Air Cargo Overbooking Model under Stochastic Weight and Volume Cancellation

N. Phumchusri, K. Roekdethawesab, M. Lohatepanont

Abstract—Overbooking is an approach of selling more goods or services than available capacities because sellers anticipate that some buyers will not show-up or may cancel their bookings. At present, many airlines deploy overbooking strategy in order to deal with the uncertainty of their customers. Particularly, some airlines sell more cargo capacity than what they have available to freight forwarders with beliefs that some of them will cancel later. In this paper, we propose methods to find the optimal overbooking level of volume and weight for air cargo in order to minimize the total cost, containing cost of spoilage and cost of offloaded. Cancellations of volume and weight are jointly random variables with a known joint distribution. Heuristic approaches applying the idea of weight and volume independency is considered to find an appropriate answer to the full problem. Computational experiments are used to explore the performance of approaches presented in this paper, as compared to a naïve method under different scenarios.

Keywords—Air cargo overbooking, offloaded capacity, optimal overbooking level, revenue management, spoilage capacity.

I. INTRODUCTION

REVENUE management (RM) is a technique of dealing with consumer behaviors in order to increase firm's profitability from perishable products such as hotel rooms, and airline seats. Over recent years, revenue management has become more accepted and implemented in various industries, especially in hotel and transport industry. RM was originated in the airline industry in term of passenger bookings problem. In 1980s, American Airlines was the first airline who implemented revenue management and its revenue was increased significantly by approximately 40% from this implementation [1]. Although revenue management has been widely used in the airline industry, it has been observed that the cargo segment just have received increase attention [2]. Also, there are relatively small numbers of revenue management related articles that have been published in air cargo industry.

Air cargo industry is one of sectors in airline industry which has been expanded over the past decades due to rapid growth in economy and international trade. Although the world trade growth is not much expanded, air cargo is still essential shipment means for time-sensitive goods. The high-speed and reliability advantages of air freight guarantee that it will still play an important role in the global economy.

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Shippers, the freight forwarders (FFs) and the airlines are three major players in air cargo supply chains. The activities in this chain starts when the shippers either contact freight forwarders, or book by themselves to ship goods from one airport to another. A booking request will contain data such as details of goods, date of shipment, origin, destination, weight and volume. Then the airline will accept the booking request if the total volume and weight of current bookings, including the booking request, does not exceed the overbooking level. Once accepted, the process is finished and that booking request is converted into a booking.

In practice, bookings may be canceled prior to the shipment date and this cancellation reduces the booking capacity. In other words, it increases the available capacity for the airline. When a cancellation is made, penalties may incur to shippers or freight forwarders. Nevertheless, many shippers or freight forwarders can negotiate for no penalty cost by doing long-term contract [3]. If a booking does not show up at the flight departure time, it is called "no-show", while the total bookings that show up at the departure time is called "show up booking".

The cargo capacity has all the characters for revenue management strategy to be successful: it is lost after the plane takes off, it has limited resources, and it can be offered at different price rates depending on the service offered [4]. Overbooking is one of RM tools. It is an approach of selling more goods or services than available capacities where sellers foresee that some buyers will not show-up or may cancel the bookings before the departure time. The objective of overbooking is to minimize the total offloaded and spoilage costs, or to maximize the expected net revenue (which is revenue minus expected costs). Offloaded costs are incurred when the show up booking is greater than the available capacity while spoilage costs are considered as revenues lost when airlines are not able to fill the capacity. Thus, overbooking decision is a trade-off strategy between offloaded and spoilage costs. Offensive strategy (e.g., booking much higher than their available capacity) can incur high offloaded costs, while risk-averse strategy such as no overbooking or allowing too few overbooking levels can lead to loss of opportunity to sell canceled capacity to others. Thus, setting the overbooking level should be watchfully determined.

Air cargo overbooking strategy received more attention because it helps airlines getting addition revenue without much investment. Reference [5] pointed out that the implementation of overbooking strategy is expected to generate 40% additional revenue. Passenger overbooking was the first strategy that airlines implemented. Consequently,

airlines seek to adapt the same techniques to air cargo business. However, smaller numbers of overbooking literatures have been published in air cargo. Reference [1] explained the differences between air cargo and passenger overbooking, and developed a method for cargo revenue management. Reference [6] developed an optimal overbooking model for stochastic demand. Reference [4] developed an overbooking model in aspect of minimizing the sum of offloaded and spoilage cost. Reference [2] compared discrete and normal show-up-rate estimators. Reference [7] considered the capacity allocation problem with random volume and weight as a Markov decision process. They developed the booking heuristic for overbooking process in term of weight and volume. Moreover, extensive simulation experiments in their paper suggested that optimal overbooking level computing separately for each dimension offered the most beneficial results. Reference [3] introduced two-dimensional (weight and volume) overbooking problems arising mainly in the cargo revenue management, and compared them with one-dimensional problems.

Even though there are a few papers proposed two-dimensional overbooking model, joint distribution between weight and volume cancellation has not yet been considered. In practice, weight and volume cancellation are highly correlated and it can generate different amount of offloaded and spoilage costs. The model presented in this paper considers these effects and proposes a method to determine the optimal number of overbooking levels for both weight and volume aspects in order to minimize the total cost: offloaded cost and spoilage cost.

This paper is organized as follows. In the next section, we present model description and formulations, as well as theoretical results of overbooking models for cargo overbooking. Computational experiments on how solutions are affected by key model parameters and the performance of the two-dimensional model are explored in Section IV. We conclude by summarizing important results and providing managerial insights in Section V. The proofs of all results are provided in the Appendix.

II. MODEL DESCRIPTION

This section describes the formulation of cargo overbooking model. The flight considered in this paper is a single-leg flight with cargo carried aircraft with no passengers. The situation when the show-up cargo exceeds actual capacity is called offload. In other words, it is when the no-show capacity is lower than overbooking level. When this situation happens, the airline may handle these offloaded items by shipping them via another airline in its network. Thus, the offloaded cost are comprised of 1) addition cost of storage and handling, 2) addition cost needed to pay another airline network, 3) penalty from delayed shipping, etc. On the other hand, spoilage cost virtually incurs when the available capacity are underutilized (no-show capacity is higher than overbooking level). This spoilage cost can be considered as an opportunity cost from not selling this capacity to other customers.

Let X_v and X_w be the quantity of volume and weight capacity cancelled or no-show, respectively. Let $f(x_v, x_w)$ be the joint probability density function of cancelled capacity or no-show of volume and weight. Let $f_v(x_v)$ and $f_w(x_w)$ be the marginal probability density function of cancelled capacity or no-show of volume and weight, respectively, $F_v(x_v)$ and $F_w(x_w)$ be the cumulative density function of cancelled capacity or no-show of volume and weight. Define csv and csw as spoilage costs of volume and weight, while cov and cow are the offloaded costs of volume and weight. The decision variables of the model are Q_v and Q_w, which are the amount of the overbooking capacity of volume and weight, respectively.

Generally, the overbooking model considers the loss of revenue and cost from the following two possible cases: (1) spoilage capacity, and (2) offloaded capacity. The spoilage capacity occurs when the no-show or cancelled capacity is higher than the overbooking capacity ($X_v > Q_v$ or $X_w > Q_w$). While the offloaded capacity occurs when the cancelled or no-show capacity is lower than the overbooking capacity ($X_v < Q_v$ or $X_w < Q_w$). To be specific, let us consider the potential situations at departure date. According to the fact that the total cargo capacity has volume and weight dimensions, there are weight and volume overbooking level. Particularly, four cases can be presented at departure date.

1. Both cancelled or no-show volume and weight are higher than the volume and weight overbooking level, the weight and volume capacity are spoiled.
2. Cancelled or no-show volume is higher than the volume overbooking level, while cancelled or no-show weight is lower than the weight overbooking level. The volume capacity is spoiled, while the weight capacity is offloaded.
3. Cancelled or no-show volume is lower than the volume overbooking level, while cancelled or no-show weight is higher than the weight overbooking level. The weight capacity is spoiled and the volume capacity is offloaded.
4. Cancelled or no-show volume and weight show-up are lower than the volume and weight overbooking level, the weight and volume capacity are offloaded.

The four cases above can be summarized as shown in Table I.

In this research, we consider two models. The first one is called "two-dimensional model with joint probability function". In the second model, we consider simpler model that disregard the correlation between cancelled volume and weight capacity, called "two-dimensional model with independence property between volume and weight cancelled or no-show".

TABLE I
FOUR POSSIBLE SITUATIONS OF WEIGHT AND VOLUME CAPACITY

Case	Volume capacity	Weight capacity
1	Spoiled $x_v > Q_v$	Spoiled $x_w > Q_w$
2	Spoiled $x_v > Q_v$	Offloaded $x_w < Q_w$
3	Offloaded $x_v < Q_v$	Spoiled $x_w > Q_w$
4	Offloaded $x_v < Q_v$	Offloaded $x_w < Q_w$

A. Two Dimensional Model with Joint Probability Function

From Table I, the situations on the departure date can be presented in four possible cases. Let $TC_{[a.b]}$ be the cost of model a occurred from case b when $a = 1,2$ and $b=1,2,3,4$. For case 1 of model 1, both cancelled or no-show volume and weight are higher than the volume and weight overbooking level, the weight and volume capacity are spoiled and the cost can be summarized as:

$$TC_{[1.1]} = \int_{Qw}^{\infty} \int_{Qv}^{\infty} \left(csv(x_v - Q_v) + csw(x_w - Q_w)\right) * f(x_v, x_w)dx_v dx_w$$

For case 2, cancelled or no-show volume is higher than volume overbooking level, while cancelled or no-show weight is lower than weight overbooking level. The volume capacity is spoiled and the weight capacity is offloaded. The cost for this case can be summarized as:

$$TC_{[1.2]} = \int_{0}^{Qw} \int_{Qv}^{\infty} \left(csv(x_v - Q_v) + cow(Q_w - x_w)\right) * f(x_v, x_w)dx_v dx_w$$

For case 3, cancelled or no-show volume is lower than volume overbooking level, while cancelled or no-show weight is higher than weight overbooking level. The weight capacity is spoiled and the volume capacity is offloaded, and the cost can be summarized as:

$$TC_{[1.3]} = \int_{Qw}^{\infty} \int_{0}^{Qv} \left(cov(Q_v - x_v) + csw(x_w - Q_w)\right) * f(x_v, x_w)dx_v dx_w$$

For case 4, cancelled or no-show volume and weight show-up are lower than volume and weight overbooking level, the weight and volume capacity are offloaded. The cost for this last case can be summarized as:

$$TC_{[1.4]} = \int_{0}^{Qw} \int_{0}^{Qv} \left(cov(Q_v - x_v) + cow(Q_w - x_w)\right) * f(x_v, x_w)dx_v dx_w$$

Considering all possible situations, the expectation of total cost of model 1 is equal to:

$$
\begin{aligned}
TC_1 &= TC_{[1.2]} + TC_{[1.2]} + TC_{[1.3]} + TC_{[1.4]} \\
&= \int_{Qw}^{\infty} \int_{Qv}^{\infty} \left(csv(x_v - Q_v) + csw(x_w - Q_w)\right) * f(x_v, x_w)dx_v dx_w \\
&+ \int_{0}^{Qw} \int_{Qv}^{\infty} \left(csv(x_v - Q_v) + cow(Q_w - x_w)\right) * f(x_v, x_w)dx_v dx_w \\
&+ \int_{Qw}^{\infty} \int_{0}^{Qv} \left(cov(Q_v - x_v) + csw(x_w - Q_w)\right) * f(x_v, x_w)dx_v dx_w \\
&+ \int_{0}^{Qw} \int_{0}^{Qv} \left(cov(Q_v - x_v) + cow(Q_w - x_w)\right) * f(x_v, x_w)dx_v dx_w.
\end{aligned}
$$

$$(1)$$

B. Two-Dimensional Model with Independence Property

When the distributions of volume and weight cancellation are uncorrelated, the joint probability is equal to the product between marginal probability function of volume and weight

cancellation. For case 1, both cancelled or no-show volume and weight are higher than volume and weight overbooking levels, the weight and volume capacity are spoiled and the cost of model 2 can be summarized as:

$$
\begin{aligned}
TC_{[2.1]} &= P(X_w > Q_w) \int_{Qv}^{\infty} csv(x_v - Q_v)f_v(x_v)dx_v \\
&+ P(X > Q_v) \int_{Qw}^{\infty} csw(x_w - Q_w)f_w(x_w)dx_w
\end{aligned}
$$

For case 2 to 4, the cost of model 2 can be written as follows:

$$
\begin{aligned}
TC_{[2.2]} &= P(X_w < Q_w) \int_{Qv}^{\infty} csv(x_v - Q_v)f_v(x_v)dx_v \\
&+ P(X_v > Q_v) \int_{0}^{Qw} cow(Q_w - x_w)f_w(x_w)dx_w \\
TC_{[2.3]} &= P(X_w > Q_w) \int_{0}^{Qv} cov(Q_v - x_v)f_v(x_v)dx_v \\
&+ P(X_v < Q_v) \int_{Qw}^{\infty} csw(x_w - Q_w)f_w(x_w)dx_w \\
TC_{[2.4]} &= P(X_w < Q_w) \int_{0}^{Qv} cov(Q_v - x_v)f_v(x_v)dx_v \\
&+ P(X_v < Q_v) \int_{0}^{Qw} cow(Q_w - x_w)f_w(x_w)dx_w
\end{aligned}
$$

Considering all possible situations, the expectation of total cost of model 2 is equal to:

$$
\begin{aligned}
TC_2 &= TC_{[2.1]} + TC_{[2.2]} + TC_{[2.3]} + TC_{[2.4]} \\
&= [1 - F_w(Q_w)] \int_{Qv}^{\infty} csv(x_v - Q_v)f(x_v)dx_v \\
&+ [1 - F_v(Q_v)] \int_{Qw}^{\infty} csw(x_w - Q_w)f(x_w)dx_w \\
&+ F_w(Q_w) \int_{Qv}^{\infty} csv(x_v - Q_v)f(x_v)dx_v \\
&+ [1 - F_v(Q_v)] \int_{0}^{Qw} cow(Q_w - x_w)f(x_w)dx_w \\
&+ [1 - F_w(Q_w)] \int_{0}^{Qv} cov(Q_v - x_v)f(x_v)dx_v \\
&+ F_v(Q_v) \int_{Qw}^{\infty} csw(x_w - Q_w)f(x_w)dx_w \\
&+ F_w(Q_w) \int_{0}^{Qv} cov(Q_v - x_v)f(x_v)dx_v \\
&+ F_v(Q_v) \int_{0}^{Qw} cow(Q_w - x_w)f(x_w)dx_w.
\end{aligned}
$$

$$(2)$$

Lemma 1. *The total cost function presented in (2) is jointly convex with respect to Q_v and Q_w.*

Theorem 1. *The optimal overbooking level of weight and volume (Q_v^* and Q_w^*) from (2) can be determined by the following two equations :*

$$-csv(1 - F_v(Q_v)) + cov(F_v(Q_v)) = 0 \qquad (3)$$

and

$$-csw(1 - F_w(Q_w)) + cow(F_w(Q_w)) = 0 \qquad (4)$$

The proofs of Lemma 1 and Theorem 1 can be referred in Appendix. Theorem 2 provides two equations in order to determine the optimal overbooking level of volume and weight for the situation when the distribution between volume and weight are independent.

From both (1) and (2), it can be noticed that the solutions of the model rely on many input parameters, e.g., the distribution of cancelled or no-show weight and volume as well as the parameters of the distribution such as expectation and variance, the correlation between cancelled or no-show weight and volume, the average spoilage cost of weight and volume per unit, the average offloaded cost of weight and volume per unit. Thus, in the next section we explore how these parameters affect total cost and optimal overbooking levels obtained from (1) and (2).

III. COMPUTATIONAL EXPERIMENTS

In this section, we perform computational experiments for the two-dimensional cargo overbooking model that we describe in the former section. The goals of section are to:

1) Understand how the total cost and optimal overbooking levels change when model parameters are varied.
2) Test the performance of solutions obtained from the model with independence property as compared to the model with joint probability function and a naïve approach.

Fig. 1 shows the optimal overbooking level of volume obtained from model 1 (Two dimensional model with joint probability function) as csv/cov increases. We can observe that the optimal overbooking level of volume increases as csv/cov increases. This is because as the spoilage cost increases as compared to the offloaded cost for volume, it is more costly if volume capacity is unused. The model, then, suggests increasing the overbooking level for volume to reduce that risk. Even though this result may be intuitive, the model will help suggesting the right amount of increasing of overbooking level when the ratio of cost parameters csv/cov increase.

On the other hand, Fig. 2 shows the optimal overbooking level of volume obtained from model 1 as cov/csv increases. The result shows that the optimal overbooking level of volume decreases as cov/csv increases. A reason behind this is as the offloaded cost increases as compared to the spoilage cost for volume, it is more costly if there is not enough volume to serve when the canceled volume is less than the overbooking capacity. This might occur when there is no other similar flight in the same departure date to absorb the show-up bookings if the canceled or no-show bookings are less than the overbooking capacity. The airline then needs to pay high extra fee for the offloaded capacity. So, the model's solution recommends decreasing the overbooking level for volume to reduce that risk.
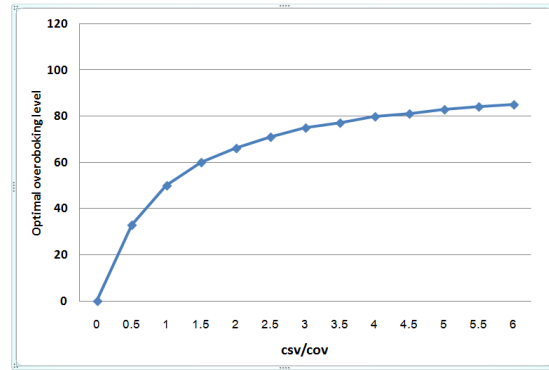

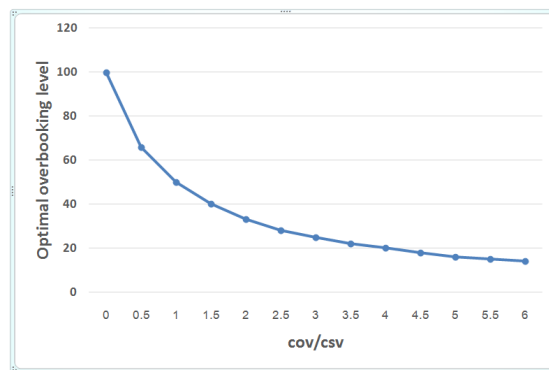Fig. 1 The optimal overbooking level of volume as csv/cov increases

Fig. 2 The optimal overbooking level of volume as cov/csv increases

Next, we aim to explore how the solution of our models performs as compared to a naïve method. Let us consider the following situations:

1) Method 1 represents solutions solved from the model 1, described in Section II *A* (Two dimensional model with joint probability function)
2) Method 2 represents solutions solved from the model 2, described in Section II *B* (Two-dimensional model with independence property)
3) Method 3 represents a naïve solution, using the average of volume cancellation and weight cancellation as the overbooking levels. This method is currently used in some airlines when there is no historical data analysis of volume and weight cancellations.

The goal of this computational experiment is to find the total cost occurred from 3 different methods explained above. Basically, if there are some correlations between weight and volume cancellation, but the airlines use solutions from simpler model (model 2, Two-dimensional model with independence property), it will incur higher cost than using the actual optimal solution obtained by solving model 1 (Two dimensional model with joint probability function). However, since we have proved that the objective function of model 2 is jointly convex with overbooking decisions (volume and weight overbooking levels), this model is then easier to be solved. So, it is interesting to learn its performance as compared to the full model 1.

Also, another point of this experiment is to identify the total cost if the airlines simply use their average of volume and weight cancellations as their overbooking solutions. It is because some airlines' data recording software is not able to automatically identify or analyze the historic cancellation distribution. Thus, those airlines may simply determine their overbooking level by using the average no-show or cancelled capacity they have observed in the past. This experiment will also point out which situation is fine to use naïve method and which situation is more beneficial to apply our proposed models to solve for the optimal solutions.

The distributions we considered in this research are: (1) bivariate uniform distribution and (2) bivariate normal distribution, which are common distributions for cargo cancellations. In the first case, it is known that if X and Y follow bivariate uniform distribution on (0, y) × (0, z), then X and Y are independent. Thus, if the cancelled or no-show volume and weight capacity (X_v and X_w) follow the bivariate uniform distribution on (0, y) × (0, z), the optimal solutions obtained from model 1 and model 2 will be the same. This is because the total cost shown in equation (1) can be written as equation (2) in this case. Fig. 3 presents the optimal total cost of the overbooking at different values of *csv/cov*. It can be seen that the optimal total cost obtained by solving (1) is equal to that of (2). Furthermore, the cost occurred from method 3 which uses only the average of cancelled or no-show capacity to determine the overbooking level tends to provide poorer performance as the ratio of *csv* and *cov* increases. This is because when the spoilage cost is very high compared to the offloaded cost, it is more beneficial to increase the volume overbooking level. Thus, using the average of volume cancellation as the overbooking level leads to higher cost. Another observation is that the cost occurred by using method 3 is closer to the cost obtained by method 1 or 2 when *csv/cov* is equal to 1. It implies using the average of volume cancellation as the overbooking level is fine in that case but not others. Similar observation can be noticed in Fig. 4 which presents the optimal total cost of the overbooking at different values of *cov/csv*.
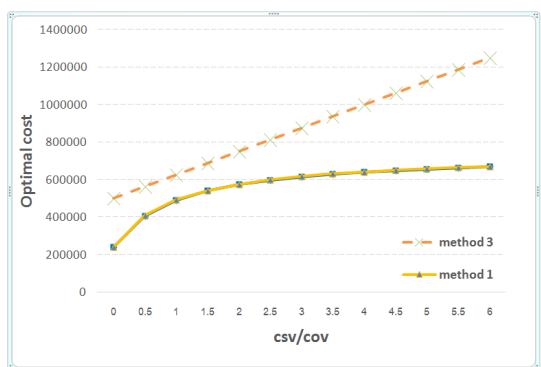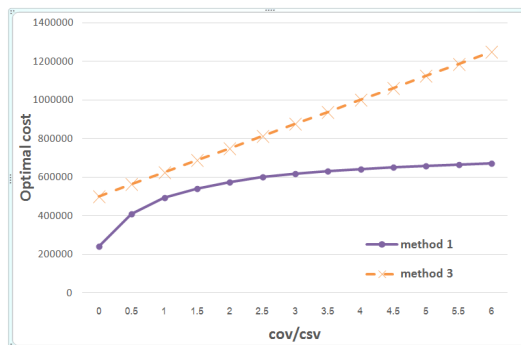

Fig. 4 The optimal total cost at different values of *cov/csv* (when *csv*=10000, *csw*=40000, *cow*=10000)

Fig. 5 presents the optimal overbooking level of volume computed from method 1, 2 and 3 under different values of the average cancelled or no-show of volume capacity (μ_{Xv}). Note that optimal overbooking level obtained by solving (1) is equal to (2) since the cancelled or no-show volume and weight capacity (X_v and X_w) in this experiment follow the bivariate uniform distribution. When μ_{Xv} increases, the optimal overbooking level of volume increases because it is believed that the cancelled or no-show capacity will be higher and it is wiser to allow higher overbooking level to fill the leftover capacity. An interesting observation is that the optimal volume overbooking level suggested by method 3 increases at a lower rate as compared to that of method 1 and 2. Let us consider the cost aspect, Fig. 6 shows the optimal total cost of the overbooking model at different values of μ_{Xv}. It can be seen that when μ_{Xv} is low (only little volume cancellation are expected), the difference between optimal cost of method 1 and 3 are also low. However, as μ_{Xv} increases the optimal cost computed by method 3 provides worse result. It implies that when higher cancellation or no-show are likely to happen, using average of past cancellation as overbooking level is not recommendable.
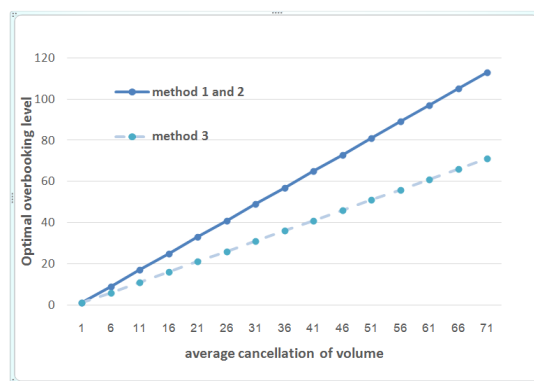

Fig. 3 The optimal total cost at different values of *csv/cov* (when *cov*=10000, *csw*=40000, *cow*=10000)

Fig. 5 The optimal overbooking level of volume computed from method 2 and 3 at different values of μ_{Xv} (when *csv*=40000, *cov*=10000, *csw*=40000, *cow*=10000)
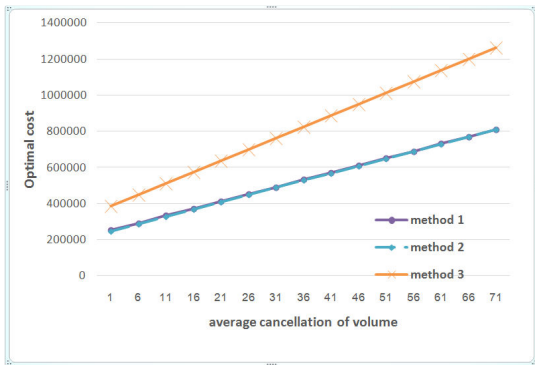
Fig. 6 The optimal total cost at different values of μ_v (when csv=40000, cov=10000, csw=40000, cow=10000)

After obtaining insights on model solutions' characteristic for bivariate uniform distribution, now the bivariate normal distribution is studied. A flight considered has weight capacity of 100 tons and 500 cubic meter. The average cancelled or no-show of weight capacity is 10 tons and that of volume capacity is 50 cubic meter. Let (X_v, X_w) follow bivariate normal distribution. Let μ_v= 50 cubic meter, μ_w= 30 tons, σ_w= 15 tons, and σ_v= 20 cubic meter. Fig. 7 presents the optimal total cost obtained by method 1 and 3 at different values of cov/csv. Also, for method 1, we study two cases: (1) when correlation of X_v and X_w is high ($\rho = 0.9$) and (2) when there is no correlation ($\rho = 0$). From Fig. 7, when cov/csv is higher than 1, the gap between the optimal cost from method 1 and method 3 is larger. It can also be observed that when the correlation of X_v and X_w is high ($\rho = 0.9$), the optimal cost is a bit higher than when there is no correlation of X_v and X_w. However, the difference is not significant as compared to the high cost obtained from using the average of volume cancellation and weight cancellation as the overbooking levels.
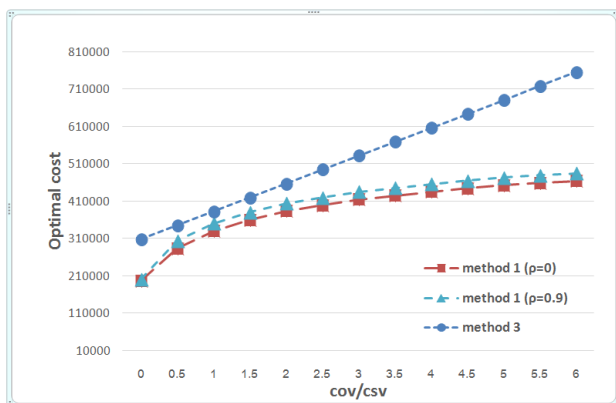

Fig. 7 The optimal total cost at different values of cov/csv (when csv=10000, csw=40000, cow=10000)

Fig. 8 shows the optimal total cost of the overbooking at different values of the standard deviation of cancelled or no-show of volume capacity(σ_v), ranging from 1 to 66. It can be seen that even if the correlation between cancelled or no-show

of volume and weight capacity (ρ) is as high as 0.9, the optimal cost is still not much different from when $\rho = 0$. In addition, the optimal cost obtained from method 3 is not much higher than that of method 1 when σ_v is low. That if there is not much uncertainty about cancelled or no-show of volume capacity, using method 3 (the average of volume cancellation and weight cancellation as the overbooking levels) is acceptable. Nevertheless, if there is uncertainty about cancelled or no-show of volume capacity, using method 3 will be more costly as σ_v increases. In that case, it is worth using method 1 (Two dimensional model with joint probability function).
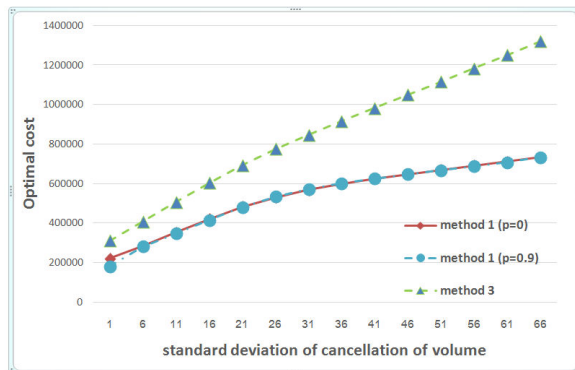

Fig. 8 The optimal total cost at different values of σ_v (when csv=40000, cov=10000, csw=40000, cow=10000)

IV. CONCLUSION

Overbooking policy for the airlines is one of the important revenue management strategies. In this policy, it is essential for airlines to find an appropriate method to determine the optimal overbooking level. Since there are two dimensions in the capacity of air cargo: volume and weight, the cancellation or no-show are also random in both properties. In this paper, we present the two model approaches to determine the overbooking level: (1) Two dimensional model with joint probability function and (2) Two-dimensional model with independence property. We proved that the total cost function of the second model is jointly convex with respect to the decision variables that are the overbooking level of weight and volume. The equations to determine the optimal values of those variables are also provided.

Two distributions of the cancelled or no-show volume and weight capacity are considered: (1) bivariate uniform distribution and (2) bivariate normal distribution. Computational experiment are conducted to understand how the total cost and optimal overbooking levels change when model parameters are varied and to test the performance of solutions obtained from the model with independence property as compared to the model with joint probability function and the naïve approach. It is observed that it is worthwhile to apply the two dimensional model with joint probability function when (1) there is a significant difference between the spoilage cost and the offloaded cost, (2) large number of cancellation or no-show are likely to happen (average value of

cancellation or no-show capacity is high), and (3) much variation in cancellation or no-show are expected. This is because the solution from a naïve method using the average value of past cancellation or no-show causes much higher cost than the proposed models in those cases.

There are several possible extensions of this paper. First, it can be noticed from the computational results that other important parameters apart from the distribution of cancelled or no-show capacity are cost of spoilage and cost of offloaded. It is interesting to identify a practical mean to determine those parameters for future model implementation. Since a cargo carrier flight is the main focus in this paper, a second possible extension is to extend the idea in this paper to passenger carrier flight having uncertainty of both passenger seats and luggage cancellations. In addition, it can be interesting to explore whether there are other methods to solve for the optimal overbooking level under different cancellation distributions, other than what were considered in this paper.

APPENDIX

Proof of Lemma 1 and Theorem 1

$$Total\ Cost\ = [1 - F_w(Q_w)]\int_{Qv}^{\infty} csv(x_v - Q_v)f(x_v)dx_v$$

$$+ [1 - F_v(Q_v)]\int_{Qw}^{\infty} csw(x_w - Q_w)f(x_w)dx_w$$

$$+ F_w(Q_w)\int_{Qv}^{\infty} csv(x_v - Q_v)f(x_v)dx_v$$

$$+ [1 - F_v(Q_v)]\int_{0}^{Qw} cow(Q_w - x_w)f(x_w)dx_w$$

$$+ [1 - F_w(Q_w)]\int_{0}^{Qv} cov(Q_v - x_v)f(x_v)dx_v$$

$$+ F_w(Q_w)\int_{Qw}^{\infty} csw(x_w - Q_w)f(x_w)dx_w$$

$$+ F_w(Q_w)\int_{0}^{Qv} cov(Q_v - x_v)f(x_v)dx_v$$

$$+ F_v(Q_v)\int_{0}^{Qw} cow(Q_w - x_w)f(x_w)dx_w.$$

Making use of Leibniz Integral Rule, the objective function could be minimized at a relative ease. Taking the first order derivative on both sides of the objective function:

$$\frac{\partial TC}{\partial Q_v} = [1 - F_w(Q_w)]\left(-\int_{Qv}^{inf} csv * f(x_v)dx_v\right)$$

$$+(-f(x_v))\int_{Qw}^{inf} csw(x_w - Q_w) * f(x_w)dx_w$$

$$+ F_w(Q_w)\left(-\int_{Qv}^{inf} csv * f(x_v)dx_v\right)$$

$$+(-f(x_v))\int_{0}^{Qw} cow(Q_w - x_w) * f(x_w)dx_w$$

$$+[1 - F_w(Q_w)]\left(\int_{0}^{Qv} cov * f(x_v)dx_v\right)$$

$$+(f(x_v))\int_{Qw}^{inf} csw(x_w - Q_w) * f(x_w)dx_w$$

$$+ F_w(Q_w)\left(\int_{0}^{Qv} cov * f(x_v)dx_v\right)$$

$$+(f(x_v))\int_{0}^{Qw} cow(Q_w - x_w) * f(x_w)dx_w$$

The above equation can be reduced into:

$$\frac{\partial TC}{\partial Q_v} = \left(-\int_{Qv}^{inf} csv * f(x_v)dx_v\right) + \left(\int_{0}^{Qv} cov * f(x_v)dx_v\right)$$

$$\frac{\partial TC}{\partial Q_v} = -csv(1 - F(Q_v)) + cov(F(Q_v))$$

Using the same method, the derivation of the total cost respect to Q_w is:

$$\frac{\partial TC}{\partial Q_w} = -csw(1 - F(Q_w)) + cow(F(Q_w))$$

Next, the second derivatives of objective function with respect to Q_w and Q_v are:

$$\frac{\partial^2 TC}{\partial Q_v^2} = csv(f(x_v)) + cov(f(x_v))$$

$$\frac{\partial^2 TC}{\partial Q_w^2} = csw(f(x_w)) + cow(f(x_w))$$

$$\frac{\partial^2 TC}{\partial Q_v Q_w} = \frac{d^2 TC}{dQ_w Q_v} = 0$$

Thus we have $\frac{\partial^2 TC}{\partial Q_v^2} > 0$ and

$$\left(\frac{\partial^2 TC}{\partial Q_v^2}\right)\left(\frac{\partial^2 TC}{\partial Q_w^2}\right) - \left(\frac{\partial^2 TC}{\partial Q_v Q_w}\right)\left(\frac{d^2 TC}{dQ_w Q_v}\right) > 0$$

Since the second derivative of the function is nonnegative (By Hessian Matrix), the objective function is, therefore, jointly convex with respect to Q_w and Q_v. Thus, The optimal overbooking level of weight and volume (Q_v^* and Q_w^*) from (2) can be determined by the following two equations :

$$-csv(1 - F_v(Q_v)) + cov(F_v(Q_v)) = 0$$

and

$$-csw(1 - F_w(Q_w)) + cow(F_w(Q_w)) = 0.$$

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