

# Adaptive Fuzzy Control of a Nonlinear Tank Process

A. R. Tavakolpour-Saleh, H. Jokar

**Abstract**—Liquid level control of conical tank system is known to be a great challenge in many industries such as food processing, hydrometallurgical industries and wastewater treatment plant due to its highly nonlinear characteristics. In this research, an adaptive fuzzy PID control scheme is applied to the problem of liquid level control in a nonlinear tank process. A conical tank process is first modeled and primarily simulated. A PID controller is then applied to the plant model as a suitable benchmark for comparison and the dynamic responses of the control system to different step inputs were investigated. It is found that the conventional PID controller is not able to fulfill the controller design criteria such as desired time constant due to highly nonlinear characteristics of the plant model. Consequently, a nonlinear control strategy based on gain-scheduling adaptive control incorporating a fuzzy logic observer is proposed to accurately control the nonlinear tank system. The simulation results clearly demonstrated the superiority of the proposed adaptive fuzzy control method over the conventional PID controller.

**Keywords**—Adaptive control, fuzzy logic, conical tank, PID controller.

## I. INTRODUCTION

**I**N the process industry, the majority of the systems are highly nonlinear and hence, the system parameters may experience some uncertainties. Thus, controlling such nonlinear plants is of practical importance. The classical PID algorithms are not able to satisfy the design conditions for such systems according to their nonlinear characteristics. Many intelligent control techniques have been previously presented to cope with the problem of controlling the nonlinear processes such as neural networks, genetic algorithms, and fuzzy logic.

The liquid level control in the nonlinear tanks is an important issue in the industry due to its extensive applications. Among them, liquid level control in the conical tanks would be probably the most challenging one due to its high nonlinearity and abrupt variations of the cross-sectional area with respect to the height of the liquid in the tank. In recent years, the fuzzy logic controllers have been widely used in various nonlinear processes to achieve desired control response. It may be attributed to the fact that the fuzzy logic provides a powerful platform that allows engineers to apply human reasoning to the control algorithm [1] and this is one of the issues that the modern control theories are not able to carry out. Indeed, in human reasoning-based control systems, it is possible to transfer human experience to the control system. One of the

successful applications of fuzzy logic controllers is in the liquid-level control field. Many researchers proposed different fuzzy logic controllers to solve the level control problems in the linear and nonlinear tanks.

Haramniwas et al. [2] investigated the fluid level control in a cylindrical tank using fuzzy logic. They then compared the obtained results to those of the conventional PID controllers. Thus, they showed the superiority of the proposed fuzzy logic controller over the conventional PID controller. Arivalahan et al. [3] applied a fuzzy logic controller to a conical tank and obtained the system response in servo and regulation modes. They then compared the obtained results with the simulation outcomes of the conventional PID controller. It was found that the fuzzy logic controller had a better performance compared to the conventional PID controller. Suresh et al. [4] studied the performance of a fuzzy logic controller for three cylindrical tanks. They showed that the proposed control system was robust to disturbance under different conditions. Disha et al. [5] simulated a water level control system in a tank using fuzzy logic Toolbox in MATLAB.

A number of researchers strived to control the nonlinear tank process by adaptive control techniques such as gain scheduling (GS) adaptive control, iterative learning control (ILC), model reference adaptive control (MRAC) etc. Dinesh Kumar et al. [6] implemented a nonlinear control system for two spherical tanks, which were connected together by a pipe using gain-scheduled PI controller. Then, the performance of the proposed control system was evaluated in terms of some performance indices. Barriljawatha [7] applied an adaptive control technique to two conical tanks and then, the performance of the control system was evaluated.

Based on the outlined literature, there was no published paper in which a gain-scheduled adaptive controller incorporating a fuzzy logic observer is applied to control a nonlinear conical tank process. Consequently, this research strives to present another alternative to liquid-level control of a conical tank system based on the proposed adaptive fuzzy control scheme. The effectiveness of the proposed adaptive control scheme is then demonstrated through simulation. Finally, the simulation results are compared to those of the conventional PID controller through which the effectiveness of the proposed adaptive fuzzy controller is demonstrated.

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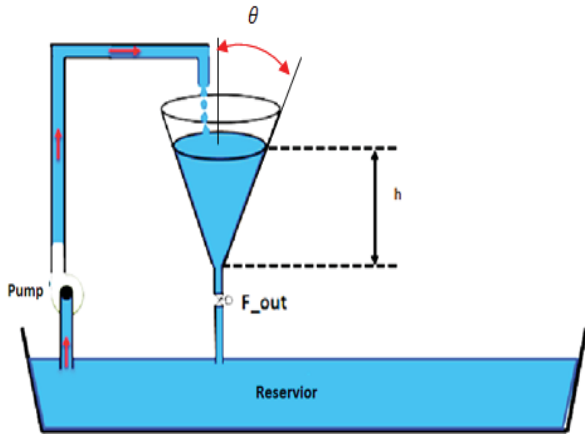


Fig. 1 Schematic diagram of the conical tank system

## II. MODELING OF THE TANK SYSTEM

The nonlinear tank process considered in this investigation includes a conical tank in which a specific liquid is pumped from the top side with the flow rate of  $F_{in}$  (see Fig. 1). The fluid flow then leaves the conical tank through a pipe connected to the bottom of the tank. The aim of this work is thus to control the liquid level in the presented nonlinear tank. In other words, it is desirable to keep the height of fluid at a specific level, which is defined by the operator. In designing an effective controller, the first step is to obtain the mathematical model governing the nonlinear tank system as a differential equation. Using the mass balance and taking into account the steady state flow of the fluid, the mathematical equations governing the tank system can be expressed as follows:

The principle of mass conservation states that:

$$\frac{\text{Mass accumulation}}{\text{time}} = \frac{\text{input mass}}{\text{time}} - \frac{\text{output mass}}{\text{time}}$$

Thus,

$$\frac{dm}{dt} = \frac{dm_{in}}{dt} - \frac{dm_{out}}{dt} \quad (1)$$

where  $m$  is the accumulated mass in the tank,  $m_{in}$  is the input mass, and  $m_{out}$  is output mass of liquid. It is assumed that the fluid is incompressible and the density of the fluid is constant. Thus, it is obvious that  $m = \rho V$ . Consequently, (1) can be simplified as:

$$\frac{dV}{dt} = F_{in} - F_{out} \quad (2)$$

where  $V$  is volume of liquid accumulated in the conical tank,  $F_{in}$  and  $F_{out}$  are respectively inlet and outlet volumetric flow rates as shown in Fig. 1. Since  $dV = Adh$ , then (2) can be written as:

$$\frac{Adh}{dt} = F_{in} - F_{out} \quad (3)$$

where,  $A$  is the cross-sectional area of the conical tank, which

is a function of the liquid height in the tank (denoted by  $h$ ).

Considering a steady state flow, the energy equation between fluid free surface in the tank and the discharge point is expressed as:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_f \quad (4)$$

where,  $P_1 = P_2 = P_a$  is atmospheric pressure,  $V_1$  is fluid the velocity at the tank inlet,  $V_2 = \frac{F_{out}}{A_p}$  is the fluid velocity at the tank outlet and  $h_f$  is the pressure loss due to the gradual contraction of outlet flow and the hydraulic resistance of the valve which can be written as follows:

$$h_f = K_c \frac{V_2^2}{2g} + K_v \frac{V_2^2}{2g} \quad (5)$$

Simplifying (5) yields,

$$h_f = (K_c + K_v) \frac{V_2^2}{2g} \quad (6)$$

where, dimensionless coefficients  $K_c$  and  $K_v$  are factors indicating the head loss due to the gradual flow contraction and the valve resistance respectively.

In this work, the viscous friction between the fluid and the inner surface of the pipe is neglected because of its small value compared to other considered resistances. Substituting (6) into (4) yields:

$$h = \frac{V_2^2}{2g} + (K_c + K_v) \frac{V_2^2}{2g} \quad (7)$$

or

$$h = (1 + K_c + K_v) \frac{V_2^2}{2g} \quad (8)$$

The volumetric flow rate can be found by multiplying the cross-sectional area of the pipe ( $A_p$ ) by the fluid average velocity at the tank outlet ( $V_2$ ) as:

$$F_{out} = A_p V_2 \quad (9)$$

Thus,  $V_2$  can be acquired using (8) and it can be substituted into (9) to compute the output flow rate as follows:

$$F_{out} = A_p \sqrt{\frac{2gh}{1 + K_c + K_v}} \quad (10)$$

Equation (10) can be represented in a compact form as:

$$F_{out} = \beta \sqrt{h} \quad (11)$$

where,

$$\beta = A_p \sqrt{\frac{2g}{1 + K_c + K_v}} \quad (12)$$

The values of  $K_v$  and  $K_c$  were considered as 0.2 and 0.05 respectively [8]. In this study, the cone angle was 30 degree ( $\theta = 30^\circ$ ). Therefore, the radius of liquid free surface in the tank was  $h \tan(30)$ . Consequently, the area of the fluid free surface in the tank ( $A$ ) as a function of the liquid height ( $h$ ) can be expressed as:

$$A = \pi(h \tan(30))^2 \tag{13}$$

Substituting (11) and (13) into (3), yields the final nonlinear differential equation governing the tank process considered in this investigation,

$$F_{in} - \beta\sqrt{h} = \pi \tan^2(30)h^2 \frac{dh}{dt} \tag{14}$$

It can be clearly seen that the considered dynamic model is nonlinear. The obtained dynamic model was thus utilized to

design the control system in the next sections.

### III. OPEN-LOOP SIMULATION OF THE PROCESS

Table I represents the required parameters for the simulation study.

Parameters	Value
gravitational acceleration (g)	9.81 (m/s <sup>2</sup> )
Valve opening area ( $A_p$ )	$\pi(.02)^2 = 0.001256$ (m <sup>2</sup> )
Sensor gain	15.7 (V/m)
Pump gain	$13.571 \times (10)^{-6}$ (m <sup>3</sup> /sV)

According to obtained mathematical model in the previous section and the considered parameters, the open loop system was simulated. Fig. 2 demonstrates the block diagram of the open-loop system.

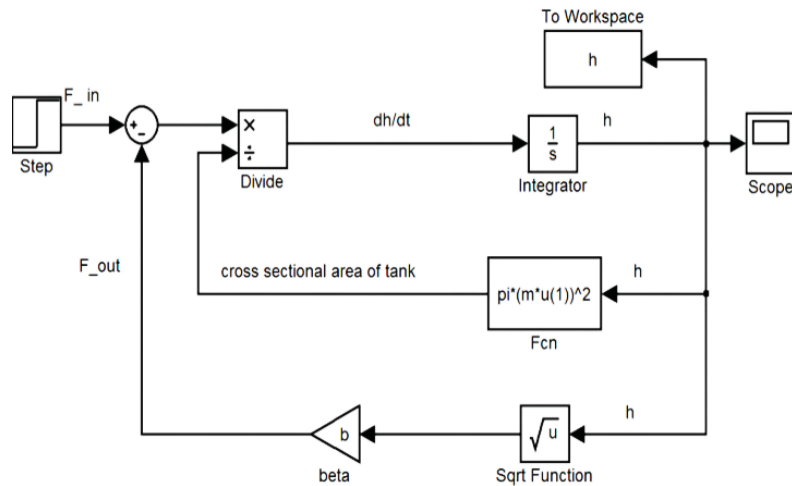


Fig. 2 Block diagram of the open-loop system

Figs. 3 and 4 respectively represent the step input and the corresponding tank response obtained from the simulation study. As can be seen in Fig. 4, the tank system responded too slow to the step input which is not desired. To cope with this latest problem, a feedback PI control system was devised.

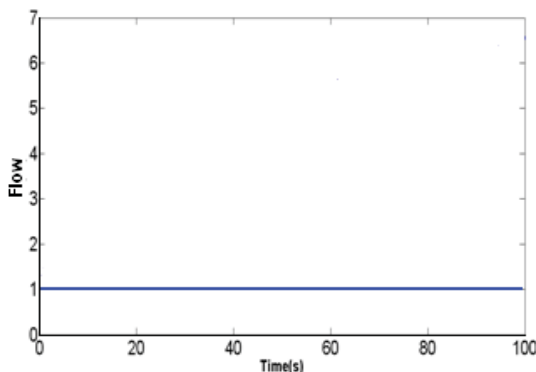


Fig. 3 Step input to the open-loop system

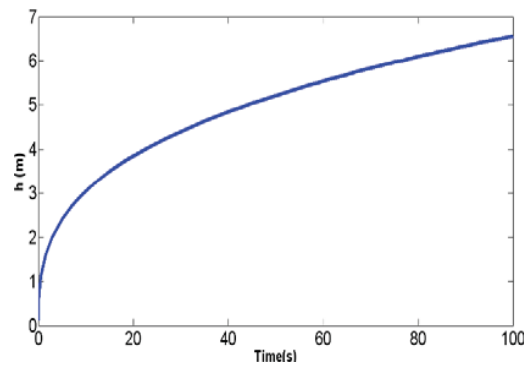


Fig. 4 Response of the open-loop system to step input

### IV. CLOSED-LOOP SIMULATION OF THE PROCESS USING CLASSICAL PI CONTROLLER

First, a PID control system was proposed to control the fluid level in the nonlinear tank process. It was firstly chosen because of its simplicity. The objective of the controller design was to

achieve a time constant of 10 s. The block diagram of the closed-loop PI control system was demonstrated in Fig. 5.

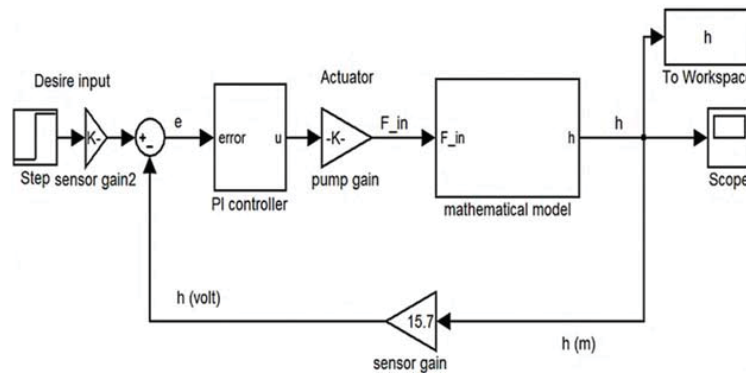


Fig. 5 Block diagram of the closed-loop PI control system

The parameters of the PI controller ( $K_p$ ,  $K_i$ ) were found through a trial and error scheme so that the desired time constant was obtained. The proposed PI control parameters were shown in Table II. These parameters were selected so as to fulfill the design criteria corresponding to the unit step input. Since the considered tank system was highly nonlinear, it was important to investigate the performance of the designed PI controller for other step inputs with different amplitudes. It is found that if the desired height varies to 5 meter, the controller

performance with the fixed parameters will degrade considerably in comparison with the results obtained with the unit step input as shown in Fig. 6.

TABLE II  
OBTAINED PI CONTROLLER PARAMETERS

Parameter	$K_p$	$K_i$
Value	1400	500

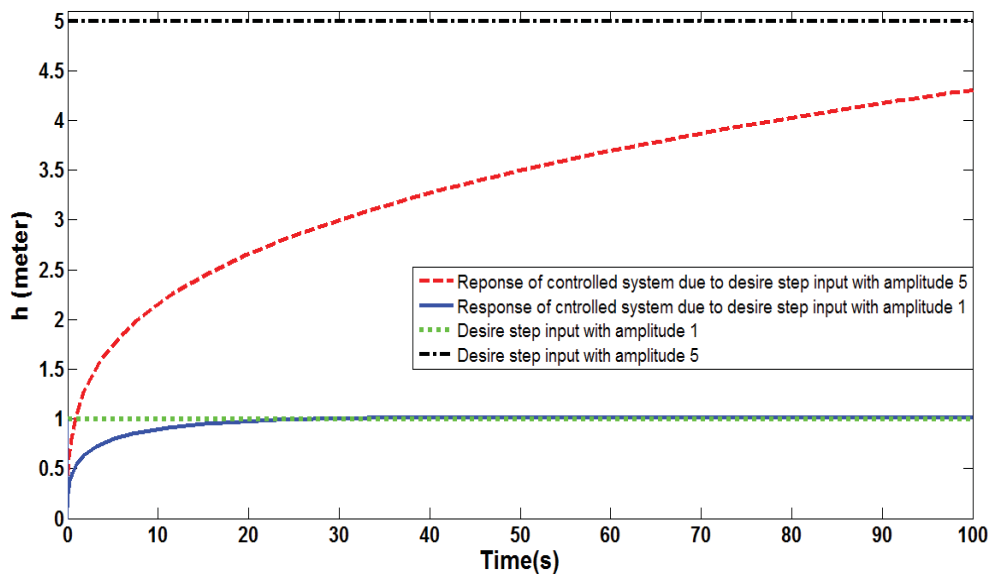


Fig. 6 Response of the PI control system to step inputs with amplitudes of 1 m and 5 m

According to Fig. 6, it can be clearly seen that the PI controller is not efficient to control such a highly nonlinear system over a wide range of the reference input. As a result, a gain-scheduled adaptive control scheme was devised to effectively control the mentioned nonlinear tank process. Since, the height of liquid in the conical tank was a measurable parameter that affected the nonlinear terms in the tank system, it was considered as the gain scheduling variable in the adaptive

controller. Thus, a gain scheduling table was organized in order to find the approximate relationships between the considered gain scheduling variable and the required proportional and integral gains of the PI controller. Next, a fuzzy logic observer was trained based on the obtained mathematical relationships from the gain scheduling table. After initially training the fuzzy observer, the control system performance was gradually enhanced by changing the center positions of the membership

functions over the output variables of the fuzzy controller. Consequently, the PI control parameters e.g.  $K_p$  and  $K_i$  could vary according to the liquid level in the tank system. One should keep in mind that the fuzzy logic controllers are known as efficient function estimators through which any nonlinear functions can be estimated. Thus, a fuzzy logic observer was used together with the proposed adaptive gain scheduling controller to tune the PI controller gains based on the measured liquid level in the tank.

#### V. DESIGN OF FUZZY LOGIC CONTROLLER TO ADAPTIVELY ESTIMATE THE CONTROLLER PARAMETERS

Fuzzy logic attempts to mimic human reasoning and decision making. Thus, it provides an intuitive way to carry out control systems, decision making and diagnostic systems in various branches of engineering. Indeed, fuzzy logic allows engineers to transfer their empirical knowledge as 'If-Then' rules to a functional block. These capabilities enable fuzzy logic to be employed in engineering applications such as intelligent control systems, decision making and expert systems. The concept of fuzzy set was introduced by Lotfi A. Zadeh in 1965 [10]. Then, by extending this concept, he presented the concept of "Linguistic variable" in 1975. Since then, fuzzy logic has been successfully used in various engineering applications. One of the first works done to demonstrate its capabilities in practice, was cement kiln which was presented in 1975 [10]. Nowadays, this theory is used in many applications such as washing machines, refrigerator, automobiles and etc.

A fuzzy system is represented by input and output linguistic variables along with the definitions of linguistic terms and the fuzzy IF-THEN rule bases. The fuzzy inference engine is a mechanism that computes the output according to the input values and based on some well-defined criteria. Any fuzzy systems contain fuzzification, fuzzy inference and defuzzification processes. The fuzzification process allows converting a crisp value into fuzzy one with the two common ways namely singleton and fuzzy set. The fuzzy inference, which works based on adopted techniques such as max, sum and etc., is used when the number of rules is more than one. The defuzzification process is the inverse of the fuzzification process in which a fuzzy set is converted to a crisp value. Note that, there are many techniques to perform defuzzification operation one of which is known as centroid.

As mentioned previously, in this paper, the controller parameters including  $K_p$  and  $K_i$  are calculated adaptively through the use of the fuzzy logic observer. The basic block diagram of the proposed adaptive fuzzy controller is shown in Fig.7. As can be seen, the fuzzy logic estimator consists of three parts. First, the crisp value of controlled output is converted to the corresponding fuzzy value over the universe of discourse as mentioned previously. It was called Fuzzification. Second part is called the fuzzy inference engine which works according to the rules defined by the designer and finally, the third part was called defuzzification in which the fuzzy value is converted into its equivalent crisp value that produces the control parameters  $K_p$  and  $K_i$ .

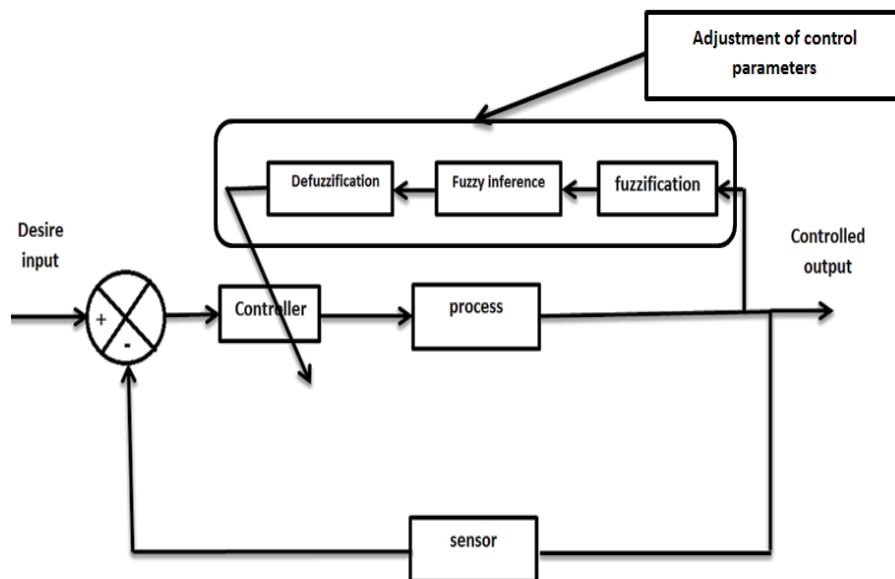


Fig. 7 Block diagram of the adaptive Fuzzy control system

In this work, the adjustment mechanism for the PI control parameters was based on a fuzzy logic observer which had one input variable namely the height of accumulated fluid in the conical tank ( $h$ ) and two output variables namely proportional and integral gains ( $K_p$  and  $K_i$ ) of the PI controller. Furthermore, four membership functions were defined for input and output

variables because of positive sign of the input variable ( $h$ ). Therefore, the following notations were designated to the input-output membership functions:

- AZ: absolutely zero
- PS: positive small
- PM: positive medium

- PB: positive big

#### A. Membership Functions for Input Variables

As mentioned earlier, the fuzzy observer possessed only one input variable. Symmetrical triangular membership functions with linear distribution were thus considered for the input

variable ( $h$ ) as shown in Fig. 8. Note that the peak of first membership function was at 0.1 m because of the assumed initial condition of  $h = 0.1$  in this work. The proposed membership functions for the input variable are depicted in Fig. 8.

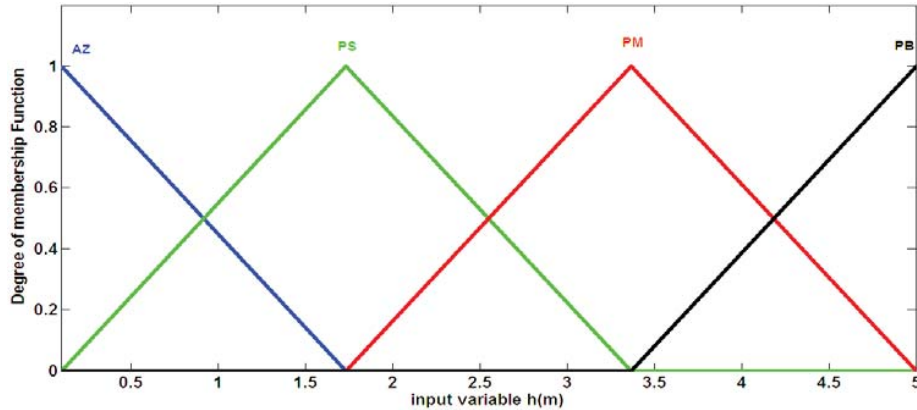


Fig. 8 Membership functions of the input variable (labeled as height of fluid in conical tank)

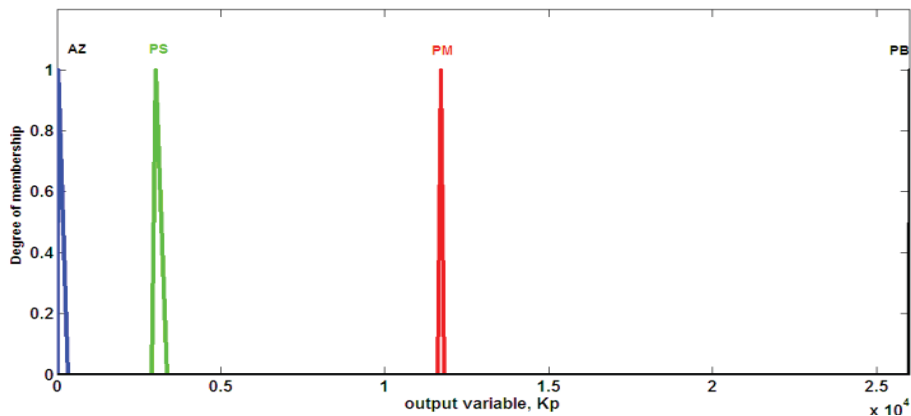


Fig. 9 Membership functions of the first output variable denoted by  $K_p$

#### B. Membership Functions for Output Variables

Four singleton membership functions were defined for the output variable  $K_p$ . The center points of the membership functions were initially found from the previously obtained gain scheduling table. Then, through a trial and error scheme the center positions of the singletons were shifted so that a better performance of the control system was found in comparison with the initial gain scheduling adaptive controller. Fig. 9 represents the considered membership functions for the first output variable labeled as  $K_p$  (the proportional gain).

For the second fuzzy output variable e.g. the integral gain  $K_i$ , four singleton-type membership functions were considered as shown in Fig. 10.

#### C. The Fuzzy Rules

In this investigation four fuzzy rules were introduced to the fuzzy inference engine as follows:

1. If  $h$  is AZ, Then  $K_p$  is AZ and  $K_i$  is AZ,
2. If  $h$  is PS, Then  $K_p$  is PS and  $K_i$  is PS,
3. If  $h$  is PM, Then  $K_p$  is PM and  $K_i$  is PM,
4. If  $h$  is PB, Then  $K_p$  is PB and  $K_i$  is PB.

The interpretation of the fuzzy inference process is represented in Fig. 11. The presented rule viewer also shows how the shape of certain membership function affects the overall result. This plot is often so helpful to understand the working mechanism of the fuzzy system because it demonstrates the fuzzy inference for every rule. For instance, as shown in Fig. 11, when  $h = 2.55$  m, the output variables  $K_p$  and  $K_i$  are 7230 and 13.3 respectively.



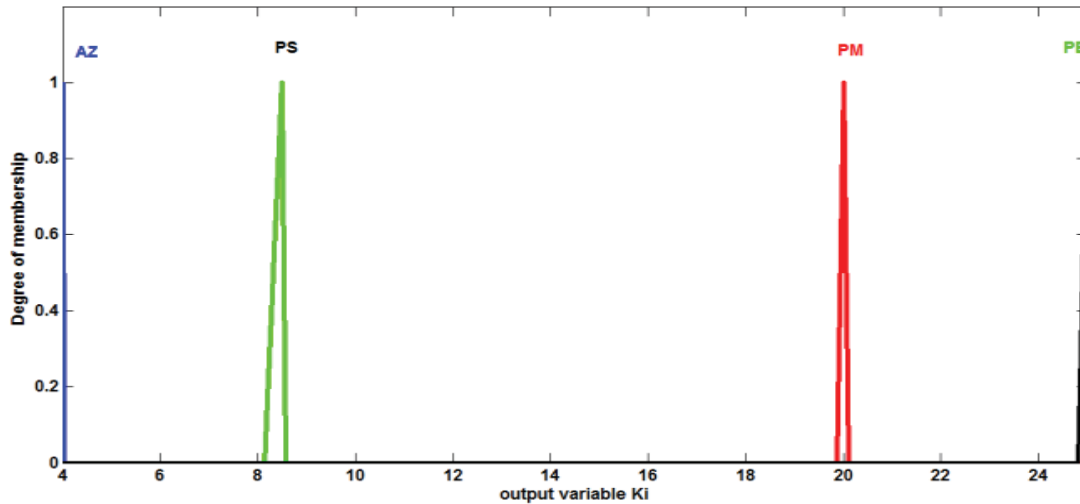


Fig. 10 Membership functions of the second output variable denoted by  $K_i$

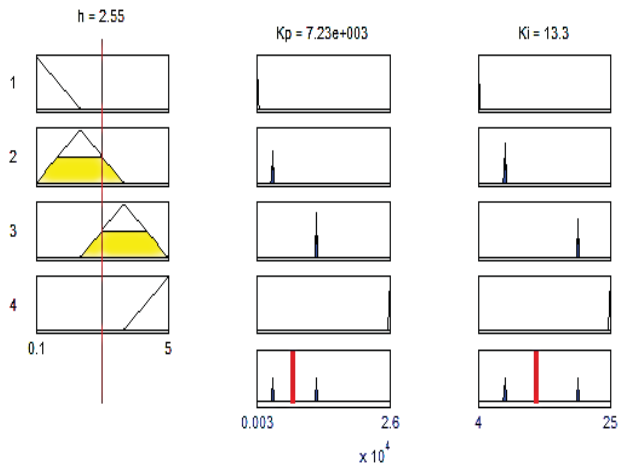


Fig. 11 Representation of the fuzzy inference using the rule viewer

The surface viewer of fuzzy logic toolbox in MATLAB can be used to present a performance curve and sometimes, a surface according to the number of the input variables. This plot represents the mapping between the input and the outputs of the fuzzy system. The obtained results were demonstrated with two curves as shown in Figs. 12 and 13 taking into consideration the proposed fuzzy block possessed one input variable. These curves represent the proportional and integral gains versus the liquid height in the conical tank. Indeed, these curves serve to clearly show the variations of the PI controller parameters corresponding to the change in the fluid level. In other words, the presented curves reveal the adaptation laws of the adaptive PI controller based on the use of the fuzzy logic observer.

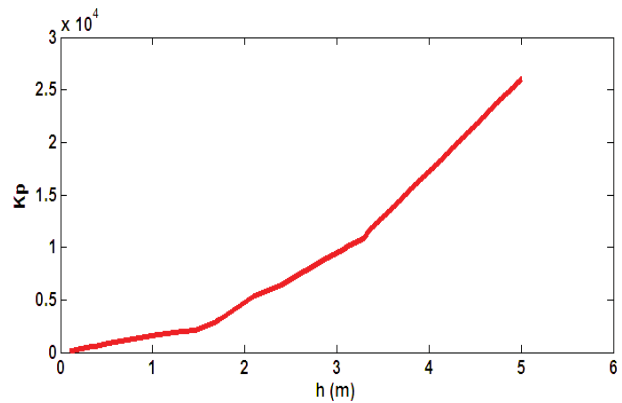


Fig. 12 Variation of the proportional gain versus liquid height based on the fuzzy logic observer

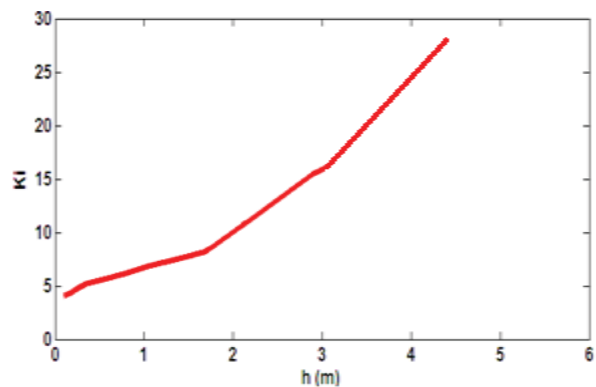


Fig. 13 Variation of the integral gain versus liquid height based on the fuzzy logic observer

## VI. SIMULATION OF THE ADAPTIVE FUZZY CONTROL SYSTEM

One of the objectives of this work, as discussed earlier, was to keep the fluid height in the tank at an arbitrary desired value so as to satisfy the expected performance. Besides, as discussed in Section IV, the classical PID controller could not achieve this

requirement due to the nonlinear characteristics of the dynamic system. Consequently, an adaptive fuzzy controller was proposed and designed in the previous section. In this section, the presented adaptive fuzzy controller is applied to the fluid level control problem of the nonlinear tank and dynamic

response of the proposed feedback control system is investigated through simulation. The block diagram of the closed-loop adaptive fuzzy PI control system is demonstrated in Fig. 14.

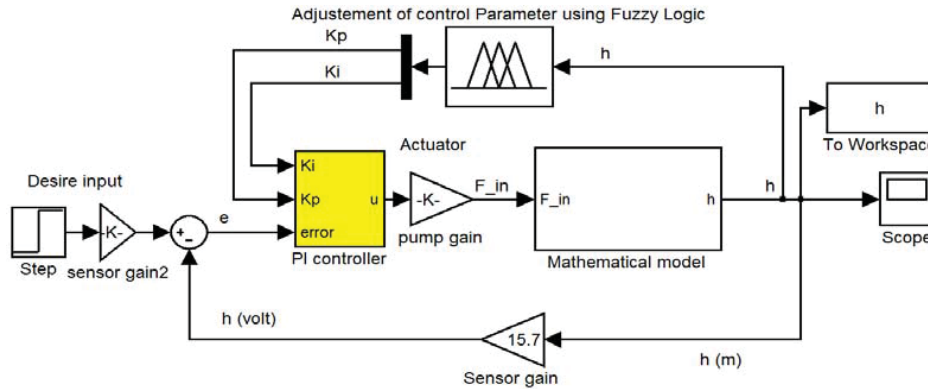


Fig. 14 Block diagram of the closed-loop control system with adaptive PI controller incorporating the fuzzy logic observer

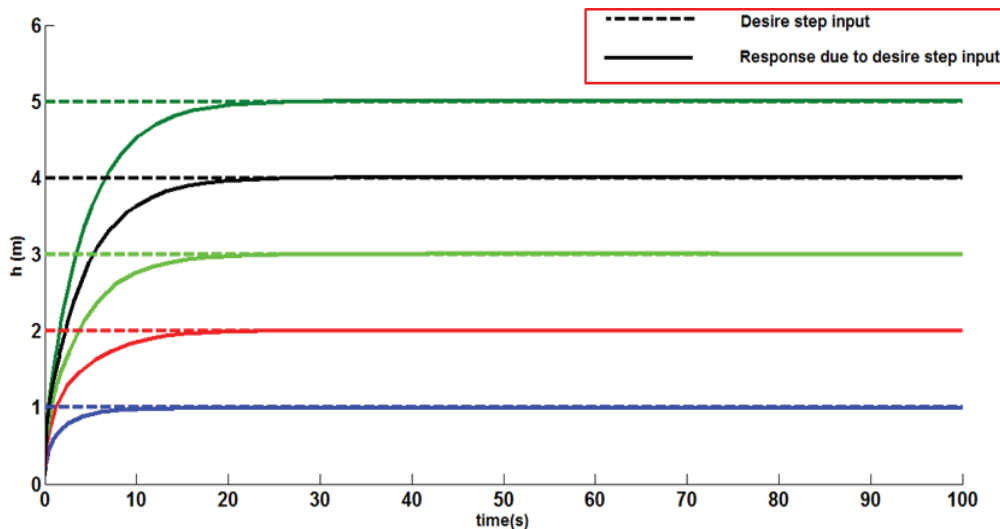


Fig. 15 Responses of the adaptive fuzzy regulator to different step inputs

The simulation study was carried out considering two operating modes of the controller. First, the controller was considered in regulation mode and the response of the controller to a fixed desired reference was investigated. Then, the servo operation of the control system was investigated.

#### A. Simulation Results of Adaptive Fuzzy Control System in Regulation Mode

In regulation mode, the set point of the control system needs to be constant while the process is varying [3]. The performance of the proposed control system in regulation mode was thus simulated considering some step references with different amplitudes within the operating range up to five meter. The simulation results are represented in Fig. 15.

Comparing to the results obtained from the conventional PI controller (see Section IV), one can clearly see the superiority

of the proposed adaptive fuzzy control scheme over the classical PI controller for all considered set points. Besides, an acceptable time constant corresponding to each set point was found through which the effectiveness of the proposed controller was demonstrated.

#### B. Simulation Results of the Adaptive Fuzzy Control System in Servo Mode

In servo mode, the set point value is variable [3]. In this section, the simulation results were acquired under the variations of the set point value. Fig. 16 reveals the capability of the adaptive fuzzy control system to follow a reference trajectory. It can be seen that alteration of the reference amplitudes results in a significant degradation of performance in the classical PI controller. In contrast, the proposed adaptive fuzzy controller is not sensitive to the reference input



variations.

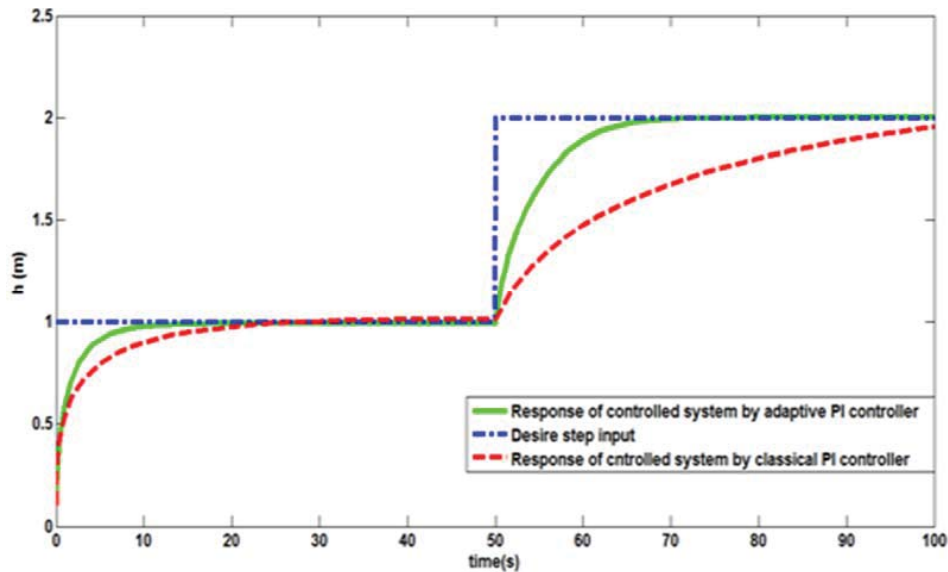


Fig. 16 Comparative responses of the adaptive fuzzy and conventional PI controllers in servo mode

## VII. CONCLUSIONS

In this paper, a highly nonlinear conical tank process was modeled and then controlled by an adaptive fuzzy scheme. According to the simulation results in servo and regulator modes, it was found that the adaptive PI controller incorporating fuzzy logic observer was able to compensate the nonlinearities presented in the dynamic system. Furthermore, it was observed that the adaptive fuzzy controller was superior compared to the classical PID controller. Consequently, the classical PID controller was not adequate for controlling the highly nonlinear processes such as liquid-level control in the conical tank system due to abrupt variation of system parameters which was unavoidable according to the presented nonlinearities. Since, a small number of fuzzy membership functions for input and output variables were considered in the fuzzy controller then the proposed control method can be used in real-time applications i.e. controlling the highly nonlinear tank system. Further research is devoted to experimentally validate the performance of the proposed control scheme.

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