

# Adomian's Decomposition Method to Generalized Magneto-Thermoelasticity

Hamdy M. Youssef, Eman A. Al-Lehaibi

**Abstract**—Due to many applications and problems in the fields of plasma physics, geophysics, and other many topics, the interaction between the strain field and the magnetic field has to be considered. Adomian introduced the decomposition method for solving linear and nonlinear functional equations. This method leads to accurate, computable, approximately convergent solutions of linear and nonlinear partial and ordinary differential equations even the equations with variable coefficients. This paper is dealing with a mathematical model of generalized thermoelasticity of a half-space conducting medium. A magnetic field with constant intensity acts normal to the bounding plane has been assumed. Adomian's decomposition method has been used to solve the model when the bounding plane is taken to be traction free and thermally loaded by harmonic heating. The numerical results for the temperature increment, the stress, the strain, the displacement, the induced magnetic, and the electric fields have been represented in figures. The magnetic field, the relaxation time, and the angular thermal load have significant effects on all the studied fields.

**Keywords**—Adomian's Decomposition Method, magneto-thermoelasticity, finite conductivity, iteration method, thermal load.

## I. INTRODUCTION

It was assumed that the interactions between the two fields take place according to the Lorentz forces embedded in the equations of motion. By using Ohm's law, the electric field produced by the velocity of a material particle, moves in a magnetic field [1], [2]. The basics of the magnetoelasticity were introduced by Knopoff [3] and developed by Kaliski and Petykiewicz [4]. The most important types of thermoelasticity are Biot model of coupled thermoelasticity [5] and Lord-Shulman model of generalized thermoelasticity [6] in which many authors considered the generalized magneto-thermoelasticity equations [1], [2], [7]-[13]. Currently, more attention has been concentrated to the numerical methods which do not need discretization of time-space variables or to linearization of the nonlinear equations [14]-[17]. The solution can be verified to any degree of approximation. Adomian decomposition method has been used to get the formal solutions to a wide class of partial and ordinary differential equations [18]-[29]. Adomian modeled as systems of nonlinear differential equations by the ADM the dynamic interaction of immune response with a population of viruses, bacteria, antigens or tumor cells [17]. Adomian decomposition

method (ADM) separates the given equation to linear and nonlinear parts, invert the highest-order derivative in both sides, and find the successive terms of the series solution by recurrence relation [14], [26]. Many modifications have been done to ADM to improve the accuracy or expand the application of the original method [23], [25], [29]. Recently, the decomposition method has been used in fractional differential equations [30]-[32].

## II. FORMULATION OF THE PROBLEM

The mathematical model of a thermoelastic half-space  $0 \leq x < \infty$  has been considered. A magnetic field with constant intensity  $H_0$  acts normal to the bounding plane. All the quantities considered will be functions of the time variable  $t$ , and the distance  $x$  starts from the bounding plane. The medium is assumed initially quiescent. The primary magnetic field  $H_0$  generates an induced magnetic field  $h$  and an induced electric field  $E$ . We assume that both  $h$  and  $E$  are small in magnitude by the assumptions of the linear theory of thermoelasticity [1], [2].

Thus, the displacement vector will take the components

$$u_x = u(x, t), u_y = u_z = 0 \quad (1)$$

The components of the magnetic intensity vector are

$$H_x = 0, H_y = H_0 + h(x, t), H_z = 0 \quad (2)$$

The electric intensity vector is perpendicular to both the magnetic intensity and the displacement vectors. Thus,  $E$  has the following component:

$$E_x = E_y = 0, E_z = E(x, t) \quad (3)$$

The current density vector  $J$  must be parallel to  $E$ , hence

$$J_x = J_y = 0, J_z = J(x, t) \quad (4)$$

Maxwell's equations in vector form can be written as

$$\text{curl } h = J + \varepsilon_0 \frac{\partial E}{\partial t} \quad (5)$$

$$\text{curl } E = -\mu_0 \frac{\partial h}{\partial t} \quad (6)$$

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$$\operatorname{div} h = 0, \quad \operatorname{div} E = 0 \quad (7)$$

$$B = \mu_0 (H_0 + h), \quad D = \varepsilon_0 E \quad (8)$$

where  $\varepsilon_0$  and  $\mu_0$  are the electric and magnetic permeability's, respectively.

By Ohm's law, the current density is defined as [1]-[3], [11]:

$$J = \sigma_0 \left( E + \mu_0 \frac{\partial u}{\partial t} \times H \right) \quad (9)$$

where  $\sigma_0$  is the electric conductivity.

The strain components are given by,

$$e_{xx} = \frac{\partial u}{\partial x}, \quad e_{yy} = e_{zz} = e_{xz} = e_{yz} = e_{xy} = 0 \quad (10)$$

The stress components are given by the relation:

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e \delta_{ij} - \gamma (T - T_0) \delta_{ij} \quad (11)$$

$$\sigma_{xx} = \sigma = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma (T - T_0) \quad (12)$$

$$\sigma_{yy} = \sigma_{zz} = \lambda \frac{\partial u}{\partial x} - \gamma (T - T_0) \quad (13)$$

$$\sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0 \quad (14)$$

where  $\lambda$  and  $\mu$  are Lamé's constants,  $T$  is the absolute temperature of the medium,  $\gamma$  is a material constant given by  $\gamma = (3\lambda + 2\mu)\alpha_T$ ,  $\alpha_T$  being the coefficient of linear thermal expansion, is Dirac's delta function and  $T_0$  is a reference temperature chosen such that  $|(T - T_0)/T_0| \leq 1$ .

Equations of motion take the form [6],

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i \quad (15)$$

where  $\rho$  is the density and  $F$  is the Lorentz force given by:

$$F = J \times B \quad (16)$$

After linearization, Ohm's law gives

$$J = \sigma_0 \left( E + \mu_0 H_0 \frac{\partial u}{\partial t} \right) \quad (17)$$

After linearization, (16) and (17), we get,

$$F_x = -\mu_0 \sigma_0 H_0 \left( E + \mu_0 H_0 \frac{\partial u}{\partial t} \right), \quad F_y = F_z = 0 \quad (18)$$

Substituting (18) into (15), we obtain the equation of motion in the form,

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} - \sigma_0 \mu_0 H_0 E - \sigma_0 \mu_0^2 H_0^2 \frac{\partial u}{\partial t} \quad (19)$$

The equation of heat conduction has the form [6],

$$\frac{\partial^2 T}{\partial x^2} = \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left( \frac{\rho C_E}{K} T + \frac{\gamma T_0}{K} \frac{\partial u}{\partial x} \right) \quad (20)$$

where  $K$  is the thermal conductivity of the medium  $C_E$  is the specific heat at constant strain and  $\tau_0$  is the relaxation time.

Equation (5) takes the form,

$$\frac{\partial h}{\partial x} = J + \varepsilon_0 \frac{\partial E}{\partial t} \quad (21)$$

Substituting from (17), we get

$$\frac{\partial h}{\partial x} = \varepsilon_0 \frac{\partial E}{\partial t} + \sigma_0 E + \sigma_0 \mu_0 H_0 \frac{\partial u}{\partial t}. \quad (22)$$

Equation (6) takes the form,

$$\frac{\partial E}{\partial x} = \mu_0 \frac{\partial h}{\partial t}. \quad (23)$$

For simplifications we shall use the following non-dimensional variables [1], [2]:

$$x' = c_0 \eta x, \quad u' = c_0 \eta u, \quad \sigma'_{ij} = \frac{\sigma_{ij}}{\lambda + 2\mu}, \quad \theta = \frac{\gamma (T - T_0)}{\lambda + 2\mu}, \quad (24)$$

$$t' = c_0^2 \eta t, \quad \tau'_0 = c_0^2 \eta \tau_0, \quad E' = \frac{\eta}{\sigma_0 \mu_0^2 H_0 c_0} E, \quad h' = \frac{\eta}{\sigma_0 \mu_0 H_0} h$$

$$\text{where } \eta = \frac{\rho C_E}{K}, \quad c_0^2 = \frac{\lambda + 2\mu}{\rho}.$$

The above equations reduce to (dropping the primes for simplicity):

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial \theta}{\partial x} - \nu^2 \varepsilon_2 E - \nu \varepsilon_2 \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2} \quad (25)$$

$$\frac{\partial^2 \theta}{\partial x^2} = \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left( \theta + \varepsilon_1 \frac{\partial u}{\partial x} \right) \quad (26)$$

$$\frac{\partial h}{\partial x} = V^2 \frac{\partial E}{\partial t} + \nu E + \frac{\partial u}{\partial t} \quad (27)$$

$$\frac{\partial E}{\partial x} = \frac{\partial h}{\partial t} \quad (28)$$

$$\text{where } \nu = \frac{\sigma_0 \mu_0}{\eta}, \varepsilon_2 = \frac{\mu_0 H_0^2}{\lambda + 2\mu}, \varepsilon_1 = \frac{T_0 \gamma^2}{C_E \rho^2 c_0^2}, V = \frac{c_0}{c}, c^2 = \frac{I}{\varepsilon_0 \mu_0}.$$

The constitutive equations reduce to:

$$\sigma_{xx} = e - \theta \quad (29)$$

$$\sigma_{yy} = \sigma_{zz} = \left(1 - \frac{2}{\beta^2}\right) e - \theta \quad (30)$$

$$\text{where } \beta^2 = \frac{\lambda + 2\mu}{\mu}.$$

Re-writing (25)-(27) by using (10), we obtain

$$\frac{\partial^2 e}{\partial x^2} - \frac{\partial^2 \theta}{\partial x^2} - V^2 \varepsilon_2 \frac{\partial E}{\partial x} - \nu \varepsilon_2 \frac{\partial e}{\partial t} = \frac{\partial^2 e}{\partial t^2} \quad (31)$$

$$\frac{\partial^2 \theta}{\partial x^2} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) (\theta + \varepsilon_1 e) \quad (32)$$

$$\frac{\partial^2 h}{\partial x^2} = V^2 \frac{\partial^2 E}{\partial t \partial x} + \nu \frac{\partial E}{\partial x} + \frac{\partial e}{\partial t} \quad (33)$$

Eliminating  $E$  by substituting from (28) into (31) and (33), we get

$$\frac{\partial^2 e}{\partial x^2} = \frac{\partial^2 e}{\partial t^2} + \nu \varepsilon_2 \frac{\partial e}{\partial t} + V^2 \varepsilon_2 \frac{\partial h}{\partial t} + \frac{\partial^2 \theta}{\partial x^2} \quad (34)$$

$$\frac{\partial^2 h}{\partial x^2} = V^2 \frac{\partial^2 h}{\partial t^2} + \nu \frac{\partial h}{\partial t} + \frac{\partial e}{\partial t} \quad (35)$$

### III. ADM

The ADM usually defines the equation in an operator form by considering the highest-ordered derivative in the problem. We define the differential operator “L” in terms of the two derivatives contained in the problem [15]-[17], [30].

Consider (32), (34), and (35) in the operator form as:

$$L_{xx} \theta(x, t) = (L_t + \tau_0 L_{tt}) \theta(x, t) + \varepsilon_1 (L_t + \tau_0 L_{tt}) e(x, t) \quad (36)$$

$$L_{xx} e(x, t) = L_{tt} e(x, t) + \nu \varepsilon_2 L_t e(x, t) + \nu^2 \varepsilon_2 L_t h(x, t) + L_{xx} \theta(x, t) \quad (37)$$

$$L_{xx} h(x, t) = V^2 L_{tt} h(x, t) + \nu L_t h(x, t) + L_t e(x, t) \quad (38)$$

where the operators which appeared in the above equations are defined as:

$$L_t = \frac{\partial}{\partial t}, L_{tt} = \frac{\partial^2}{\partial t^2}, L_x = \frac{\partial}{\partial x}, L_{xx} = \frac{\partial^2}{\partial x^2} \quad (39)$$

Assuming that the inverse of the operator “ $L_x^{-1}, L_{xx}^{-1}$ ” exist and can conveniently be taken as definite integral with respect to “ $x$ ” from “0” to “ $x$ ” as following [15]-[17], [30]:

$$L_x^{-1} f(x) = \int_0^x f(x) dx, \quad L_{xx}^{-1} f(x) = \int_0^x \int_0^x f(x) dx dx \quad (40)$$

Thus, applying the inverse operator on both the sides of (36)-(38), we obtain

$$\theta(x, t) = \theta(0, t) + \frac{\partial \theta(x, t)}{\partial x} \Big|_{x=0} + L_{xx}^{-1} [(L_t + \tau_0 L_{tt}) \theta(x, t) + \varepsilon_1 (L_t + \tau_0 L_{tt}) e(x, t)] \quad (41)$$

$$e(x, t) = e(0, t) + \frac{\partial e(x, t)}{\partial x} \Big|_{x=0} + L_{xx}^{-1} [L_{tt} e(x, t) + \nu \varepsilon_2 L_t e(x, t) + \nu^2 \varepsilon_2 L_t h(x, t) + L_{xx} \theta(x, t)] \quad (42)$$

$$h(x, t) = h(0, t) + \frac{\partial h(x, t)}{\partial x} \Big|_{x=0} + L_{xx}^{-1} [V^2 L_{tt} h(x, t) + \nu L_t h(x, t) + L_t e(x, t)] \quad (43)$$

Now, we will decompose the unknown functions  $\theta(x, t)$ ,  $e(x, t)$  and  $h(x, t)$  by a sum of components defined by the following series:

$$\theta(x, t) = \sum_{k=0}^{\infty} \theta_k(x, t) = \theta_0 + \sum_{k=1}^{\infty} \theta_k(x, t) \quad (44)$$

$$e(x, t) = \sum_{k=0}^{\infty} e_k(x, t) = e_0 + \sum_{k=1}^{\infty} e_k(x, t) \quad (45)$$

$$h(x, t) = \sum_{k=0}^{\infty} h_k(x, t) = h_0 + \sum_{k=1}^{\infty} h_k(x, t) \quad (46)$$

The zero-components are defined by the terms that arise from the boundary conditions on the bounding plane  $x = 0$ , which give

$$e_0 = e(0, t) + \frac{\partial e(x, t)}{\partial x} \Big|_{x=0} \quad (47)$$

$$\theta_0 = \theta(0, t) + \frac{\partial \theta(x, t)}{\partial x} \Big|_{x=0} \quad (48)$$

$$h_0 = h(0, t) + \frac{\partial h(x, t)}{\partial x} \Big|_{x=0} \quad (49) \quad h(0, t) = 0, \quad \frac{\partial h(x, t)}{\partial x} \Big|_{x=0} = 0 \quad (58)$$

Substituting from (44)-(46) in (41)-(43), we get

$$\theta(x, t) = \sum_{k=0}^{\infty} \theta_k(x, t) = \theta(0, t) + \frac{\partial \theta(x, t)}{\partial x} \Big|_{x=0} + L_{xx}^{-1} \left[ (L_t + \tau_0 L_{tt}) \sum_{k=0}^{\infty} \theta_k(x, t) + \varepsilon_1 (L_t + \tau_0 L_{tt}) \sum_{k=0}^{\infty} e_k(x, t) \right] \quad (50)$$

$$e(x, t) = \sum_{k=0}^{\infty} e_k(x, t) = e(0, t) + \frac{\partial e(x, t)}{\partial x} \Big|_{x=0} + L_{xx}^{-1} \left[ L_{tt} \sum_{k=0}^{\infty} e_k(x, t) + \nu \varepsilon_2 L_{tt} \sum_{k=0}^{\infty} e_k(x, t) + \nu^2 \varepsilon_2 L_{tt} \sum_{k=0}^{\infty} h_k(x, t) + L_{xx} \sum_{k=0}^{\infty} \theta_k(x, t) \right] \quad (51)$$

and

$$h(x, t) = \sum_{k=0}^{\infty} h_k(x, t) = h(0, t) + \frac{\partial h(x, t)}{\partial x} \Big|_{x=0} + L_{xx}^{-1} \left[ V^2 L_{tt} \sum_{k=0}^{\infty} h_k(x, t) + \nu L_{tt} \sum_{k=0}^{\infty} h_k(x, t) + L_t \sum_{k=0}^{\infty} e_k(x, t) \right] \quad (52)$$

We obtain these components by  $e_k(x, t)$ ,  $\theta_k(x, t)$  and  $h_k(x, t)$  the recursive formulas [15]-[17], [30]:

$$\theta_{k+1}(x, t) = L_{xx}^{-1} \left[ (L_t + \tau_0 L_{tt}) \theta_k(x, t) + \varepsilon_1 (L_t + \tau_0 L_{tt}) e_k(x, t) \right] \quad (53)$$

$$e_{k+1}(x, t) = L_{xx}^{-1} \left[ L_{tt} e_k(x, t) + \nu \varepsilon_2 L_{tt} e_k(x, t) + \nu^2 \varepsilon_2 L_{tt} h_k(x, t) + L_{xx} \theta_k(x, t) \right] \quad (54)$$

$$h_{k+1}(x, t) = L_{xx}^{-1} \left[ V^2 L_{tt} h_k(x, t) + \nu L_{tt} h_k(x, t) + L_t e_k(x, t) \right] \quad (55)$$

We assume that the bounding plane to the surface  $x=0$  of the body is thermally loaded by harmonic heat and traction free while the induced magnetic field  $h$  is vanished in the free space. Hence, we have:

$$\theta(0, t) = \theta^0 \sin(\omega t), \quad \frac{\partial \theta(x, t)}{\partial x} \Big|_{x=0} = 0 \quad (56)$$

$$\sigma(0, t) = 0 \rightarrow e(0, t) = \theta(0, t), \quad \frac{\partial e(x, t)}{\partial x} \Big|_{x=0} = 0 \quad (57)$$

where  $\theta^0$  is constant and  $\omega$  is the angular thermal load and assumed to be constant. Thus, we have

$$\theta_0 = \theta^0 \sin(\omega t), \quad e_0 = \theta^0 \sin(\omega t), \quad h_0 = 0 \quad (59)$$

Substituting from (59) into (50)-(52), we get the complete of the iteration formulas.

The first components of the iteration take the forms:

$$\theta_1(x, t) = \frac{\omega(1 + \varepsilon_1)}{2} (\cos(\omega t) - \omega \sin(\omega t)) x^2 \quad (60)$$

$$e_1(x, t) = \frac{\omega}{2} (\varepsilon_2 \nu \cos(\omega t) - \omega \sin(\omega t)) x^2 \quad (61)$$

$$h_1(x, t) = \frac{\omega \cos(\omega t)}{2} x^2 \quad (62)$$

The rest components of the iteration formulas (53)-(55) have been calculated by using the MAPLE 17. Moreover, the decomposition series solutions (53)-(55) are convergent in real physical problems very rapidly [23]-[26].

In an algorithmic form, the ADM can be expressed and implemented in linear generalized magneto-thermoelasticity models with the suitable value for the tolerance  $Tol = 10^{-6}$  and  $k$  is the iteration index, as follows [23]-[26]:

#### Algorithm

**Step 1:** Compute the initial approximations  $\theta_0 = \theta(0, t)$ ,  $e_0 = e(0, t)$  and  $h_0 = h(0, t)$  given by (59).

**Step 2:** Use the calculated values of  $\theta_k$ ,  $e_k$ , and  $h_k$  to compute  $\theta_{k+1}$ ,  $e_{k+1}$  and  $h_{k+1}$  from (53)-(55).

**Step 3:** If  $\max|\theta_{k+1} - \theta_k| < Tol$ ,  $\max|e_{k+1} - e_k| < Tol$  and  $\max|h_{k+1} - h_k| < Tol$  stop, otherwise continue and go back to step 2.

**Step 4:** Calculating the stress from (29), (50)-(52) as follows:

$$\sigma(x, t) = \sum_{k=0}^{\infty} e_k(x, t) - \sum_{k=0}^{\infty} \theta_k(x, t) \quad (63)$$

**Step 5:** Calculating the displacement from (10) and (51) as:

$$u(x, t) = \int_0^x e(\xi, t) d\xi = \int_0^x \sum_{k=0}^{\infty} e_k(\xi, t) d\xi \quad (64)$$

#### IV. THE NUMERICAL RESULTS AND DISCUSSION

The copper material has been chosen for the numerical evaluations, and the material constants were taken as follows

[1], [2]:

$$K = 386 \text{ W / (mK)}, \alpha_T = 1.78 \times 10^{-5} \text{ K}^{-1}, C_E = 383.1 \text{ J / (kg K)},$$

$$\eta = 8886.73 \text{ s / m}^2, T_0 = 293 \text{ K}, \mu = 3.86 \times 10^{10} \text{ N / m}^2,$$

$$\lambda = 7.76 \times 10^{10} \text{ N / m}^2, \rho = 8954 \text{ kg/m}^3, \varepsilon_0 = 10^{-9} / 36\pi,$$

$$\sigma_0 = 5.7 \times 10^7, \mu_0 = 4\pi \times 10^{-7}, \tau_0 = 0.35 \times 10^{-14}.$$

Thus, the following non-dimensional parameters have been obtained;

$$\varepsilon_1 = 0.0168, \varepsilon_2 = 80.0, V = 0.000014, \nu = 0.008, \tau_0 = 0.05$$

We calculate the numerical solutions when the non-dimensional value of the time is  $t = 2.0$ , the non-dimensional value of the distance is  $0 \leq x \leq 1.0$ ,  $\omega = \pi$ , and  $\theta^0 = 1.0$ .

According to the above algorithm, we stopped the calculation on the 10<sup>th</sup> component  $\theta_{10}(x, t)$ ,  $e_{10}(x, t)$ , and  $h_{10}(x, t)$ .

Figs. 1-6 show the distribution of the temperature increment, the strain, the induced magnetic field, the electrical field, the stress, and the displacement, respectively, with various cases of the magnetic field when  $H_0 = 0.0$  and  $H_0 \neq 0.0$  to stand on the effect of the magnetic field on all the studied functions. The temperature increment is the only function that received a limited effect due to the magnetic field while all the rest functions received significant effects. For non-zero magnetic field, the absolute values of the strain, the induced magnetic field, the electrical field, the stress, and the displacement have increased.

Figs. 7-12 show the distribution of the temperature increment, the strain, the induced magnetic field, the electrical field, the stress, and the displacement, respectively, with various cases of the magnetic field when  $H_0 = 0.0$  and  $H_0 \neq 0.0$  to stand on the effect of the magnetic field on all the studied functions. The temperature increment is the only function that received a limited effect due to the magnetic field while all the rest functions received significant effects. For non-zero magnetic field, the absolute values of the strain, the induced magnetic field, the electrical field, the stress, and the displacement have increased.

Figs. 13-18 show the distribution of the temperature increment, the strain, the induced magnetic field, the electrical field, the stress, and the displacement, respectively, with various cases of the magnetic field when  $H_0 = 0.0$  and  $H_0 \neq 0.0$  to stand on the effect of the magnetic field on all the studied functions. The temperature increment is the only function that received a limited effect due to the magnetic field while all the rest functions received significant effects. For non-zero magnetic field, the absolute values of the strain, the induced magnetic field, the electrical field, the stress, and the displacement have increased.

## V. CONCLUSION

A mathematical model of generalized thermoelasticity with one relaxation time of a half-space conducting medium has been constructed taking into account a constant magnetic field acts normal to the bounding plane. ADM has been used to solve the model. The results show that; the magnetic field, the relaxation time, and the angular thermal load have significant effects on all the studied fields, and ADM is a useful method and very suitable for that types of mathematical models.

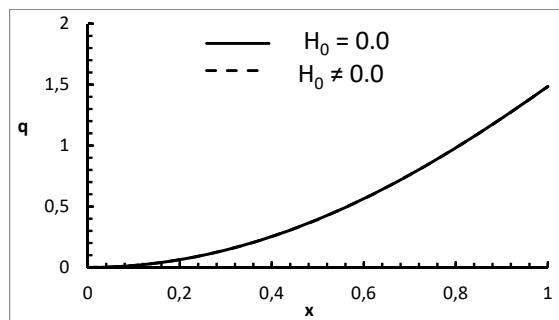


Fig. 1 The temperature increment distribution with various cases of magnetic field

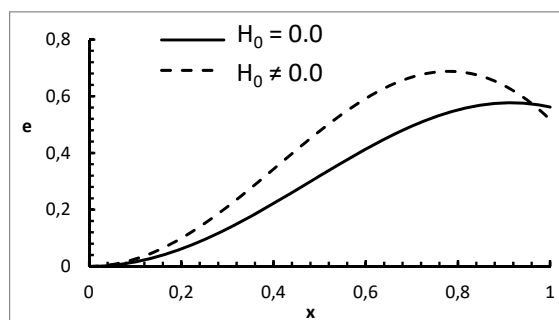


Fig. 2 The strain distribution with various cases of magnetic field

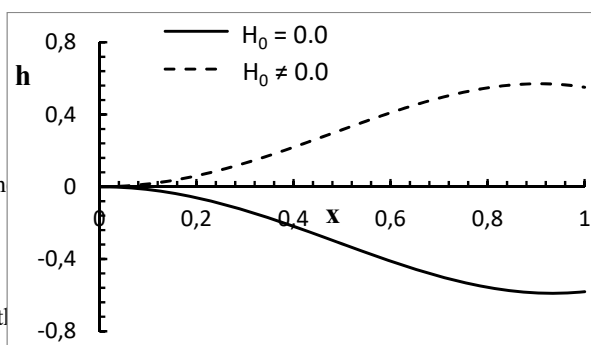


Fig. 3 The induced magnetic field distribution with various cases of magnetic field

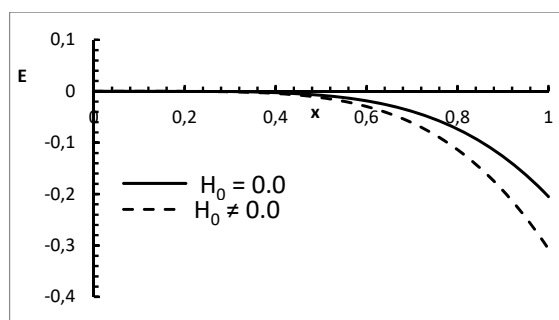


Fig. 4 The electric field distribution with various cases of magnetic field

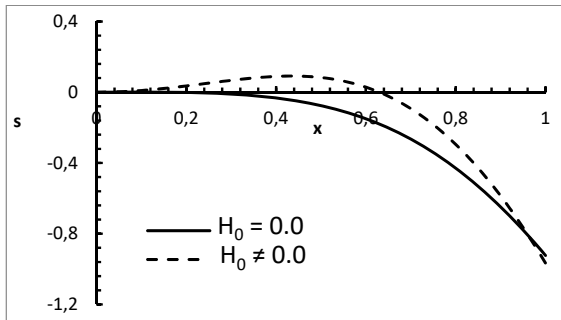


Fig. 5 The stress distribution with various cases of magnetic field

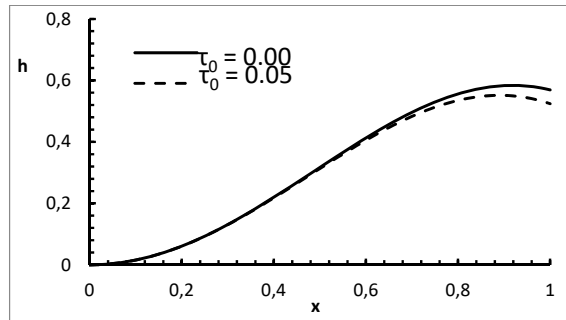


Fig. 9 The induced magnetic field distribution with various values of relaxation time

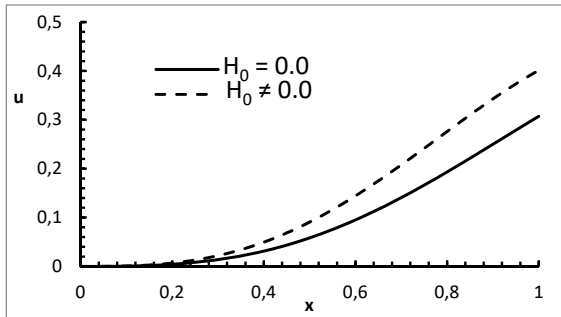


Fig. 6 The displacement distribution with various cases of magnetic field

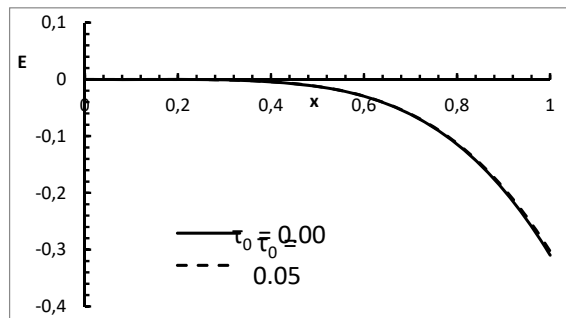


Fig. 10 The electric field distribution with various values of relaxation time

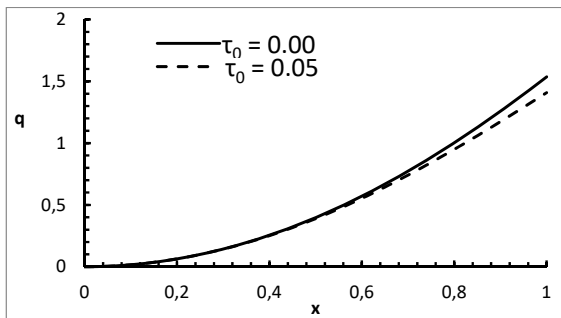


Fig. 7 The temperature increment distribution with various values of relaxation time

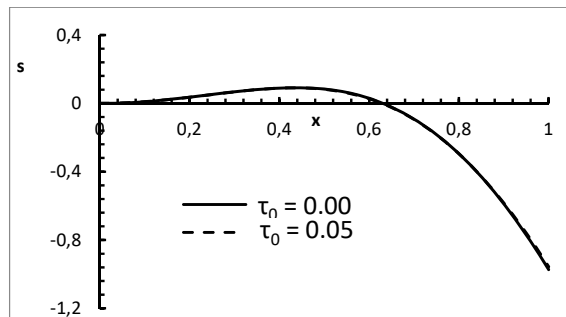


Fig. 11 The stress distribution with various values of relaxation time

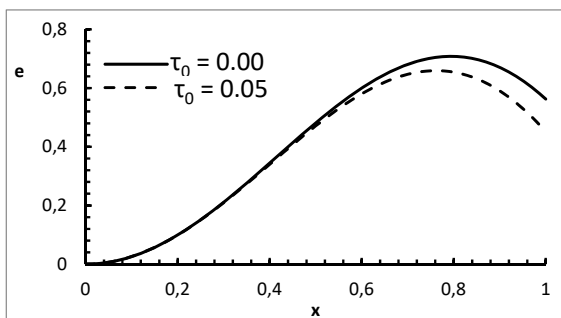


Fig. 8 The strain distribution with various values of relaxation time

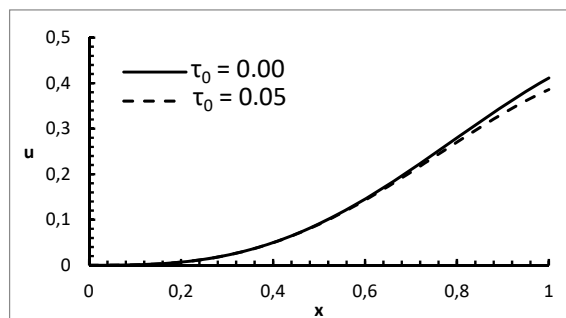


Fig. 12 The displacement distribution with various values of relaxation time

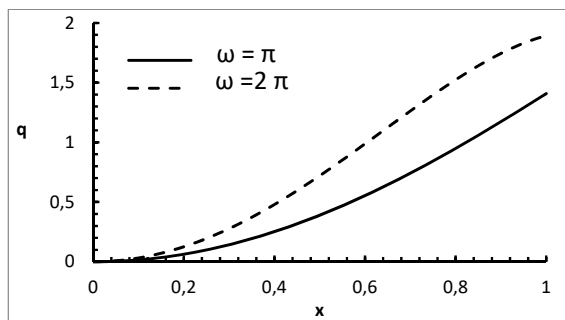


Fig. 13 The temperature increment distribution with various values of angular thermal load

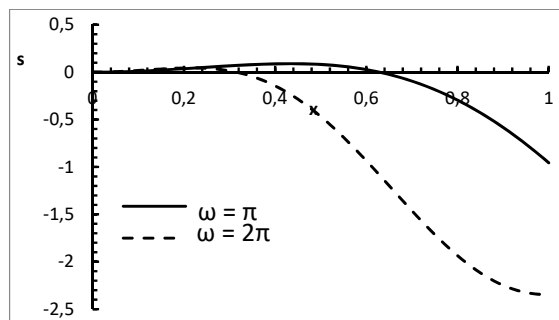


Fig. 17 The stress distribution with various values of angular thermal load

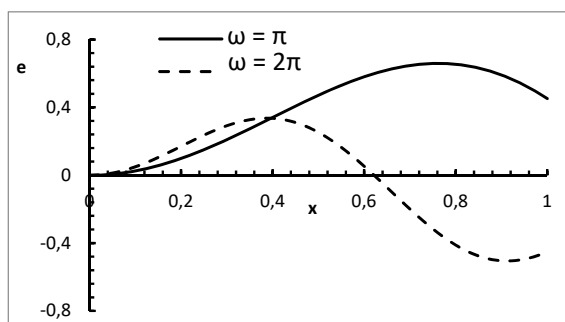


Fig. 14 The strain distribution with various values of angular thermal load

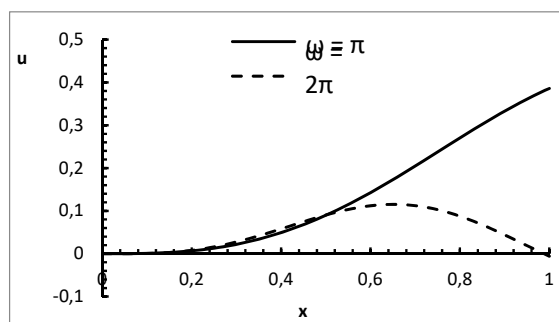


Fig. 18 The displacement distribution with various values of angular thermal load

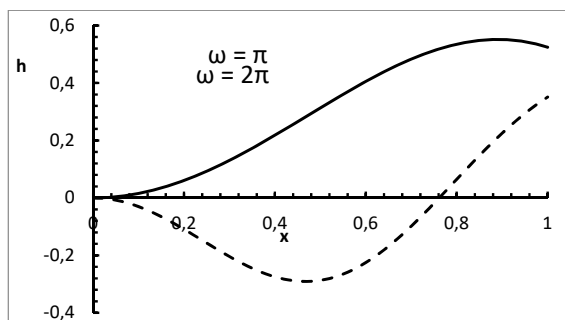


Fig. 15 The induced magnetic field distribution with various values of angular thermal load

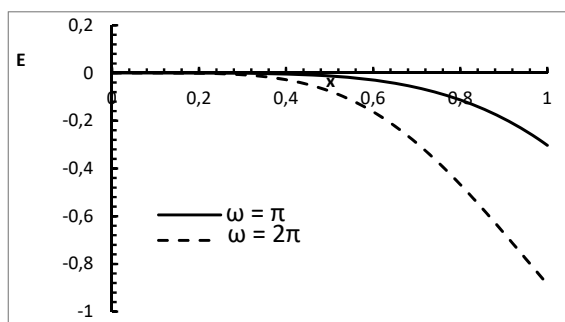


Fig. 16 The electric field distribution with various values of angular thermal load

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