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Active Control Improvement of Smart Cantilever Beam by Piezoelectric Materials and On-Line Differential Artificial Neural Networks

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***Abstract*—**The main goal of this study is to test differential neural network as a controller of smart structure and is to enumerate its advantages and disadvantages in comparison with other controllers. In this study, the smart structure has been considered as a Euler Bernoulli cantilever beam and it has been tried that it be under control with the use of vibration neural network resulting from movement. Also, a linear observer has been considered as a reference controller and has been compared its results. The considered vibration charts and the controlled state have been recounted in the final part of this text. The obtained result show that neural observer has better performance in comparison to the implemented linear observer.

***Keywords*—**Smart material, on-line differential artificial neural network, active control, finite element method.

I. Introduction

USING piezoelectric materials in controlling smart structures have always been one of the challenges of the today's science. In the conducted researches on the control of vibration of smart structures, due to the sensitivity of piezoelectric materials, control methods have been used based on optimal control. Cantilever beams are used as a base model for many industrial structures such as plane wings, helicopters propellers, and robot armholes. Researchers have mainly used two analytical and finite element methods for modeling beams. But so far, the online control logic has not been used by smart neural networks to control these structures. Therefore, in this research, a brief introduction and an overview of the mentioned underlying concepts have been discussed. First, previous researches that were devoted to the modeling and controlling of the vibrations of cantilever beams are presented. At first, beam modeling is presented with the finite element method and Euler-Bernoulli beam theory, then explanation of the active vibration control, its advantages and disadvantages, a brief description of the smart structures and a variety of smart materials, especially piezoelectric ones, are provided. In the following, a brief description of the differential online neural networks and the benefits of its efficiency, and finally, the results are expressed.

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Since 1970, finite element methods have been used to model smart structures. Finite element methods are based on analytical methods and are used in problems of complexity in form and material.

Culshaw has discussed the concept of smart structure, its advantages and uses [1]. Rao and Sunar, in an article, showed how to use piezoelectric materials as sensors and actuators to measure vibrations [2]. Hubbard and Baily investigated the application of piezoelectric materials as sensors and actuators in flexible structures [3]. Hanagud and colleagues studied a finite element model (FEM) for a beam, along with many different piezoceramic sensors and actuators [4]. Fanson and colleagues carried out experiments on beam equipped with piezoelectric and its vibration control using positive conditional feedback [5]. The experimental evaluation of piezoelectric actuator for vibration control in a cantilever beam was presented by Burdess and Faucett [6]. Brennan and colleagues conducted tests on the beam for various actuation technologies [7]. Yang and Lee investigated the optimization of feedback gain in designing a control system for structures [8]. They described an analytical model for controlling the vibration of the structures in which the locating of the sensor and the non-centered actuator and control feedback gain were considered as independent variables. Crawley and Luis presented the expansion of the piezoelectric sensors and actuators as elements of the smart structures for the first time [9]. The continuous time and discrete time algorithms for controlling a thin piezoelectric structure, were proposed by Bona et al. A brief overview of recent years' studies in this area has been selected and summarized as follows. Gaudenzi et al. studied controlling of the lateral vibrations of the cantilever beam using the finite element method and Euler-Bernoulli beam modeling [10]. In this reference, state feedback and speed feedback are used to control vibrations. Sung examined controlling of a simply supported beam with a moving mass [11]. He, with the hypothetical modes approach, transformed the dynamical equations based on the Euler-Bernoulli beam theory, to the space state form and via a multi-input multi-output controller, reduced the beam curvature due to the moving mass. To determine the optimal location of piezoelectric actuators, Sung used a second-order linear regulator function and determined the position of two actuators connected to the bottom of the beam. Xu and Koko monitored the vibration control of the beam [12]. In this study, piezoelectric materials have been used as the stimuli and, by introducing an index as the controllability index, the optimal location of the stimuli has been identified. Lin and Nien investigated the vibrations of the cantilever beam and its active control in an analytical manner [13]. They used six pairs of piezoelectric patches as sensors and actuators to control vibrations. Vasquez et al. studied the active vibration control of the beam with limited element modeling [14]. They compared classical control methods and optimal control methods for the Gaussian second-order linear regulator and examined the advantages and disadvantages of each of them. Ducarne et al. investigated the inactive vibration reduction with piezoelectric patches by an analytical method [15]. By proposing the optimization method based on maximizing the electromechanical coupling coefficient between the beam and the piezoelectric actuator, they optimized the piezoelectric location, length and thickness. Among active structural vibration control methods, the use of piezoelectric stimuli and sensors, and the design of a controller with various controlling laws have a significant prevalence, while the system equations of these structures are fairly complex and require a great deal of time and accuracy to solve them relatively precisely. The advantage of using the neural network, is to predict the state of the system without the use of heavy mathematical equations and thus, to control the vibrations of the system with high precision. Neural networks can be trained in an online manner in parallel with the system and they can reduce their error to a very marginally amount, which makes the neural network state feedback controller to be highly accurate and, especially, to be distinguished from other controllers in controlling nonlinear systems.

II. Finite Element Method Modeling

Consider the homogeneous steel beam of Fig. 1. This beam is assumed to a cantilever one and, piezoelectric sensors and actuators are arranged in discrete positions along the length of the beam. This is shown in Fig. 2. An external force of f_{ext} is applied on the beam and therefore, the beam vibrates.

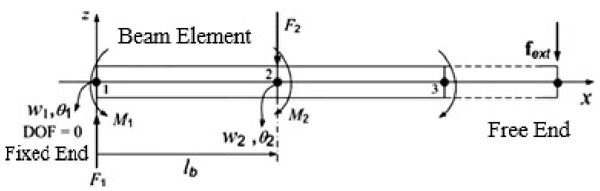

Fig. 1 Aluminum cantilever beam element

For modeling the beam mathematically, first, the usual model of a beam element, as well as the usual model of a piezo element are obtained for a two nodes finite element.

The flexible beam is divided into a number of finite elements (for example, four elements, Fig. 3), and piezoelectric elements are placed on one of them.

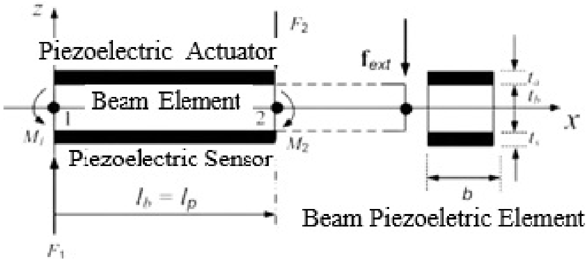

Fig. 2 Smart cantilever beam element

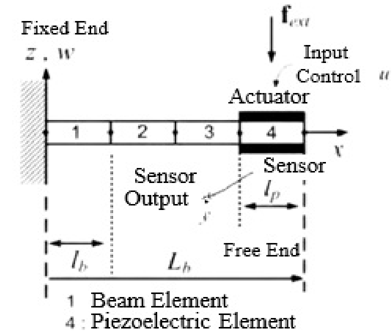

Fig. 3 A flexible beam divided into four finite elements

Subsequently, the general model of the smart beam is extracted using piezoelectric elements and usual beam elements, derived from the theory of Euler-Bernoulli beam and finite element method. The usual beam elements are used for areas with no sensor or actuator on them. The equation of motion for the usual beam element, after applying the finite element method, its application to the Euler-Bernoulli beam theory and simplifying it, is obtained as follows [16]:

$$\frac{\rho_b A_b l_b}{420}\begin{bmatrix} 156 & 22l_b & 54 & -13l_b \\ 22l_b & 4l_b^2 & 13l_b & -3l_b^2 \\ 54 & 13l_b & 156 & -22l_b \\ -13l_b & -3l_b^2 & -22l_b & 4l_b^2 \end{bmatrix}\begin{bmatrix} \ddot{\omega}_1 \\ \ddot{\theta}_1 \\ \ddot{\omega}_2 \\ \ddot{\theta}_2 \end{bmatrix} + \frac{E_b I_b}{l_b}\begin{bmatrix} 12/l_b^2 & 6/l_b & -12/l_b^2 & 6/l_b \\ 6/l_b & 4 & -6/l_b & 2 \\ -12/l_b^2 & -6/l_b & 12/l_b^2 & -6/l_b \\ 6/l_b & 2 & -6/l_b & 4 \end{bmatrix}\begin{bmatrix} \omega_1 \\ \theta_1 \\ \omega_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{bmatrix} \tag{1}$$

Here, F_1 and F_2 are forces, and M_1 and M_2 are the bending moments applied on nodes 1 and 2 of the beam element.

Piezoelectric elements can be used as actuators and sensors in flexible structures. These elements, such as the beam element, are considered as two degrees of freedom in each knot, and a degree of electrical freedom is assigned to them as a voltage.

Mass and stiffness matrices of M^p and K^p of piezoelectric element have been extracted such as the usual beam element [16].

$$M^p = \frac{\rho_p A_p l_p}{420}\begin{bmatrix} 156 & 22l_p & 54 & -13l_p \\ 22l_p & 4l_p^2 & 13l_p & -3l_p^2 \\ 54 & 13l_p & 156 & -22l_p \\ -13l_p & -3l_p^2 & -22l_p & 4l_p^2 \end{bmatrix} \tag{2}$$

$$K^p = \frac{E_p I_p}{l_p}\begin{bmatrix} 12/l_p^2 & 6/l_p & -12/l_p^2 & 6/l_p \\ 6/l_p & 4 & -6/l_p & 2 \\ -12/l_p^2 & -6/l_p & 12/l_p^2 & -6/l_p \\ 6/l_p & 2 & -6/l_p & 4 \end{bmatrix} \quad (3)$$

where in, $_p\rho$ is the piezoelectric material density, A_p is the piezoelectric patch cross section, l_p is the piezoelectric element length, b is the piezoelectric piece and beam width, Ep is the piezoelectricity elastic modulus, Ip is the moment of inertia of the piezoelectric layer, tp is the piezoelectric layer thickness, and tb is the thickness of the beam. It should be noted that the matrices of mass and stiffness of the piezoelectric element, are the same for both the sensor and actuator layers of the same dimensions.

III. Governing Equations of Smart Beam

With regard to Fig. 4, in which the beam is divided into four elements, and the piezoes are placed in a co-centered manner in the first element, we consider the matrices M^{bi}, M^{pi}, K^{bi} and K^{pi} (i and j from 1 to 4), as the mass and stiffness matrices of the beam and piezoelectrics. Each of these matrices is as 4 × 4. It is assumed that the first two vibrational modes are intended.

The overall motion equation of the beam, as well as the sensor output equation are written as follows:

$$M\ddot{q} + Kq = f_{ext} + f_{ctrl} = f^t \quad (4)$$

$$y(t) = V^s(t) = P^T\dot{q} \quad (5)$$

where in, $\ddot{q}$, f_{ext}, f_{ctrl}, f^t and P are respectively, the vectors of displacement, acceleration, external force, control force, equivalent force and constant vector of the beam. Using the conversion $q = Tg$, (4) and (5) are modified as follows.

$$MT\ddot{g} + KTg = f_{ext} + f_{ctrl} = f^t \quad (6)$$

$$y(t) = V^s(t) = P^T\dot{q} = P^T T\dot{g} \quad (7)$$

$$T^T MT\ddot{g} + T^T KTg = T^T f_{ext} + T^T f_{ctrl} = T^T f^t \quad (8)$$

$$M^*\ddot{g} + K^* g = f^*_{ext} + f^*_{ctrl} \quad (9)$$

where the matrices M* and K* are the general matrices of mass and stiffness of the equation. Ultimately, the basic equations of the smart structure are as follows:

$$M^*\ddot{g} + C^*\dot{g} + K^* g = f^*_{ext} + f^*_{ctrl} \quad (10)$$

$$y(t) = V^s(t) = P^T\dot{q} = P^T T\dot{g} \quad (11)$$

where the modal damping matrix of the system will be based on using the Riley modal damping method.

IV. State Space Equation of Smart Beam

Considering the g = x, (10) is obtained as a state space:

$$g = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{(2\times1)} = x \quad (12)$$

$$\dot{g} = \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}_{(2\times1)}, \quad \ddot{g} = \ddot{x} = \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}_{(2\times1)} \quad (13)$$

$$\begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = -M^{*-1}K^*\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - M^{*-1}C^*\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + M^{*-1}f^*_{ext} + M^{*-1}f^*_{ctrl} \quad (14)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} o & I \\ -M^{*-1}K^* & -M^{*-1}C^* \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ M^{*-1}T^T h \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ M^{*-1}T^T f \end{bmatrix} r(t) \quad (15)$$

The sensor voltage is considered as the output of the beam. As a result, the system output is obtained as (16):

$$y(t) = P^T T\dot{g} = \begin{bmatrix} p^T & T \end{bmatrix}\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \quad (16)$$

Finally, the one input- one output model of the system's space state is as follows:

$$\dot{x} = Ax(t) + Bu(t) + Er(t) \quad (17)$$

$$y(t) = C^T x(t) + Du(t) \quad (18)$$

where in:

$$A = \begin{bmatrix} 0 & I \\ -M^{*-1}K^* & -M^{*-1}C^* \end{bmatrix}_{(4\times4)}, B = \begin{bmatrix} 0 \\ M^{*-1}T^T h \end{bmatrix}_{(4\times1)} \quad (19)$$

$$C^T = \begin{bmatrix} 0 & p^T T \end{bmatrix}_{(1\times4)}, \ D = 0, \ E = \begin{bmatrix} 0 \\ M^{*-1}T^T f \end{bmatrix}_{(4\times1)} \quad (20)$$

V. Differential Neural Network

In general, the goal of controlling by neural networks is to build a model based on the given system and then, to obtain the appropriate control rule from this model. Usually, neural networks are used to achieve this crucial aim. This is done via a nonlinear algebraic mapping. However, in the present research, dynamic neural networks are used for this purpose. In dynamic neural networks, the neural network model is expressed by a differential equation for the continuous time state or a differential equation for the discrete time mode. These networks are used to estimate the state of the system as an accurate observer so that, using this estimation, the proper feedback can be formed for the closed loop system. For the states that neural networks are used to identify and control the system, the most common used structures are shown in Fig. 4.

The given neural network can be used in different ways in the system. The schematic diagram that is used in this text is shown in Fig. 5.

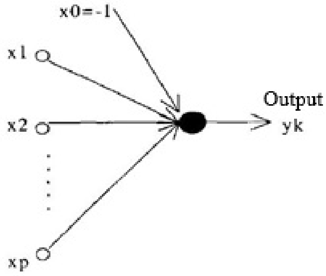

Fig. 4 Simplified structure used for neural networks

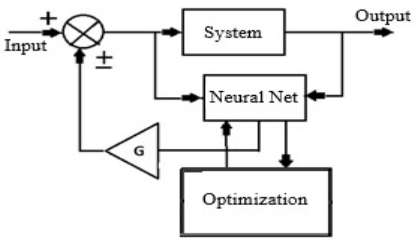

Fig. 5 Schematic of the examined system with the neural network as the observer

The following space state system is considered:

$$\dot{x} = f(x,t) + g(\tilde{x},t)u \tag{21}$$

$$y = h(x,t) \tag{22}$$

where $x \in R^n$ is the system state vector and t is the time. The function f represents the system, function g is the input coefficient of system, and function h is the output formation function of the system, which the matrix equations of these two functions are shown previously. Therefore, it is assumed that these functions are completely known.

For the given system, the following neural network can show a correct approximation of the states and behavior of the system [17]:

$$\dot{\tilde{x}} = A\tilde{x} + W_{1,t}\sigma(\tilde{x}) + W_{2,t}\varphi(\tilde{x})\gamma(u) \tag{23}$$

where $\tilde{x}$ denotes the state of the neural network. In addition, we have: $W_{1,t} \in R^{n\times k}$ as the weight matrix of the feedback state and $W_{2,t} \in R^{n\times r}$ as the input weights matrix. Also, the function σ and φ are selected in an exponential manner and their elements are displayed as follows:

$$\sigma_i(x) = a_i(1 + e^{-b_i x_i})^{-1} - c_i \tag{24}$$

$$\varphi_{ij}(x) = \bar{a}_{ij}(1 + e^{-\sum_p \bar{b}_{ij}{}^p x_p})^{-1} - \bar{c}_{ij} \tag{25}$$

For the given system, if the system dynamics is completely clear, then for the corresponding neural network of it, weights can be found that the neural network below, exactly reflects the system response:

$$\dot{\tilde{x}} = A\tilde{x} + W_1{}^*\sigma(\tilde{x}) + W_2{}^*\varphi(\tilde{x})\gamma(u) \tag{26}$$

As a result, the Riccati inequality has a positive resolution of $P=P^T>0$ as follows.

$$A^T P + PA + PRP + Q < 0 \tag{27}$$

The above mentioned *LMI* matrix equation which finally, leads to finding *P*, on the condition of *Q* to be positive and determined, and the following assumption for *R*, will prove the existence of a positive result for (27):

$$R = \overline{W_1} + \overline{W_2} \tag{28}$$

$$Q = Q_0 + D_\sigma + D_\varphi \bar{u} \tag{29}$$

where in, $\overline{W_1}$ and $\overline{W_2}$ are the upper matrix limits for the weight matrices of the neural network equation. The learning rules of online neural networks are written as follows [17]:

$$\dot{W}_{1,t} = -K_1 P \Delta \sigma(\tilde{x})^T \tag{30}$$

$$\dot{W}_{2,t} = -K_2 P \Delta \gamma(u)^T (\tilde{x})^T \tag{31}$$

The initial weights are given as $W_{1,0}$ and $W_{2,0}$. K_1 and K_2 are two positive determined matrices and, P is obtained from solving the Riccati equation according to the above-mentioned statements. The obtained weights from the aforementioned learning law, lead to the upper limit for the weight matrices at each step and eventually, [17]:

$$\lim_{t\to\infty} \dot{W}_{1,t} = 0 \tag{32}$$

$$\lim_{t\to\infty} \dot{W}_{2,t} = 0 \tag{33}$$

Also, with the obtained weights it can be shown that:

$$\lim_{t\to\infty} \Delta = 0 \tag{34}$$

where in:

$$\Delta = \tilde{x} - x \tag{35}$$

VI. Neural Network Design as an Observer

For the following state space system:

$$\dot{x} = f(x,t) + g(\tilde{x},t)u \tag{36}$$

$$y = h(x,t) \tag{37}$$

where in, $x \in R^n$ the system state vector and t is the time. The function f represents the system, function g is the input coefficient of system, and function h is the output formation function of the system, where the matrix equations of these two functions are shown previously. Therefore, it is assumed that these functions are completely known.

If a system that is modeled based on a neural network is observable, based on the proposed classical techniques for estimating the system states, the following equation, which in general can be nonlinear, is expressed as the state estimator:

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$$\dot{\tilde{x}} = f(\tilde{x}, t) + g(\tilde{x}, t)u + K[y - h(\tilde{x}, t)] \tag{38}$$

It should be noted that in the absence of uncertainty in the system, K is independent of time and constant.

For the neural observers that are used in the given system, the equation of the used neural network system is as follows:

$$\dot{\tilde{x}} = A\tilde{x} + W_{1,t}\sigma(\tilde{x}) + W_{2,t}\varphi(\tilde{x})\gamma(u) + K[y - \tilde{y}] \tag{39}$$

$$\tilde{y} = C_0\tilde{x} \tag{40}$$

where $\tilde{x}$ represents the state of the neural network and K is the gain of the observer.

VII. Results

The obtained results from the simulation of the system using the proposed equations, are presented in this section. In order to compare the results, a linear state observer, which is previously studied, was first applied as the control feedback to the system and the validity of the results was investigated. Then, the obtained results from this model were compared with the proposed neural model in this text and finally, both of the results from the system controlling, were compared with the free vibration of a cantilever beam. The intended constants for the piezoelectric beam and material are shown in Table I.

TABLE I
THE INTENDED CONSTANTS TO PERFORM SIMULATIONS

Constant	Piezoelectric	Beam
$L(m)$	0.05	0.2
$h(m)$	0.001	0.01
$b(m)$	0.001	0.001
$\rho\ (kg/m^3)$	1800	7800
$E(Gpa)$	2	200
$d_{31}\ (c/N)$	2.2×10^{-11}	-
$g_{31}\ (c.m/N)$	0.216	-

The simulated system of the cantilevered beam, will respond to the displacement of the tip of the beam as the initial condition, as in Fig. 6.

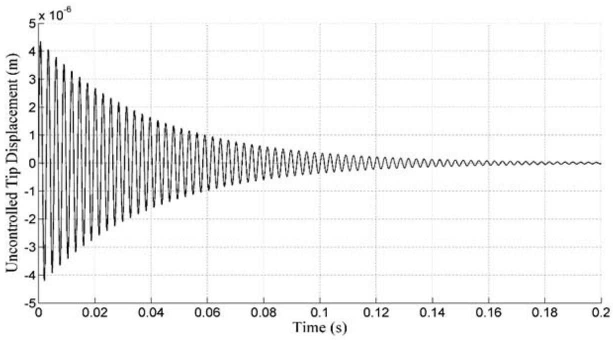

Fig. 6 Free vibrations of a cantilever beam with two piezoelectric layers

It can be seen that the effect of the applied damping to the dynamic equations of the system, after about two tenths of a second, results in zeroing the vibrations of the beam, which is consistent with previous research results. Fig. 7 shows the decrease of these vibrations after controlling the linear state feedback.

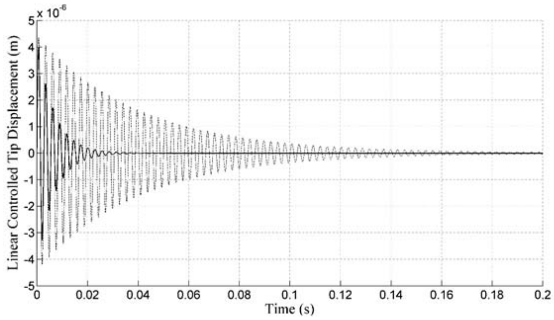

Fig. 7 Vibrations of a cantilever beam with two piezoelectric layers under the linear feedback control

Here, the vibrations of the tip of the given beam are intended and presented in the results. As shown in this figure, the state feedback control by the linear observer will significantly reduce vibrations. Fig. 8 shows the vibrations of the very system under the control of a differential neural network.

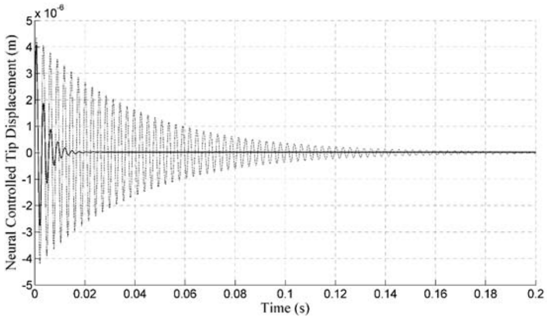

Fig. 8 Vibrations of a cantilever beam with two piezoelectric layers under the neural feedback control

In order to compare the effects of both observers on the vibrations of the beam, the two above graphs are plotted as as in Fig. 9:

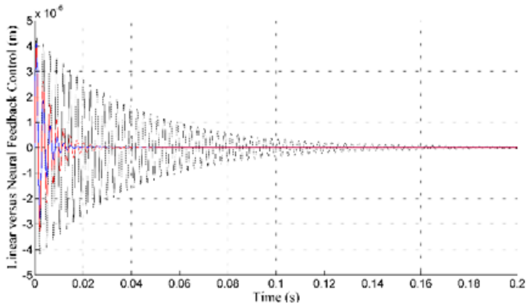

Fig. 9 Comparison of the control function of a cantilever beam with two piezoelectric layers under the linear feedback control and neural control

As it is obvious, the differential neural network is much faster than the linear observer, with eliminating the vibrations of the beam and improving the result by about 50%.

Using the criterion RSSE, errors from all three systems are plotted as in Fig. 10:

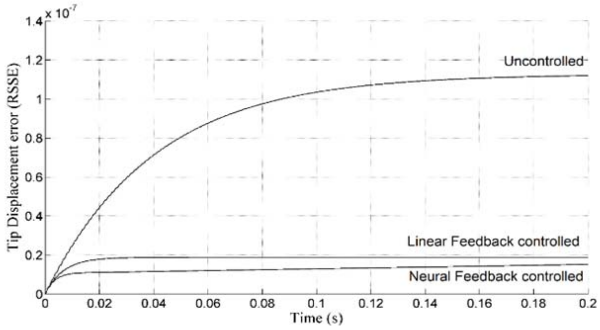

Fig. 10 Vibration Control Error of a cantilever beam with two piezoelectric layers under the linear feedback control and neural control, and comparing it with free vibration of the beam using the RSSE criterion

In Figs. 11 and 12, another criterion for the comparison of these two observers is shown, which is the system output voltage of the piezoelectric actuator.

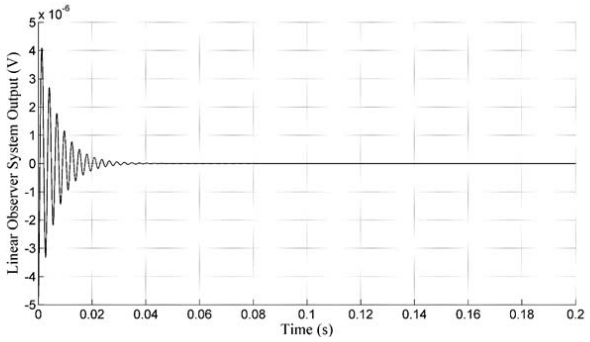

Fig. 11 Voltage output of a cantilever beam with two piezoelectric layers under the linear feedback control

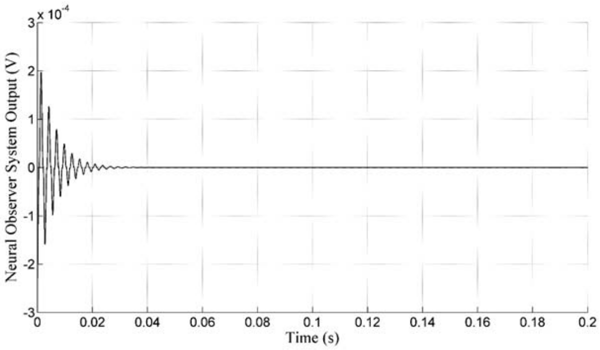

Fig. 12 Voltage output of a cantilever beam with two piezoelectric layers under the neural feedback control

According to the results shown in these two charts, although the neural observer has a higher rate, it uses more control energy to restore the beam to the desired state, which is one of the disadvantages of the neural observer in compare to the linear observer.

VIII. Conclusion

In the present paper, we tried to reduce the vibrations of a piezoelectric cantilever beam with two piezoelectric material layers, using the introduction of a new observer. The presented control logic was based on a differential online neuronal observer and its stability was investigated using solving a matrix inequality based on the Riccati equation. The obtained results from this simulation were compared to a system with a linear observer.

The results show that the convergence rate in the differential neuronal observer is much higher than the linear observer. On the contrary, more control energy is spent for this convergence. In fact, if the piezoelectric material, has the ability to transfer and create this the voltage (which is not a high amount), the use of the differential neural network as an observer would be more reasonable than using the linear observer.

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