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A Super-Efficiency Model for Evaluating Efficiency in the Presence of Time Lag Effect

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Abstract**—**In many cases, there are some time lag between the consumption of inputs and the production of outputs. This time lag effect should be considered in evaluating the performance of organizations. Recently, a couple of DEA models were developed for considering time lag effect in efficiency evaluation of research activities. Multi-periods input(MpI) and Multi-periods output(MpO) models are integrate models to calculate simple efficiency considering time lag effect. However, these models can't discriminate efficient DMUs because of the nature of basic DEA model in which efficiency scores are limited to '1'. That is, efficient DMUs can't be discriminated because their efficiency scores are same. Thus, this paper suggests a super-efficiency model for efficiency evaluation under the consideration of time lag effect based on the MpO model. A case example using a long term research project is given to compare the suggested model with the MpO model.

Keywords**—**DEA, Super-efficiency, Time Lag.

I. Introduction

DEA evaluates the relative performances of comparable units having the authority to make decisions. DEA models use multiple input and output factors of DMUs in efficiency assignments. The efficiency of a DMU is defined as the ratio of weighted outputs to weighted inputs. Since the first Data Envelopment Analysis (DEA) model was developed by Charnes, Cooper and Rhodes (CCR) [1], there are various extensions of the original CCR model [2]-[6]. In special, Banker, Charnes, and Cooper (BCC) extended the CCR model to accommodate technologies that exhibit variable returns to scale [2]. There are many extended models of the basic DEA model. A Generalized DEA model (GDEA) was developed by Yu, Wei and Brockett [7]. Post and Spronk proposed an interactive DEA procedure (IDEA) and introduced a combined DEA model that solves a single linear program to compute the efficiency values of all DMUs [8].

DEA models generally assume that inputs are converted to outputs in the same period. However, this assumption may not be appropriate to some situations such as the performance evaluation of R&D activity and educational activity. Consider academic papers published or patents applied or registered as an output of a research activity. A published journal paper may result from research efforts over several years because the review process of an article for a journal could take some time. That is, production lead time is required to produce the output, such as journal papers or patents, from starting a research project. Furthermore, this production lead time varies according to research areas. In other words, there is a certain length of time lag between the consumption of inputs and the production of outputs. The length of time lag must be dependent on research areas on which are considered to obtain efficiency of research activity.

In this case, general DEA models cannot be applied to obtain efficiency value of DMUs because inputs in a given period may contribute to outputs of several subsequent periods. Özgür Özpeynirci and Murat Kökslan [9] developed the Multi-period Input (MpI) model to capture the time lag between the inputs and outputs in DEA. Zhang and Jeong [10] developed a Multi period output (MpO) model to consider the time lag effect in view of input period. This model was developed under assumption that the inputs of a period contribute the outputs of one or more subsequent periods from the input period.

The results of MpO model classify DMUs into two sets of those that are efficient and constitute a Pareto frontier and of those that are inefficient. This model doesn't give any idea to compare performances of the efficient DMUs. Anderson and Petersen [11] suggested a super-efficiency model based on CCR model to obtain the discrimination among efficient DMUs. The approach enables an extreme efficient unit *k* to achieve an efficiency score greater than one by removing the k^{th} constraint limiting the score under 0-1 in the primal formulation. Extension of MpO model is also available to get super-efficiency considering time lag effect.

The purpose of this paper is to modify the MpO model in order that efficient DMUs can be discriminated. That is, the suggested model allows efficient DMUs to have larger efficiency value than '1'. To do this, this paper suggests a super-efficiency model based on the MpO model. Section II describes the suggested super-efficiency MpO model. A brief case example and discussions are given in Section III.

II. Super-Efficiency Multi-Period Output Model

The MpO model suggested by Zhang and Jeong [10] is to get efficiency values considering time lag effect, based on the assumption that the inputs of a period can contribute to the outputs not only of the same period but also one or more subsequent periods of the period. That is, the MPO model assumes that the inputs of a period contribute partially to the outputs from the input period to PM periods after the input period.

Let h_{jt} be the efficiency of the input used in the t^{th} period by DMU j. Let x_{ijt} denote the amount of the i^{th} input consumed by DMU j in the period t. Let v_{ijt} be the weight of the i^{th} input consumed by DMU j in the period t. Let $Y_{rj(t+p)}$ represent the

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amount of the r^{th} output produced by DMU j in the period t + p. Let u^{p}_{rjt} denote the weight of the r^{th} output produced by DMU j p periods after from the period t. Since there is a time lag of PM periods, the outputs corresponding to the input of the 1^{st} period are from the first period to the $(PM+1)^{th}$ period. The following MPO model finds the efficiency scores for periods [1, T-PM]. When PM = 0, the model MpO is equivalent to the combined DEA model.

[MpO]

$$\max \sum_{t=1}^{T-PM} \sum_{j=1}^{n} h_{jt} = \sum_{p=0}^{PM} \sum_{t=1}^{T-PM} \sum_{j=1}^{n} \sum_{r=1}^{s} u^{p}_{rjt} Y_{rj(t+p)}$$

S.T.

$$\sum_{i=1}^{m} v_{ijt} X_{ijt} = 1, \quad j = 1,\ldots,n;\ t = 1,\ldots,T-PM;$$

$$\sum_{p=0}^{PM} \sum_{r=1}^{s} u^{p}_{rjt} Y_{ij(t+p)} - \sum_{i=1}^{m} v_{ijt} X_{ijt} \le 0, \quad k = 1,\ldots,n;\ j = 1,\ldots,n;$$

$$t = 1,\ldots,T-PM;$$

$$v_{ijt}, u^{p}_{rjt} \ge \varepsilon, \quad \forall p, r, i, j, t;$$

The second constraint set of the above model restricts efficiency scores of all DMU not greater than '1'. Thus, the MpO model doesn't have any means to rank efficient DMUs because all efficient DMUs have same value of '1'. That is, we can't compare the performance of efficient DMUs using the results of MpO model. To resolve this problem, this paper suggests a modified MpO model to get super-efficiency based on the super-efficiency model suggested by Andersen & Peterson [11].

[Model I]

$$\text{Max } h_{j_0 t_0} = \sum_{p=0}^{PM} \sum_{r=1}^{s} u^{p}_{rj_0 t_0} Y_{rj_0 (t_0+P)}$$

S.T.

$$\sum_{i=1}^{m} v_{ij_0 t_0} X_{ij_0 t_0} = 1;$$

$$-\sum_{i=1}^{m} v_{ij_0 t_0} X_{ikt_0} + \sum_{p=0}^{PM} \sum_{r=1}^{s} u^{p}_{rj_0 t_0} Y_{rk(t_0+P)} \le 0, k = 1,\ldots,n \;\&\; k \ne j_0;$$

$$u^{p}_{rj_0 t_0}, v_{ij_0 t_0} \ge 0, \forall p, r, i;$$

The model I is a model to calculate super-efficiency of the inputs consumed at period t_0 by DMU j_0. The super-efficiency is obtained by maximizing the weighted outputs for periods (t_0-t_0+PM) to reflect time lag effect. The constraint for the DMU, of which we are to get super-efficiency, is removed from the second constraint set. Thus, some efficient DMUs can have efficiency value larger that '1'. That is, the Model I is a super-efficiency model for calculating super-efficiency for the input of DMU at a period under consideration of time lag effect.

However, the Model I is a model for a single period and a single DMU. That is, the model has to be implemented n(T-PM) times to get super-efficiency for all DMUs and entire periods. To get super-efficiency values of all DMUs for entire periods, an integrated super-efficiency MpO model can be established as follows.

[Model II]

$$\text{Max} \sum_{t=1}^{T-PM} \sum_{j=1}^{n} h_{jt} = \sum_{t=1}^{T-PM} \sum_{j=1}^{n} \sum_{p=0}^{PM} \sum_{r=1}^{s} u^{p}_{rjt} Y_{rj(t+P)}$$

S.T.

$$\sum_{i=1}^{m} v_{ijt} X_{ijt} = 1, j = 1,\ldots,n; t = 1,\ldots,T-PM;$$

$$-\sum_{i=1}^{m} v_{ijt} X_{ikt} + \sum_{p=0}^{PM} \sum_{r=1}^{s} u^{p}_{rjt} Y_{rk(t+P)} \le 0, j = 1,\ldots,n; k = 1,\ldots,n \;\&\; k \ne j;$$

$$t = 1,\ldots,T-PM;$$

$$u^{p}_{rjt}, v_{ijt} \ge 0, \forall p, r, i, j, t;$$

The Model II is an integrated model to calculate super-efficiency scores of all DMUs for entire periods at once. The Model II can be decomposed into Model I.

III. Case Example

TABLE I
DATA OF A PROJECT IN RESEARCH CENTER SG09

yr	Fund	researcher			papers		Patent	
	(m ₩)	Ph.D	MS	BS	SCI	Non SCI	appl	Reg.
1	300	0	10	13	0	0	2	0
2	300	0	9	12	0	1	3	2
3	300	4	12	6	3	3	4	0
4	150	5	5	2	3	0	2	0
5	180	5	5	2	2	2	5	3
6	180	5	5	2	8	7	8	0
7	180	3	0	6	1	0	6	4
8	180	3	0	3	5	0	3	6
9	180	0	3	5	2	0	9	2
10	180	0	2	4	9	0	5	2

The 21st century frontier R&D program is a long-term national R&D program of Korea, for selective and intensive development of strategic technologies to enhance national scientific competitiveness to the level of advanced countries. Currently, 24 research centers are being supported by this program since two project centers were launched in 1999. The number of subprojects supported by these 24 centers is totally 2,670 subprojects. However, the long term projects are required to apply the suggested super-MpO model considering time lag effect. The input and output data set of a center (SG09) among these 24 centers is used to show the implementation of the suggested model. The center SG09 has 18 subprojects which are supported for 10 years. For instance, Table I shows a sample data set for a subproject supported by the research center SG09. There are four input variables, research expenditure (million Korean Won), the number of researchers (Ph.D, MS, BS). The number of published papers (SCI journal and non SCI journal) and the number of applied and registered patents are considered as outputs of research activity. The data set of the center SG09 is used to compare the MpO model and the suggested super-MpO model. We used LINGO to run these models.

Tables II and III show the results of the MpO model and the suggested model using the data set. The inefficient DMUs get same efficiency scores in two models. Table III shows that efficient DMUs can have efficiency score greater than “1”. However, unbounded problems were obtained in some cases, for example, DMU A for periods 1, 2, 7. This problem comes from that some output variables have zero value. Thus, this problem should be resolved in the future research.

TABLE II
EFFICIENCY SCORES OF 18 PROJECTS IN CENTER SG09 FOR EACH PERIOD

project	1	2	3	4	5	6	7	8
A	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
B	0.0000	0.0000	0.3205	1.0000	1.0000	1.0000	1.0000	1.0000
C	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.6723
D	1.0000	1.0000	1.0000	0.8887	1.0000	1.0000	1.0000	1.0000
E	1.0000	1.0000	1.0000	0.5515	0.6380	1.0000	1.0000	0.8062
F	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
G	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
H	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7997
I	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000
J	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
K	0.5125	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
L	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9213	0.8181
M	1.0000	1.0000	1.0000	0.7368	0.7368	0.8363	0.9458	0.9458
N	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.7545	1.0000
O	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
P	1.0000	1.0000	0.9441	0.7221	0.6007	1.0000	0.5280	0.2877
Q	0.8338	1.0000	1.0000	1.0000	1.0000	0.5576	0.7336	0.7029
R	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE III
SUPER-EFFICIENCY SCORES OF 17 PROJECTS IN CENTER SG09 FOR EACH PERIOD

project	1	2	3	4	5	6	7	8
A	Un	Un	2.0359	3.0453	3.0147	2.3333	Un	17.9987
B	0.0000	0.0000	0.3205	2.3333	2.3333	6.3998	1.9999	1.1538
C	2.1662	6.1233	5.9964	2.2049	2.1442	5.0367	3.1256	0.6723
D	1.9997	2.9995	3.1426	0.8887	2.8437	2.8202	3.4165	3.2295
E	1.4543	1.5218	1.5572	0.5515	0.6380	2.0896	1.0344	0.8062
F	Un	2.1859	2.4531	1.2222	1.3846	1.0100	1.3734	1.7624
G	7.0826	7.0642	1.3998	1.7999	2.0415	2.8418	1.3984	1.5415
H	3.7968	4.0221	4.7406	3.3109	3.3376	2.1487	1.4614	0.7997
I	8.4261	18.9716	15.9993	23.9989	5.9237	5.0472	Un	Un
J	0.5125	5.0943	4.4443	5.0000	1.9893	2.2583	1.3332	Un
K	Un	Un	Un	1.2171	1.1103	1.0085	0.9213	0.8181
L	3.8081	3.8579	1.5468	0.7368	0.7368	0.8363	0.9458	0.9458
M	1.0054	1.1537	1.3838	1.6661	1.9634	2.1913	0.7545	1.1999
N	1.5511	7.2794	10.4958	15.6247	3.9999	1.7391	Un	1.3125
O	2.2974	1.6640	0.9441	0.7316	0.6007	1.3468	0.5280	0.2877
P	0.8338	1.2725	2.8016	1.0481	1.1533	0.5576	0.7336	0.7029
Q	3.6677	4.6130	3.7867	Un	Un	Un	1.2158	1.2869

IV. CONCLUSION

This paper modified the MpO model to improve the discrimination power of efficient DMUs under consideration of time lag effect. The suggested model was compared with the MpO model using the data set of long term projects. However, the proposed model gives unbounded solution when the DMU to be evaluated has some zero value in output factors. Thus, further research should be focused to resolve this problem.

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REFERENCES

[1] Charnes A, Cooper WW, Rhodes E (1978) Measuring the efficiency of decision-making units. Eur J Oper Res 2:429–444

[2] Banker, R.D., Chames, A., and Cooper, W.W. (1984) "Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis", Management Science 30/9, 1078-1092.
[3] Charnes, A., and Cooper, W.W. (1985), "Preface to topics in Data Envelopment Analysis", Annals of Operations Research 2, 59-94.
[4] Chames, A., Cooper, W.W., Golany, B., Seiford, L., and Stutz, J. (1985), "Foundations of Data Envelopment Analysis for Pareto-Koopmans efficient empirical production functions", Journal of Econometrics 30, 91-107.
[5] Chames, A., Cooper, W.W., Wei, Q.L., and Huang, Z.M. (1990), "Fundamental theorems of non-dominated solutions associated with cones in normed linear space", Journal of Mathematical Analysis and Applications 150/1, 54-78.
[6] Seiford, L.M., and Thrall, R.M. (1990), "Recent developments in DEA, the mathematical programming approach to Frontier Analysis", Journal of Econometrics 46, 7-38.
[7] Yu, G., Wei, Q.L., and Brockett, P. (1996), "A Generalized Data Envelopment Analysis model: A unification and extension of existing methods for efficiency analysis of decision making units", Annals of Operations Research 66.
[8] Post T, Spronk J (1999) Performance benchmarking using interactive data envelopment analysis. Eur J Oper Res 115:472–487
[9] O¨zgu¨r O¨zpeynirci and Murat Ko¨kslan(2007) Performance evaluation using data envelopment analysis in the presence of time lags. J Prod Anal 27:221–229
[10] Zhang, Y. and Jeong, B.H. (2012) "A DEA Model for Performance Evaluation in The presence of Time Lag Effect", World Academy of Science , Engineering and Technology 69, 611-615
[11] Anderson, P. and Petersen, N.C.(1993) "A Procedure for Ranking Efficient Units in Data Envelopment analysis," *Mgmt. Sci*., 39, 1261-1264

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