

# Adaptive Line Enhancement of Narrowband Signal

Young-Seok Choi

**Abstract**—The Adaptive Line Enhancer (ALE) is widely used for enhancing narrowband signals corrupted by broadband noise. In this paper, we propose novel ALE methods to improve the enhancing capability. The proposed methods are motivated by the fact that the output of the ALE is a fine estimate of the desired narrowband signal with the broadband noise component suppressed. The proposed methods preprocess the input signal using ALE filter to regenerate a finer input signal. Thus the proposed ALE is driven by the input signal with higher signal-to-noise ratio (SNR). The analysis and simulation results are presented to demonstrate that the proposed ALE has better performance than conventional ALE's.

**Keywords**—Adaptive filter, adaptive line enhancer, noise, feedback.

## I. INTRODUCTION

THE enhancement and detection of narrowband signals in noisy environments is recurrent problem in signal processing. When there is a lack of a priori knowledge available of possibly time-varying narrowband parameters, an adaptive solution is preferred. The adaptive line enhancer (ALE) is one possible solution to this problem [1], [2].

The purpose of ALE is to separate the narrowband component in observed data with unity gain, while reducing any broadband component. The ALE was first introduced by Widrow et al. [3]. Since then, the ALE was studied by Zeidler et al. [4], Treichler [5], and Chang and Glover [6] represent some of these contribution.

The conventional ALE is an adaptive filter, and in some applications the length of the filter may be larger, requiring hundreds of taps. The disadvantages associated with using such long adaptive filters include high computational cost and poor performance of the adaptive algorithm in terms of convergence speed and gradient noise. In addition, the large lengths may also preclude the use of certain adaptive algorithms such as the RLS algorithm, which yields fast convergence at the increased computational complexity.

In this paper, we propose novel ALE methods to improve the performance without increasing the computational load. The proposed methods are motivated by the fact that the output of an ALE is an estimate of the input signal with the broadband component suppressed. We preprocess the input signal using the ALE filter itself. By doing this, we can regenerate the finer input signal, i.e., the ALE operates with the input signal of higher signal-to-noise ratio (SNR). Therefore, the performance of the ALE is improved. Furthermore by simple modification of the regenerating process of the input signal, the ALE structure

changes from finite impulse response (FIR) filter to infinite impulse response (IIR) filter. It is well known that an IIR filter can achieve a sharper frequency response with fewer weights than an FIR filter. Thus, the modification to IIR structure may increase the filter gain and attenuate more broadband noise component.

The remainder of this paper is organized as follows. Section II describes the system model and the behavior of the ALE. Then Section III proposes novel ALE methods and its modification to IIR structure. Section IV evaluates performance of the proposed methods by computer simulation. Finally conclusions are presented in Section V.

## II. ADAPTIVE LINE ENHANCER

A schematic diagram of the ALE is shown in Fig. 1. The ALE can be conceived as a degenerate form of adaptive noise canceller. In the ALE the second or reference input of the noise canceller, instead of being separately derived, is a delayed version of the primary input. The delayed input is processed with an adaptive transversal filter and subtracted from the primary input to produce the error signal. The weighting coefficients of the filter are recursively adjusted by an adaptive algorithm so as to minimize a certain cost function.

### A. System Model

If The input data  $x(n)$  consists of sinusoids corrupted by noise, i.e.

$$x(n) = s(n) + v(n) \quad (1)$$

where  $s(n)$  is the signal composed of sinusoids and  $v(n)$  which is assumed to be a zero mean noise sequence with variance . The input signal to the ALE is a delayed version of  $x(n)$ , i.e.  $x(n - \Delta)$ , where  $\Delta (\Delta \geq 1)$  is the decorrelation delay parameter. As shown in Fig. 1, the ALE consists of one FIR filter that has  $N$  taps. In this paper, the FIR filter is expressed as the vector that is denoted by  $\mathbf{w}^T = [w_0 \ w_1 \ \dots \ w_{N-1}]$ . From Fig. 1, it is straightforward to verify that the ALE output is given by



Fig. 1 Configuration of the conventional ALE

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$$y(n) = \sum_{k=0}^{N-1} w_k(k)x(n-\Delta-k). \quad (2)$$

The ALE output  $y(n)$  in (2) is an estimate of  $s(n)$ . The FIR filter coefficients  $w_k(n)$  are time-varying and are adapted based on a certain criterion. We shall consider here the well known least-mean square (LMS) algorithm that has been widely used due to its simplicity and robustness.

Let a linear prediction error of the ALE be

$$e(n) = x(n) - y(n). \quad (3)$$

The LMS algorithm updates coefficients so that the mean square error (MSE)  $E[e^2(n)]$  is minimized. The resulting update equation for the filter coefficients is given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n)\mathbf{x}(n-\Delta), \quad (4)$$

where  $\mathbf{x}^T(n-\Delta) = [x(n-\Delta)x(n-\Delta-1) \cdots x(n-\Delta-N+1)]$  and  $\mu$  is the step size that controls convergence of the adaptation process. Global convergence is guaranteed because there is only a single global minimum in the constrained MSE surface. The optimal solution is that set of weights which minimizes the MSE.

### B. ALE Operation

The ALE's behavior as a signal separator can be understood by considering its response to an input consisting of a sinusoid in white noise. The delay  $\Delta$  decorrelates the noise components in the filter input with respect to those in the primary input while introducing a simple phase shift between the sinusoidal components. The adaptive filter, in seeking to minimize the error power, compensates for the phase shift so that the sinusoidal components cancel each other at the summing junction. Since it cannot compensate for the decorrelation of the noise components, it removes the noise from its own output so that only the noise in the primary input appears in the error output. The ALE adaptive filter acts as a self-tuning filter, creating a peak in its transfer function at the sinusoidal frequency.

To give better insight for the ALE operation, we present simple simulation results demonstrating the separation of a sinusoid from white background noise. To simplify our discussion we assume that the input signal to the ALE has the form

$$x(n) = a \sin 2\pi f_0(n) + v(n) \quad (5)$$

where  $a$  and  $f_0$  are constant. The adaptive process minimizes the MSE. Then the resulting impulse response of the ALE is equivalent to the matched filter response, i.e., a sampled sinusoid whose frequency is  $f_0$  [4].

Assume that  $a = 1$  without loss of generality and let the peak value in the sinusoidal output of the ALE be  $\hat{a}$ . Then when the ALE converges to the matched filter response, the optimal

value of  $\hat{a}$  is given by [3], [5]

$$\hat{a} = \frac{\text{SNR}_{in} \cdot \left(\frac{N}{2}\right)}{1 + \text{SNR}_{in} \cdot \left(\frac{N}{2}\right)} \quad (6)$$

where  $\text{SNR}_{in}$  is the input signal-to-noise ratio, i.e.,  $E[s^2(n)]/\sigma_v^2$ . At high  $\text{SNR}_{in}$ ,  $\hat{a} \approx 1$  and at low  $\text{SNR}_{in}$ ,  $\hat{a} < 1$ . Low  $\text{SNR}_{in}$  conditions can be dealt with by using a large number of adaptive weights at the expense of the computational load. In this paper, we propose a method to increase  $\text{SNR}_{in}$  at the ALE input without increasing the filter length  $N$ .

### III. ADAPTIVE LINE ENHANCEMENT BASED ON SELF-TUNING FRAMEWORK

To fully achieve the benefits of the ALE the basic idea is to use the ALE filter as a preprocessing filter for the input signal  $x(n)$ . By doing this, we can increase  $\text{SNR}_{in}$  at the ALE input.

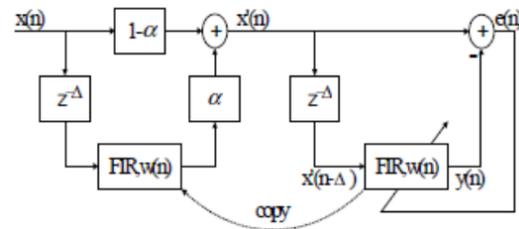


Fig. 2 The proposed ALE with tuning filter

#### A. The ALE with Self-Tuning Prefilter

As shown in Fig. 2, we regenerate the input signal from a weighted average of the primary input  $x(n)$  and the filtered version of  $x(n)$ . For filtering of  $x(n)$ , the ALE filter itself is used. So, the actual input signal to the ALE becomes

$$x'(n) = (1-\alpha)x(n) + \alpha \mathbf{x}^T(n-\Delta)\mathbf{w}(n) \quad (7)$$

where  $\alpha(0 < \alpha < 1)$  is a weighting factor. Since  $\mathbf{x}^T(n-\Delta)\mathbf{w}(n)$  in (6) corresponds to the filter output of the conventional ALE in Fig.1, the broadband noise component in  $x'(n)$  is smaller than  $x(n)$ . It means that the proposed ALE operates with a cleaner sinusoidal signal, i.e., higher  $\text{SNR}_{in}$ . We investigate this fact quantitatively as follows. We can represent  $x'(n)$  as

$$x'(n) = s(n) + v'(n), \quad (8)$$

where  $v'(n)$  is filtered noise. We define  $\text{SNR}'_{in}$  at the ALE filter input as

$$\text{SNR}'_{in} = \frac{\sigma_s^2}{E[v'^2(n)]} \quad (9)$$

The filtered noise  $v'(n)$  can be rewritten as

$$v'(n) = x'(n) - s(n) \quad (10)$$

Using (8), (10) becomes

$$v'(n) = (1-\alpha)v(n) + \alpha \left( s(n) - z^{-\Delta} \sum_{k=0}^{N-1} w_k(k)x(n-\Delta-k) \right), \quad (11)$$

Let  $\Delta s(n)$  be  $\left( s(n) - z^{-\Delta} \sum_{k=0}^{N-1} w_k(k)x(n-\Delta-k) \right)$ . Then it follows that

$$E[v'^2(n)] = (1-\alpha)^2 \sigma_v^2 + \alpha^2 E[(\Delta s(n))^2]. \quad (12)$$

From the result in (6), (12) is rewritten as

$$E[v'^2(n)] = (1-\alpha)^2 \sigma_v^2 + \alpha^2 \sigma_s^2 (1-\hat{a})^2. \quad (13)$$

So, the  $SNR'_{in}$  becomes

$$\begin{aligned} SNR'_{in} &= \frac{\sigma_s^2}{(1-\alpha)^2 \sigma_v^2 + \alpha^2 \sigma_s^2 (1-\hat{a})^2} \\ &= \frac{SNR_{in}}{(1-\alpha)^2 + \alpha^2 SNR_{in} \left( 1 - \frac{SNR_{in} \left( \frac{N}{2} \right)}{1 + SNR_{in} \left( \frac{N}{2} \right)} \right)^2} \\ &= \frac{SNR_{in}}{(1-\alpha)^2 \left( 1 + SNR_{in} \left( \frac{N}{2} \right) \right)^2 + \alpha^2 SNR_{in}} \left( 1 + SNR_{in} \left( \frac{N}{2} \right) \right)^2. \end{aligned} \quad (14)$$

As expected, when  $\alpha = 0$  the  $SNR'_{in}$  is the same as  $SNR_{in}$ .

When  $\alpha = 1$ , (14) reduces to

$$SNR'_{in} = \left( 1 + SNR_{in} \left( \frac{N}{2} \right) \right)^2.$$

For  $0 < \alpha < 1$ , it satisfies that

$$SNR'_{in} > SNR_{in}, \quad (15)$$

in case of  $N \geq 2$ .

The proposed ALE starts up in the conventional ALE configuration, with initially  $\alpha$  set to zero. After the ALE convergence is reached,  $\alpha$  is changed to a value between 0 and 1, usually around 0.8. The actual input signal to the ALE is then a mix of the primary input and the filter output. Since the proposed ALE operates with a finer signal, the broadband noise component is progressively reduced. The closer is to one, the finer input signal is obtained.

#### B. The Proposed ALE with Feedback

The proposed ALE in Fig. 2 requires an additional filtering

to generate  $x(n)$ . The additional filtering can be removed by simple modification. If we substitute  $\mathbf{x}^T(n-\Delta)\mathbf{w}(n)$  in (7) with  $\mathbf{x}^{T'}(n-\Delta)\mathbf{w}(n)$ , the actual input signal to the ALE becomes

$$\begin{aligned} x'(n) &= (1-\alpha)x(n) + \alpha \mathbf{x}^{T'}(n-\Delta)\mathbf{w}(n) \\ &= (1-\alpha)x(n) + \alpha y(n). \end{aligned} \quad (16)$$

Then the proposed ALE structure in Fig. 2 changes to the structure shown in Fig. 3. To generate  $x(n)$ , the ALE output is reused as a feedback signal. So, the filter structure changes from FIR to IIR when  $\alpha > 0$ . It is well known that an IIR filter can achieve a sharper frequency response with fewer weights than an FIR filter. Thus the proposed ALE configuration in Fig. 3 may increase the filter gain and attenuate noise component, i.e., give higher SNR at  $y(n)$  without increasing the filter length. This is precisely the desired result. The simulations described later will support these claims. In the case of  $\alpha = 1$ , this configuration becomes as an adaptive oscillator to track the instantaneous frequency of the input signal.

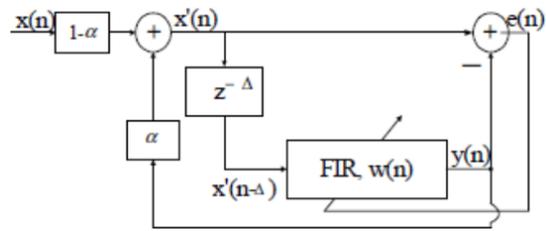


Fig. 3 The proposed ALE with feedback

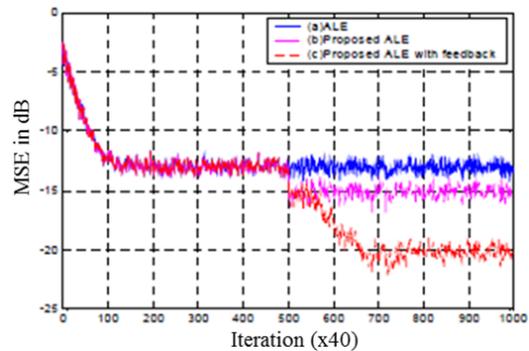


Fig. 4 MSE of sinusoidal estimation

## IV. RESULTS

We evaluate the performance of the proposed ALE methods by computer simulation in comparison with the conventional ALE. The simulation is undertaken using various combination of weighting factor  $\alpha$ . The input is a sinusoidal signal with unit amplitude for all simulations.

Fig. 4 shows the mean square of sinusoidal estimation error,  $(s(n) - y(n))^2$  in dB scale. The  $SNR_{in}$  is set to 0dB and filter

length is 40. The proposed ALEs begin in conventional ALE configuration with  $\alpha = 0$ . At the 20000th iteration,  $\alpha$  is set to 0.8. The MSE is averaged over 50 independent trials. In Fig. 4, we can see the initial exponential decay of the MSE and then a clear decrease after the proposed ALE methods are adopted. As expected, the proposed ALE with feedback shows best performance.

#### V. CONCLUSION

A novel adaptive line enhancer structure has been proposed. Compared with the conventional ALE, the performance is highly improved. This means that the filter order of the proposed ALE can be much lower than that of the ALE to achieve equivalent performance, thus reducing computational load. This improvement is achieved by preprocessing the input signal to the ALE using the ALE filter. Also, since the preprocessing filter is the ALE filter itself, the structure can be changed to IIR form with the reduced filter length. The main focus of the paper was to make the ALE operate with a finer.

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