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Adaptive Motion Planning for 6-DOF Robots Based on Trigonometric Functions

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Abstract**—**Building an appropriate motion model is crucial for trajectory planning of robots and determines the operational quality directly. An adaptive acceleration and deceleration motion planning based on trigonometric functions for the end-effector of 6-DOF robots in Cartesian coordinate system is proposed in this paper. This method not only achieves the smooth translation motion and rotation motion by constructing a continuous jerk model, but also automatically adjusts the parameters of trigonometric functions according to the variable inputs and the kinematic constraints. The results of computer simulation show that this method is correct and effective to achieve the adaptive motion planning for linear trajectories.

Keywords**—**6-DOF robots, motion planning, trigonometric function, kinematic constraints

I. Introduction

THE methods of trajectory planning for 6-DOF robots include Joint-space schemes and Cartesian-space schemes. Using Joint-space schemes, the point-to-point trajectory planning is achieved by controlling each joint motor directly. However, Joint-space schemes are difficult to control the shape of the space path, such as straight line, arc, parabola, sinusoid and etc. [1]. Using Cartesian-space schemes, the trajectory functions with time are constructed by the motion model of the end-effector and the constraints of different trajectory types. The inverse kinematics operation can then be solved in real time to obtain the corresponding motor variation to achieve the certain trajectory planning [2]-[4]. Both schemes are applicable and indispensable for trajectory planning. In this paper, the main work is to adopt an appropriate end-effector motion model under the kinematic constraints to ensure the stability and smoothness of the robotic system with Cartesian-space schemes.

In order to achieve the trajectory planning in the Cartesian coordinate system, the crucial issue is to build the acceleration and deceleration motion models for the end-effector for translation and rotation. The smoothness of the model curve mainly determines the stability and smoothness of the robotic system.

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II. Trigonometric Function Motion Model

According to the mathematical functions, the acceleration and deceleration motion models can be divided into polynomial models [5]-[7], exponential function models, trigonometric function models [8]-[10], etc.

In this paper, the constant acceleration segment, constant velocity segment and constant deceleration segment which may exist in the motion model are ignored for model simplification.

Selecting a linear polynomial or a quadratic polynomial to build the acceleration model is common in the polynomial model planning.

When the linear polynomial is chosen as the basic mathematical function to build the acceleration model, the complete acceleration model $a_l(t)$ is constructed with the following 3-segments function, as given in (1)

$$a_l = \begin{cases} Jt, & 0 < t \le \frac{T}{4} \\ \frac{JT}{2} - Jt, & \frac{T}{4} < t \le \frac{3T}{4} \\ Jt - JT, & \frac{3T}{4} < t \le T \end{cases} \tag{1}$$

where J is the acceleration curve slope, T is the total motion time.

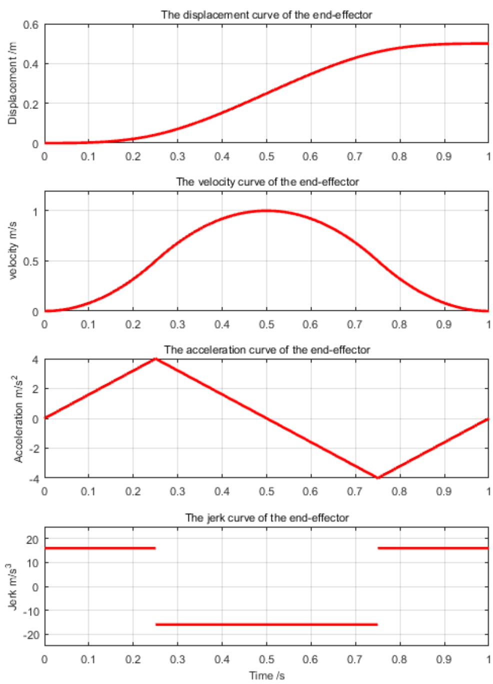

Fig. 1 The acceleration model built according to the linear polynomial

When the quadratic polynomial is chosen as the basic mathematical function to build the acceleration model, the linear polynomial becomes the basic mathematical function of the jerk model and the complete jerk model $j_q(t)$ is constructed with the 6-segments functions, as given in (2)

$$j_q = \begin{cases} Mt, & 0 < t \le \frac{T}{8} \\ \frac{MT}{4} - Mt, & \frac{T}{8} < t \le \frac{3T}{8} \\ Mt - \frac{MT}{2}, & \frac{3T}{8} < t \le \frac{T}{2} \\ \frac{MT}{2} - Mt, & \frac{T}{2} < t \le \frac{5T}{8} \\ Mt - \frac{3MT}{4}, & \frac{5T}{8} < t \le \frac{7T}{8} \\ MT - Mt, & \frac{7T}{8} < t \le T \end{cases} \tag{2}$$

where M is the jerk curve slope, T is the total motion time.

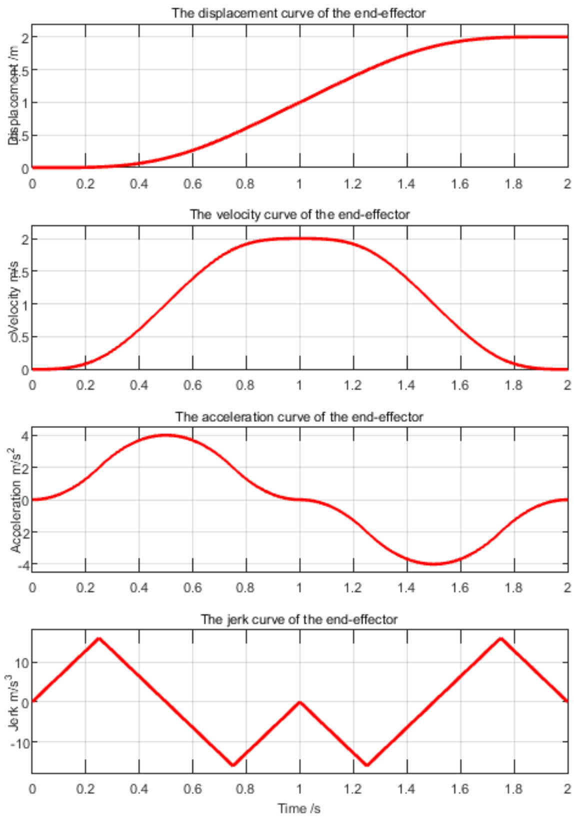

Fig. 2 the acceleration model built according to the quadratic polynomial

As seen from Fig. 1, discontinuity exists in the jerk curve when the linear polynomial model is used for motion planning, and the jerk curve is continuous when the quadratic polynomial model is applied for motion planning as shown in Fig. 2. The difference indicates that the motion model constructed with the quadratic polynomial is smoother and more stable than the one constructed with the linear polynomial. However, the increasing degree of the polynomial increases the segments of the piecewise functions and the complexity of computation.

In order to keep the stability and smoothness of motion model and reduce the segments of piecewise functions, the trigonometric function is appropriate to build the acceleration and deceleration motion model. The complete acceleration model $a_t(t)$ and jerk model $j_t(t)$ can both be constructed with the following 2-segments functions, as shown in (3) and (4)

$$a_t = \begin{cases} -\frac{A\cos\left(\frac{4\pi t}{T}\right)}{2} + \frac{A}{2}, & 0 < t \le \frac{T}{2} \\ \frac{A\cos\left(\frac{4\pi t}{T}\right)}{2} - \frac{A}{2}, & \frac{T}{2} < t \le T \end{cases} \tag{3}$$

$$j_t = \begin{cases} \frac{2\pi A\sin\left(\frac{4\pi t}{T}\right)}{T}, & 0 < t \le \frac{T}{2} \\ -\frac{2\pi A\sin\left(\frac{4\pi t}{T}\right)}{T}, & \frac{T}{2} < t \le T \end{cases} \tag{4}$$

where A is the maximum acceleration, T is the total motion time.

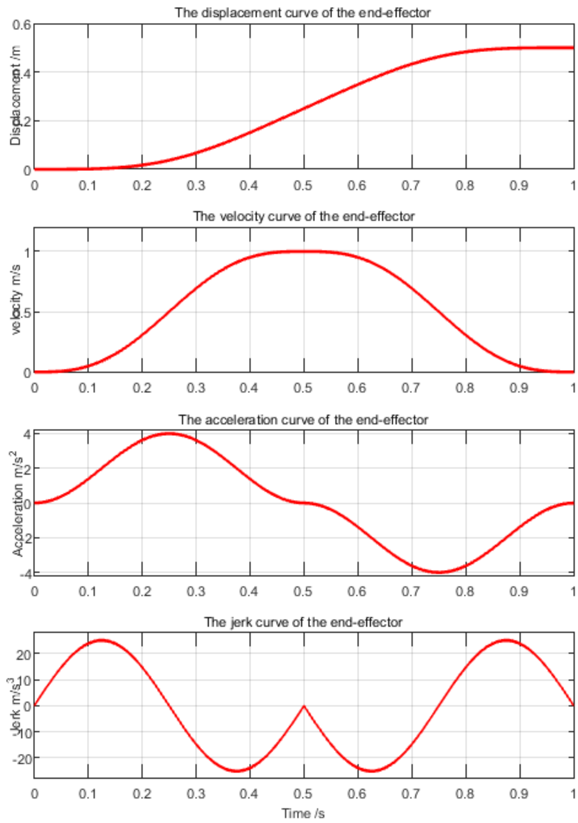

Fig. 3 the acceleration model built according to the trigonometric functions

As shown in Fig. 3, the jerk curve is continuous and the segments of the model function are much less than those in the quadratic polynomial based acceleration model. Therefore, the trigonometric function is more appropriate to build the acceleration and deceleration motion model for trajectory planning.

III. Translation and Rotation Planning

Achieving the trajectory planning with Cartesian-space schemes, the motion of the end-effector includes the translation motion and the rotation motion. Therefore, the acceleration and deceleration motion model for the position translation and the orientation rotation need to be built separately and achieve the synchronization of them to ensure the translation motion and the rotation motion accomplished simultaneously.

For the position translation planning, the trajectory expression can be defined by the input information. Straight line expression is defined by the starting point and the ending point.

With the starting position $P_s(x_s, y_s, z_s)$ and the ending position $P_e(x_e, y_e, z_e)$ of a straight line, the total distance D is

$$D = \sqrt{(x_e - x_s)^2 + (y_e - y_s)^2 + (z_e - z_s)^2} \tag{5}$$

For every point $P(x, y, z)$ in the straight line, the straight line expression is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} + k \begin{bmatrix} x_e - x_s \\ y_e - y_s \\ y_e - y_s \end{bmatrix} \tag{6}$$

where k is an unitary distance coefficient.

For the orientation rotation planning, in order to express the rotation motion more intuitively and solve the problem of gimbal lock, the quaternion method is appropriate to describe the orientation rotation [11], [12]. The quaternion expression q is

$$q = [s, v] = [s, (a, b, c)] \tag{7}$$

where s is the real number port, $v = (a, b, c)$ is the vector port.

The rotation matrix R in (8) can be transformed to the unit quaternion q according to (9).

$$R = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix} \tag{8}$$

$$\begin{cases} s = \pm \frac{\sqrt{n_x + o_y + a_z + 1}}{2} \\ a = \frac{o_z - a_y}{4s} \\ b = \frac{a_x - n_z}{4s} \\ c = \frac{n_y - o_x}{4s} \end{cases} \tag{9}$$

To simplify the space rotation planning, the motion constraints between two rotation axes that are expressed with quaternions are ignored in this paper. With the starting orientation quaternion $q_1 = [s_1, (a_1, b_1, c_1)]$ and the ending orientation quaternion $q_2 = [s_2, (a_2, b_2, c_2)]$, the orientation rotation angle θ is

$$\theta = 2 \cdot acos(s_1 s_2 + a_1 a_2 + b_1 b_2 + c_1 c_2) \tag{10}$$

When the quantity of the position translation and the quantity of the orientation rotation are known, the unified motion time that ensures the motion planning subject to the robotic kinematic constraints is obtained to achieve the synchronization of the position motion and the orientation motion.

IV. Adaptive Time Planning Under Constraints

The displacement, velocity, and jerk of the translation and rotation for the acceleration and deceleration motion model constructed by the trigonometric function can be deduced by the acceleration function.

Take the translation motion planning as an example. The translation acceleration amplitude A_d and the total translation time T_d of motion model are unknown.

The translation displacement D is calculated by using (5). And the translation displacement D, the maximum velocity V_m, and the maximum jerk J_m can be expressed by the translation acceleration amplitude A_D and the total translation time T_D, as in (11)-(13).

$$D = \frac{A_D {T_D}^2}{8} \tag{11}$$

$$V_m = \frac{A_D T_D}{4} \tag{12}$$

$$J_m = \frac{2\pi A_D}{T_D} \tag{13}$$

The velocity, acceleration, and jerk should satisfy the translational velocity constraint V_t , the translational acceleration constraint A_t, and the translational jerk constraint J_t respectively, as in (14).

$$\begin{cases} V_m \leq V_t \\ A_D \leq A_t \\ J_m \leq J_t \end{cases} \tag{14}$$

According to (14), the total translation time T_D should satisfy the following constraint, as given in (15).

$$T_D \geq \max(\frac{2D}{V_t}, \sqrt{\frac{8D}{A_t}}, \sqrt[3]{\frac{16\pi D}{J_t}}) \tag{15}$$

The translation acceleration amplitude A_D can then be calculated by using (11) when the total translation time T_D is known.

For rotation planning, the rotation acceleration amplitude A_θ and the total rotation time T_θ of model are unknown. The system rotation constraints include the rotational velocity constraint V_r, the rotational acceleration constraint A_r, and the rotational jerk constraint J_r.

The rotation angle θ can be calculated according to (10). The total rotation time T_θ should satisfy the following constraint, as shown in (16).

$$T_\theta \geq \max(\frac{2\theta}{V_r}, \sqrt{\frac{8\theta}{A_r}}, \sqrt[3]{\frac{16\pi\theta}{J_r}}) \tag{16}$$

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Therefore, by choosing an appropriate total motion time T, as in (17), the translation acceleration amplitude $A_D = \frac{8D}{T^2}$ and the rotation acceleration amplitude $A_\theta = \frac{8\theta}{T^2}$ can be calculated.

$$T \geq \max(T_D, T_\theta) \tag{17}$$

V. Simulations for a Linear Trajectory

Assume the end-effector obey the kinematic constraints: the translation velocity constraint $V_t = 2m/s$, the translation acceleration constraint $A_t = 8m/s^2$, the translation jerk constraint $J_t = 60m/s^3$, the rotation velocity constraint $V_r = 2rad/s$, the rotation acceleration constraint $A_r = 8rad/s^2$, rotation acceleration constraint $J_r = 60rad/s^3$. The original position and orientation matrix T_o of the end-effector in the robotic base coordinate system is

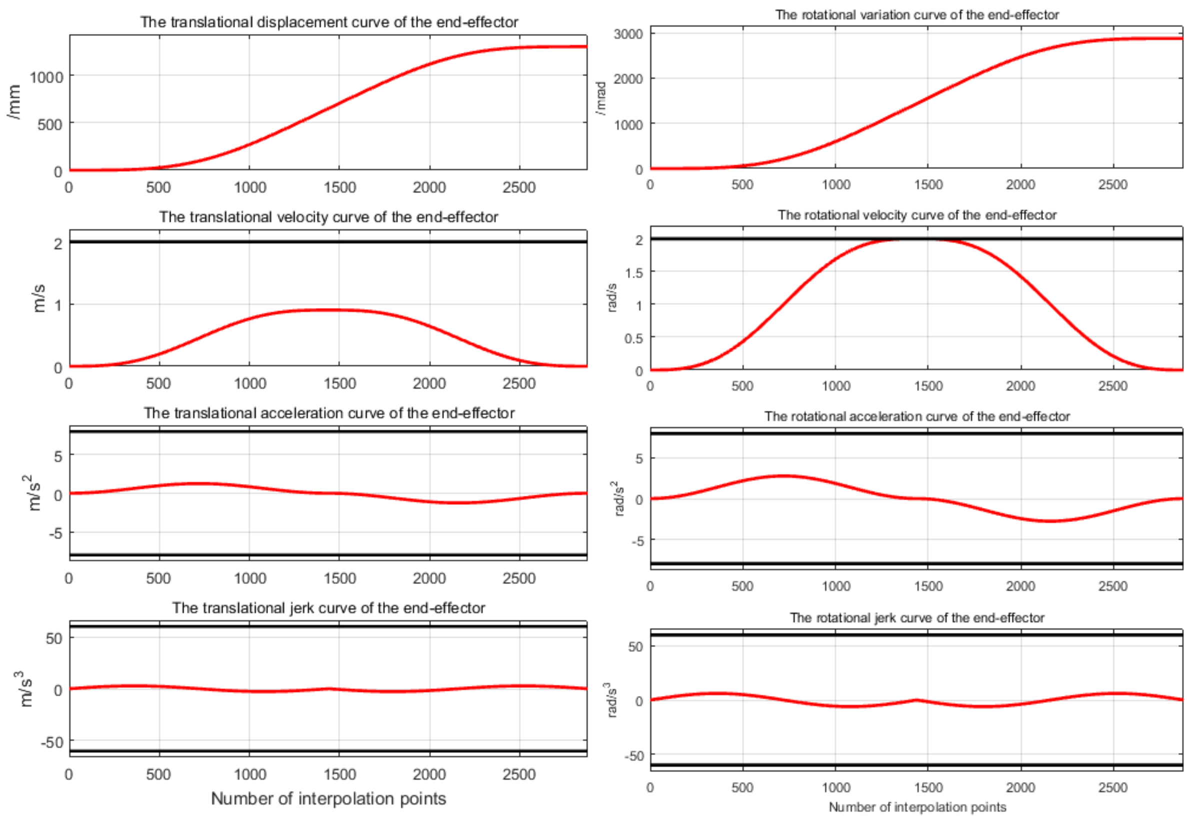
$$T_o = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 775 \\ 0 & 0 & 1 & 570 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

Giving the starting position and orientation matrix T_s and the ending position and orientation matrix T_e, as in (19) and (20).

$$T_s = \begin{bmatrix} 0.500 & -0.866 & 0 & 671.170 \\ 0.866 & 0.500 & 0 & 387.500 \\ 0 & 0 & 1 & 570 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

$$T_e = \begin{bmatrix} -0.085 & -0.912 & -0.401 & -623.717 \\ 0.753 & -0.322 & 0.573 & 312.221 \\ -0.652 & -0.253 & 0.715 & 504.095 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

The translation distance $D \approx 1298.747$mm, the rotation angle $\theta \approx 2878.835$mrad are calculated according to (5) and (10). Choose the minimum total motion time $T \approx 2878.835ms$ obtained according to (15), (16), and (17), then the translation acceleration amplitude and the rotation acceleration amplitude can be obtained as follows: $A_D \approx 1.254m/s^2$, $A_\theta \approx 2.779m/s^2$. With them, the translation motion model and the rotation motion model are constructed as in (21) and (22).



(a) (b)

Fig. 4 Translation and rotation planning: (a) Translation model planning, (b) Rotation model planning

$$a_t = \begin{cases} -\frac{A_D \cos\left(\frac{4\pi}{T}t\right)}{2} + \frac{A_D}{2}, & 0 < t \leq \frac{T}{2} \\ \frac{A_D \cos\left(\frac{4\pi t}{T}\right)}{2} - \frac{A_D}{2}, & \frac{T}{2} < t \leq T \end{cases} \tag{21}$$

$$a_r = \begin{cases} -\frac{A_\theta \cos\left(\frac{4\pi}{T}t\right)}{2} + \frac{A_\theta}{2}, & 0 < t \leq \frac{T}{2} \\ \frac{A_\theta \cos\left(\frac{4\pi t}{T}\right)}{2} - \frac{A_\theta}{2}, & \frac{T}{2} < t \leq T \end{cases} \tag{22}$$

Fig. 4 shows the acceleration and deceleration motion models based on the trigonometric function of the translation and rotation. The maximum rotation velocity equaling to the rotation velocity constraint V_r shows that the total motion time $T \approx 2878.835ms$ is the minimum motion time satisfying all the kinematic constraints.

Fig. 5 shows the translation and rotation motion of the end-effector from the starting position and orientation matrix T_s to the ending position and orientation matrix T_e with a fixed-time interpolation method in the Cartesian coordinate system.

The starting position and orientation matrix T_s and the ending position and orientation matrix T_e can be transformed to the starting angular variation q_s and the ending angular variation q_e of six motors with the inverse kinematics operation, as in (23) and (24).

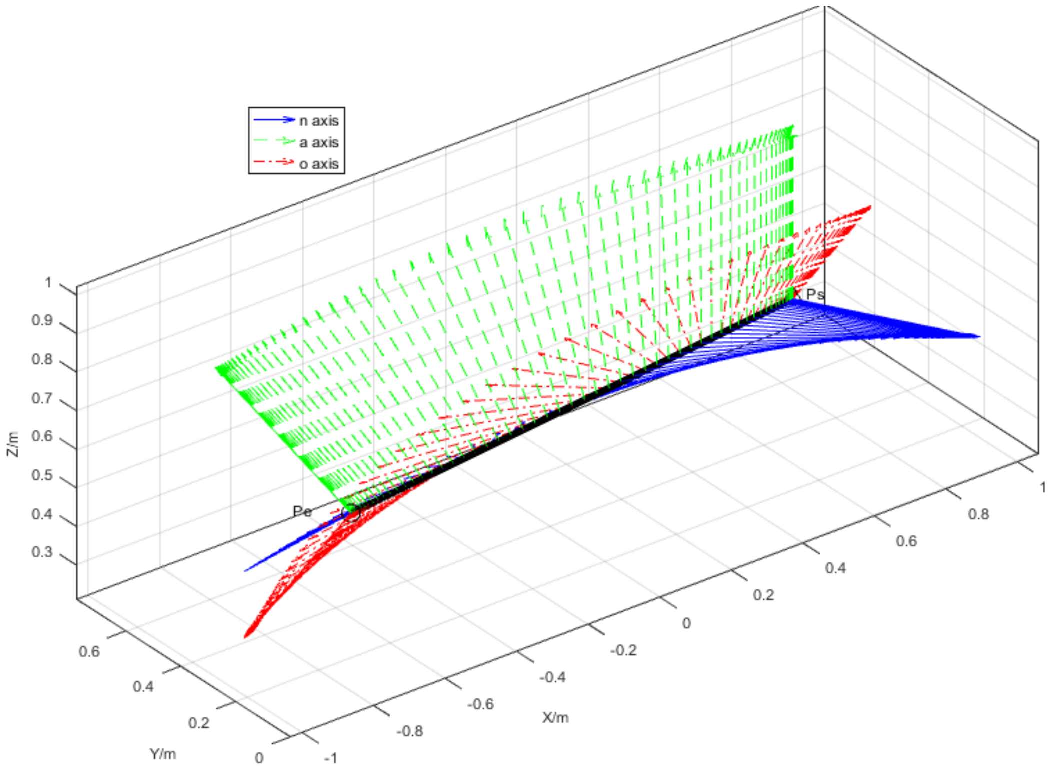
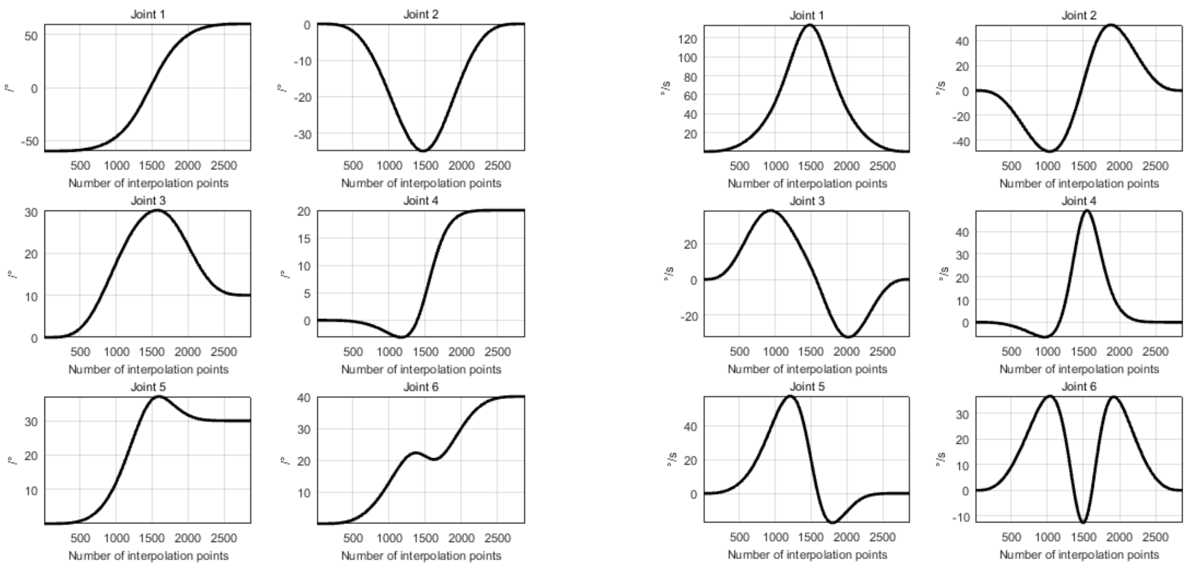
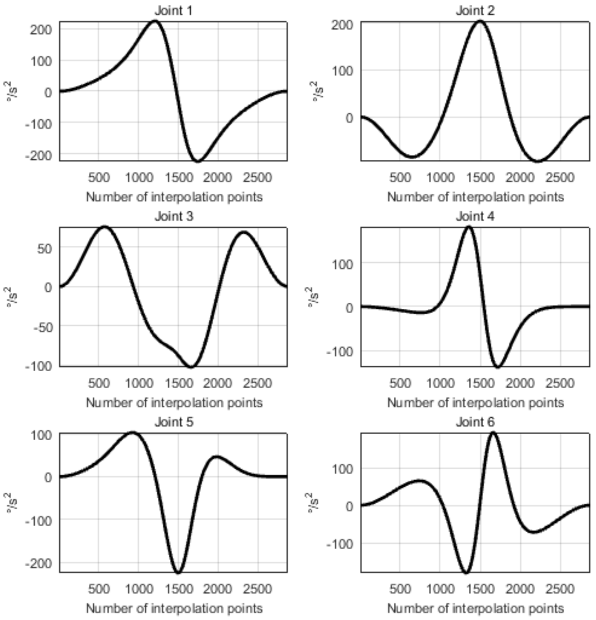


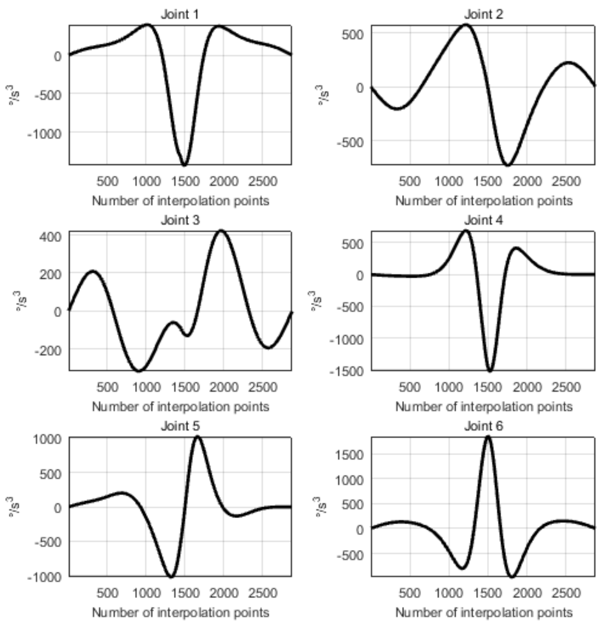
Fig. 5 Translation and rotation motion of the end-effector



(a) (b)



(c)



(d)

Fig. 6 Motion model of six motors: (a) Angular variation model, (b) Angular velocity model, (c) Angular acceleration model, (d) Angular jerk model

$$q_s = [-60 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{23}$$

$$q_e = [60 \quad 0 \quad 10 \quad 20 \quad 30 \quad 40] \tag{24}$$

Solve the inverse kinematics operation for every position and orientation matrix that is chose with a fixed-time interpolation method along the linear trajectory in real time and obtain the corresponding motor variation, then calculate the velocity, acceleration, and jerk of six joint motors. Fig. 6 shows the angular variation, angular velocity, angular acceleration, and angular jerk of six motors for a 6-DOF robot. The jerk curves are continuous and the acceleration and velocity curves are smooth, ensuring the stability of the robotic system.

VI. Conclusion

The fundamental of high-quality robotic operation is smooth and stable motion that is mainly determined by the acceleration and deceleration motion model. The motion model based on the trigonometric function ensures the jerk model of the end-effector being continuous with less segments of the model function and satisfies the kinematic constraints to achieve the synchronization of position translation and orientation rotation smoothly and stably.

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