

# A Statistical Model for the Dynamics of Single Cathode Spot in Vacuum Cylindrical Cathode

Po-Wen Chen, Jin-Yu Wu, Md. Manirul Ali, Yang Peng, Chen-Te Chang, Der-Jun Jan

**Abstract**—Dynamics of cathode spot has become a major part of vacuum arc discharge with its high academic interest and wide application potential. In this article, using a three-dimensional statistical model, we simulate the distribution of the ignition probability of a new cathode spot occurring in different magnetic pressure on old cathode spot surface and at different arcing time. This model for the ignition probability of a new cathode spot was proposed in two typical situations, one by the pure isotropic random walk in the absence of an external magnetic field, other by the retrograde motion in external magnetic field, in parallel with the cathode surface. We mainly focus on developed relationship between the ignition probability density distribution of a new cathode spot and the external magnetic field.

**Keywords**—Cathode spot, vacuum arc discharge, transverse magnetic field, random walk.

## I. INTRODUCTION

VACUUM arc plasma is widely used in many applications, such as vacuum interrupters, vacuum arc coating thin films, ion implantation, switching devices, and accelerators [1], [2]. The arc discharge usually originates from cathode spots, which is confined to a very small area, existing under extreme physical conditions (current density  $\sim 10^{11}$  A/m<sup>2</sup>, power density  $\sim 10^{16}$  W/m<sup>2</sup>) in the near cathode region [1], [2]. These applications are used by expanding plasma jet generated from cathode spots. The dynamics of current per spot is influenced by various factors, e.g., arc current strength, inter-electrode pressure, contact material, contact gap, and design of the external magnetic field [1], [2]. The motion of cathode spot (CS), especially in an external magnetic field, plays an important role in various near-electrode phenomena and influence on the arc performance in applications. An important issue is developed with relationship between mechanism of cathode spot and an external transverse magnetic field. Without the external magnetic field, the motion of CS presents a chaotic movement over the cathode surface, which is essential characteristic with isotropic random walk [3]. In an external transverse magnetic field (TMF), CS motion is directed by the opposite the Amperian motion, e.g., in the  $-j \times B$  direction [4].

The characteristics of the random walk of a CS had been studied for various cathode materials, J. E. Daalder

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deduced a one-dimensional (1-D) Rayleigh function from a two-dimensional (2-D) probability density function, which was verified by experiments [3]. In [5], a 2-D statistical model for a single CS, based on the Brownian motion model, was established to simulate the random walk and retrograde motion. In that paper, they used some physical parameter  $k$  which is closely related to the strength of  $B_{em}$  to characterize different probabilities of CS motion. In [6], they have investigated the relationship between the parameter  $B_{em}$  and the ignition probability of new cathode spot under magnetic pressure.

In this article, a three-dimensional statistical model has been established to simulate the distribution of the ignition probability of a new CS in cylindrical cathode occurring at different arcing time. We investigate the transient dynamics of single CS of vacuum arc for two typical situations, one by the pure isotropic random walk in the absence of external magnetic field, and the other by the retrograde motion in presence of an external TMF applied in parallel with the cathode surface. We mainly focus on the influence of TMF on the ignition probability density distribution of a new cathode spot in a vacuum cylindrical cathode.

## II. THE THREE-DIMENSIONAL STATISTICAL MODEL FOR DYNAMICS OF CATHODE SPOT IN THE CYLINDRICAL CATHODE

The schematic of this model is shown in Fig. 1, a single CS is assumed to be at the coordinate origin  $(x_0, y_0, z_0)$  at initial arcing time  $t = 0$ . The basic idea of this model is to simulate the dynamics of a single cathode spot in an external magnetic field applied along  $\phi$ -axis. Therefore, the retrograde motion should be in the negative  $z$  direction. We can simulate the 3D motion of a single CS by three 1D movements along the  $x$ ,  $y$ , and  $z$ -axis, respectively. By assuming the CS displacement to be stochastically independent along different directions, we can write down the 3D probability density function  $f(x, y, z)$  as

$$f(x, y, z) = f(x) f(y) f(z). \quad (1)$$

### A. Trajectory of a Single CS

In this model, the motion of CS is treated as a sequence of step-by-step jump along 3D space, with a characteristic spatial step  $s'$  and temporal step  $\tau$ . In Fig. 1, the CS in step  $s$ , is located at the position coordinate  $(x_{s-1}, y_{s-1})$  after  $s-1$  time steps  $[t = (s-1)\tau]$ , and moves to the coordinate  $(x_s, y_s)$  after

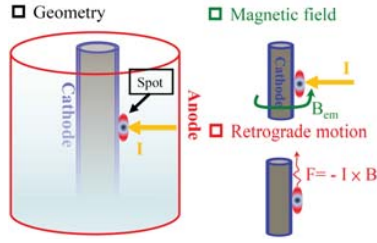


Fig. 1 Schematic diagram of the physical model

time step of  $\tau$ . Accordingly, we can describe the coordinates through the following relationship

$$\begin{cases} x_s = x_{s-1} + ms' \\ y_s = y_{s-1} + ns' \\ z_s = z_{s-1} + ts' \end{cases}, \quad (2)$$

where the values of the parameter  $m$  is equal to  $+1$  or  $-1$  with its probability  $p_x$  and  $p_{-x}$  in positive  $x$ -axis and negative  $x$ -axis. The parameter  $n$  is equal to  $+1$  or  $-1$  with its probability  $p_y$  and  $p_{-y}$  in positive  $y$ -axis and negative  $y$ -axis, and the parameters  $t$  is equal to  $1$  or  $-1$  with its probability of  $p_z$  and  $p_{-z}$  in positive  $z$ -axis and negative  $z$ -axis, respectively. These probabilities satisfy the relations

$$p_x + p_{-x} = 1, \quad (3)$$

$$p_y + p_{-y} = 1, \quad (4)$$

$$p_z + p_{-z} = 1, \quad (5)$$

respectively. In this paper, we assume that the CS moving along  $x$ - and  $y$ -axis are isotropic random walk, which means  $p_x = p_{-x} = 1/2$  and  $p_y = p_{-y} = 1/2$ . After  $s$  steps ( $t = s\tau$ ), the probability densities,  $f(x)$  and  $f(y)$  of the CS displacement along the  $x$ - and  $y$ -axis at time  $t$  can be described as

$$f(x) = \frac{1}{\sqrt{2\pi\alpha t}} \exp\left(-\frac{x^2}{2\alpha t}\right), \quad (6)$$

$$f(y) = \frac{1}{\sqrt{2\pi\alpha t}} \exp\left(-\frac{y^2}{2\alpha t}\right), \quad (7)$$

where the diffusion parameter  $\alpha = \frac{s'^2}{\tau}$ , which can be calculated through the Joule heating model [3], where the motion of CS is treated as a sequence of step-by-step jump by a characteristic step length  $s'$  and time interval  $\tau$ . Physically, the parameters  $s'$  and  $\tau$  are associated to the CS radius for continuous CS motion on cathode surface and the life cycle of the CS. In an external TMF, the CS shows retrograde motion over the cathode surface, where the probability  $p_{-z}$  to jump in the negative  $z$ -direction will be higher than the probability  $p_z$  in the positive  $z$ -axis. In this model, we describe the difference between the probabilities  $p_z$  and  $p_{-z}$  through a detuning parameter  $\Delta$  as follows

$$\begin{aligned} p_z &= 1/2 + \Delta, \\ p_{-z} &= 1/2 - \Delta. \end{aligned} \quad (8)$$

In [6], the retrograde motion theory of ignition probability of a new CS under magnetic pressure on old cathode spot

surface has been described in detail [4]. The relationship between external TMF, self-magnetic fields  $B_{sm}$  and the dimensionless parameter  $\Delta$  is denoted as

$$\Delta = \frac{B_{em}B_{sm}}{B_{em}^2 + B_{sm}^2}. \quad (9)$$

Here, the self-magnetic field is calculated as  $B_{sm} = \frac{\mu_0 I_{CS}}{2\pi d}$ ,  $I_{CS}$  is the spot current, and  $d$  describes a distance from the center of the cathode spot. Moreover, the motion of the CS along the  $z$ -axis is assumed to be a pure random walk with  $\Delta = 0$ , i.e.,  $p_z = p_{-z} = 0.5$ . With the above configuration, the probability density function  $f(z)$  of CS displacement along the  $z$ -axis at time  $t$  can be obtained as:

$$f(z) = \frac{1}{\sqrt{2\pi\alpha t(1-4\Delta^2)}} \exp\left(-\frac{\left[z - 2\frac{s'}{\tau}\Delta t\right]^2}{2\alpha t(1-4\Delta^2)}\right). \quad (10)$$

According to (1), the 3-D probability density function of the CS displacements at time  $t$  can be transformed into cylindrical coordinate as:

$$\begin{aligned} f(r, z) &= \left( \frac{1}{(2\pi\alpha t)^{3/2} (1-4\Delta^2)^{1/2}} \exp\left(-\frac{r^2}{2\alpha t}\right) \right. \\ &\quad \left. \exp\left(-\frac{\left[z - 2\frac{s'}{\tau}\Delta t\right]^2 / (1-4\Delta^2)}{2\alpha t}\right) \right). \end{aligned} \quad (11)$$

### III. SIMULATION RESULTS AND DISCUSSION

Using the above results, we investigate numerically the ignition probability density distribution of a new cathode spot in the cylindrical cathode surface. We propose to study the ignition probability of a new cathode spot through the three important parameters  $s'$ ,  $\tau$ , and  $\Delta$ . We also investigate how to develop of the ignition probability of a CS related with the strength of applied TMF. The physical parameter  $\tau$  depends on the lifetime of CS and  $s'$  depends on distance between two cathode spots generated continuously. Here we consider  $s' = 6.6 \mu m$ , and  $\tau = 0.067 \mu s$ , and  $d = r_{cs} \cong \sqrt{3}s'$  as considered in previous [3], [7]. We mainly focus on investigating probability density distribution function of cathodic spot under a cylindrical rod cathode. We simulate three different cases, i.e., the pure isotropic random walk when  $B_{em} = 0$  T, the retrograde motion for weak TMF,  $B_{em} = 0.1$  T, and the retrograde motion in strong TMF,  $B_{em} = 1$  T. In this section, we simulate our results for different values of  $\Delta$  with given values of  $B_{em}$ . The distribution of the ignition probability of a CS in vacuum cylindrical cathode is demonstrated at different arcing time.

#### A. $B_{em} = 0$ T

Fig. 1 shows the ignition probability density distribution of a new CS in vacuum cylindrical cathode with  $B_{em} = 0$ , i.e.,  $\Delta = 0$ . This corresponds to the motion of CS with isotropic random walk at different arcing time. We can see

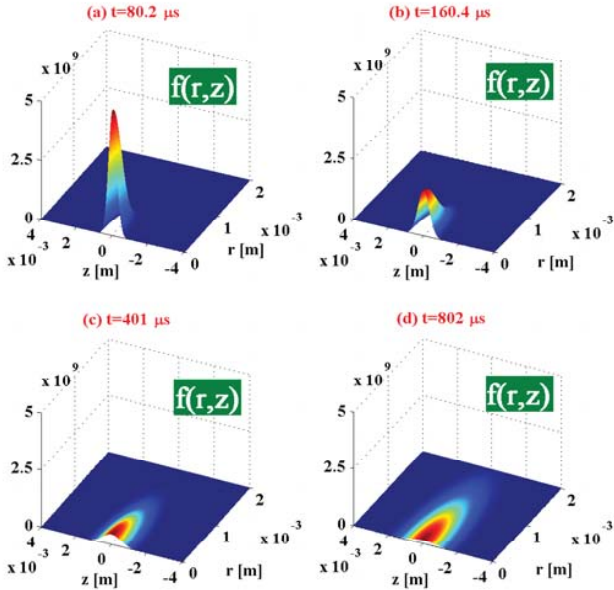


Fig. 2 The ignition probability density distribution of a CS without applying TMF is shown as a function of  $r$  and  $z$ . We draw 3D plot of  $f(r, z)$  for four different arcing time (a)  $80.2 \mu s$ , (b)  $160.4 \mu s$ , (c)  $401 \mu s$  and (d)  $802 \mu s$ . The other parameters are taken as  $I_{cs} = 100 \text{ A}$ ,  $s' = 6.6 \mu m$ , and  $\tau = 0.067 \mu s$

the ignition probability density distribution function,  $f(r, z)$ , in the isotropic random walk case that has the maximum value at  $r = 0$  and  $z = 0$ . With the increase of arcing time  $f(r, z)$  was broadened to large  $r$  and  $z$  position with decreasing the maximum value. We observe that  $f(r, z)$  become more and more broadened with increasing arcing time.

B.  $B_{em} = 0.1 T$

Fig. 2 shows the ignition probability density distribution of a new CS in vacuum cylindrical cathode with  $B_{em} = 0.1 T$ . This corresponds to the motion of CS in a relatively weak TMF. The distribution of  $f(r, z)$  at different arcing time was shown in Fig. 2 with  $r$  and  $z$  as parameters. We can see the ignition probability density distribution function has the maximum value at  $r = 0$ , and the distribution function is broadened to larger radial position coordinate as we increase the arcing time. In Fig. 2, we show that  $f(r, z)$  is no longer symmetrical about  $r$  direction and appear in retrograde motion in  $z$  direction with increasing arcing time  $t$ .

C.  $B_{em} = 1 T$

Fig. 3 shows the ignition probability density distribution of a new CS in vacuum cylindrical cathode with  $B_{em} = 1 T$ . This corresponds to the motion of CS in a relatively stronger TMF. With the increasing of  $B_{em}$ , the tendency of retrograde motion was enhanced. We can see that the ignition probability density distribution function has the maximum value at  $r = 0$ . A larger broadening of the distribution function  $f(r, z)$  is observed for stronger TMF compared to weak TMF case.

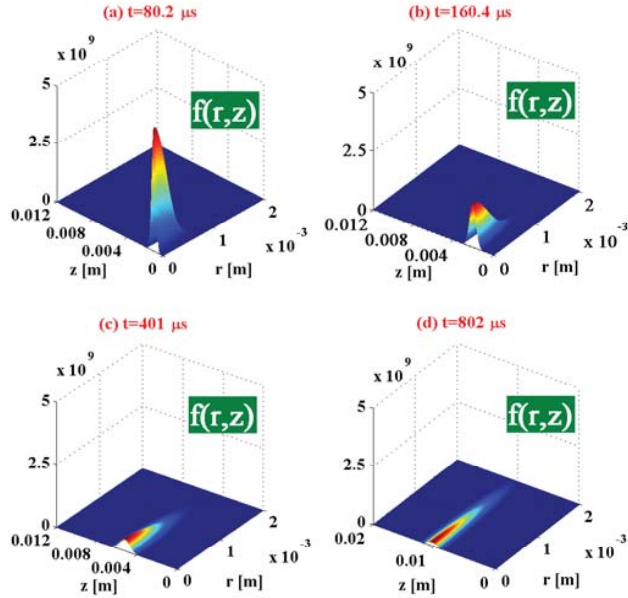


Fig. 3 The ignition probability density distribution of a new CS in a relatively weak TMF case is shown as a function of  $r$  and  $z$ . We draw 3D plot of  $f(r, z)$  for four different arcing time (a)  $80.2 \mu s$ , (b)  $160.4 \mu s$ , (c)  $401 \mu s$  and (d)  $802 \mu s$ . The other parameters are taken as  $I_{cs} = 100 \text{ A}$ ,  $s' = 6.6 \mu m$ , and  $\tau = 0.067 \mu s$

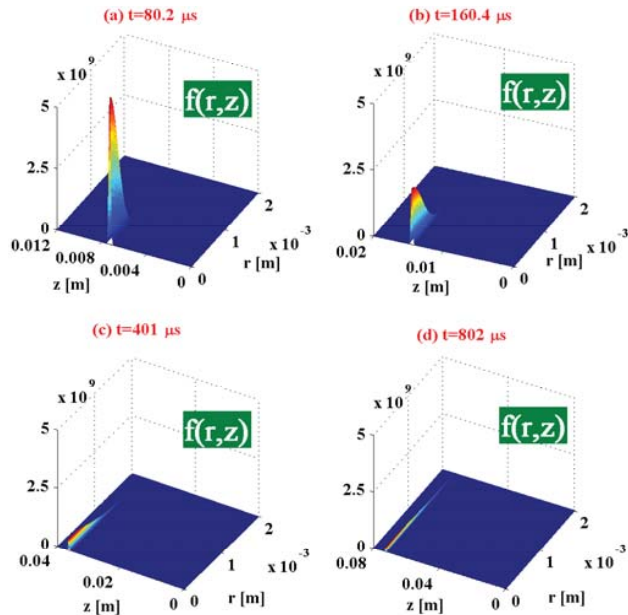


Fig. 4 The ignition probability density distribution of a new CS in a relatively stronger TMF case is shown as a function of  $r$  and  $z$ . We draw 3D plot of  $f(r, z)$  for four different arcing time (a)  $80.2 \mu s$ , (b)  $160.4 \mu s$ , (c)  $401 \mu s$  and (d)  $802 \mu s$ . The other parameters are taken as  $I_{cs} = 100 \text{ A}$ ,  $s' = 6.6 \mu m$ , and  $\tau = 0.067 \mu s$

## IV. CONCLUSION

We have investigated the distribution of the ignition probability of a new CS in vacuum cylindrical cathode occurring at different arcing time. We consider three different cases related to the strength of the applied TMF, for example, (i) in the absence of an TMF, i.e., the pure isotropic random walk,  $B_{em} = 0$  T, (ii) the retrograde motion in weak TMF,  $B_{em} = 0.1$  T, and (iii) the retrograde motion in strong TMF,  $B_{em} = 1$  T. It was shown that the dynamics of cathode spot in vacuum cylindrical cathode can be controlled with a suitable choice of TMF strength. We find that the tendency of retrograde motion in  $z$ -direction is enhanced with the increasing of  $B_{em}$ .

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