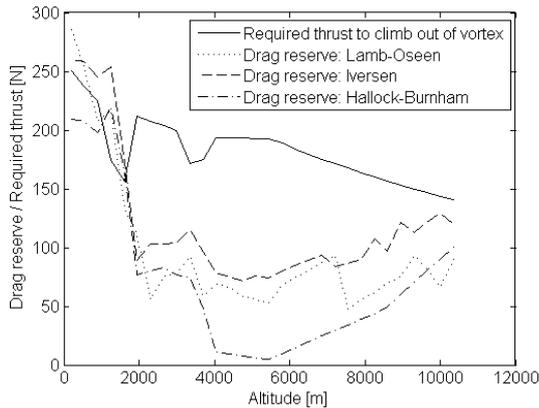


Fig. 9 Drag reserve dependent on altitude

The solid line is a required thrust necessary to follow an airliner out of the wake vortex. The thrust is depicted to demonstrate an energy excess during beginning and end of the climb. Figs. 10–12 plot results of the drag reserve for the three vortex models. The induced upwards velocity is evaluated in the optimal downstream distance behind an airliner for all altitudes. The last axis represents the span-wise direction coordinate. The zero value is situated into the vortex axis. The white colour means insufficient energy in the vortex and also a region around the vortex axis, where the WVG UAV cannot compensate the roll moment within the vortex core.

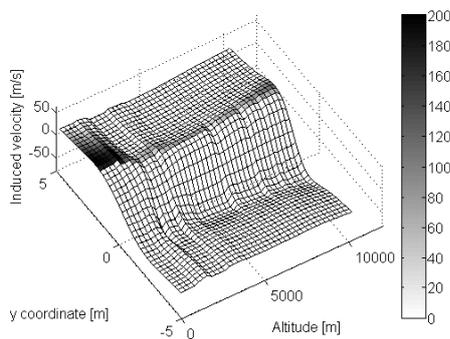


Fig. 10 Drag reserve [N] in case of Lamb-Oseen vortex model

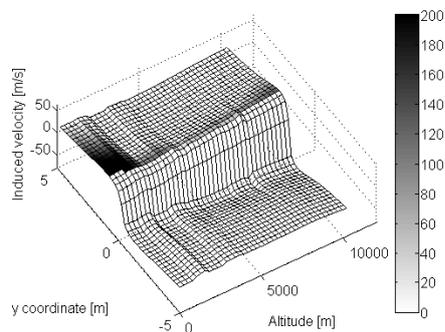


Fig. 11 Drag reserve [N] in case of Iversen vortex model

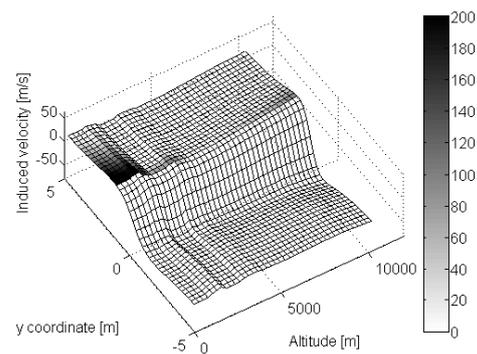


Fig. 12 Drag reserve [N] in case of Hallock-Burnham vortex model

## VI. CONCLUSION

This paper analysed a feasibility of the WVG UAV gliding in wake vortex models generated by climbing B747. The theoretical concept of the extreme formation is analysed from flight performance and controllability point of view. The WVG UAV is controllable in a region of outer-side upwards tangential speeds of wake vortex, but is not controllable near a vortex axis. The area of the sufficient energy in wake vortex is not wide and varies with rising altitude and distance behind an airliner. The wake vortex models contain enough energy to lift the WVG UAV up to a cruise level of an airliner. The critical altitude was identified as 4000 – 6000 m where the area suitable for gliding is small with low drag reserve which can be used for electrical energy generation.

## ACKNOWLEDGMENT

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