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A statistical approach for predicting and optimizing depth of cut in AWJ machining for 6063-T6 Al alloy

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***Abstract*—** In this paper, a set of experimental data has been used to assess the influence of abrasive water jet (AWJ) process parameters in cutting 6063-T6 aluminum alloy. The process variables considered here include nozzle diameter, jet traverse rate, jet pressure and abrasive flow rate. The effects of these input parameters are studied on depth of cut (h); one of most important characteristics of AWJ. The Taguchi method and regression modeling are used in order to establish the relationships between input and output parameters. The adequacy of the model is evaluated using analysis of variance (ANOVA) technique. In the next stage, the proposed model is embedded into a Simulated Annealing (SA) algorithm to optimize the AWJ process parameters. The objective is to determine a suitable set of process parameters that can produce a desired depth of cut, considering the ranges of the process parameters. Computational results prove the effectiveness of the proposed model and optimization procedure.

***Keywords*—** AWJ machining, Mathematical modeling, Simulated Annealing, Optimization

I. INTRODUCTION

AMONG different cutting techniques, AWJ is one of the most important techniques used in wide variety of industrial application. This technique relies on erosive action of abrasive laden water jet for applications of cutting, drilling, cleaning, and decaling of thick sections of very soft to very hard materials. In this method, a stream of small abrasive particles is introduced in the water jet in such a manner that water jet's momentum is partly transferred to the abrasive particles. The main role of carrier fluid (water) is primarily to accelerate large quantities of abrasive particles to a high velocity and to produce a highly coherent jet. This jet is then directed towards working area to perform cutting [1].

There are several distinguished advantages of AWJ technique. It is less sensitive to material properties and hence does not cause chatter, has no thermal effects, imposes minimal stresses on the workpiece, and has high machining versatility and flexibility. However, AWJ has some limitations and drawbacks. It may generate loud noise and a messy working environment. It may also create tapered edges on the kerf, especially when cutting at high traverse rates [2, 3].

As in the case of every machining process, the quality of process in AWJ is significantly affected by the process tuning parameters. There are several process parameters in this technique, among which water pressure, abrasive flow rate, jet traverse rate and diameter of focusing nozzle are of great importance and precisely controllable[4, 5]. The main process quality measures include attainable depth of cut, kerf width and its regularity and surface finish. Therefore, it is of great importance to study the effects of the process parameters on the process response characteristics. In this study, the depth of cut is considered as the performance measure as in many industrial application it is the main constraint on the process applicability.

A few attempts have been made to model and optimize the process parameters in AWJ. The approaches employed in this direction include design of experiments (DOE), regression modeling, analysis of variance (ANOVA), fuzzy logics and artificial neural networks. Some of these studies gave rise to various mathematical equations developed for predicting the output parameters [6]. Hashish [7] was among the first who developed a set of mathematical model to relate the process parameters settings to the process output variables in water jet technique. Later Ramulu and Arola [8] used regression analysis to predict depth of cut due to cutting and deformation wear for graphite/epoxy composite materials.

In recent years, determining an optimal set of process parameters values to achieve a certain output characteristics has been the prime interest by many researchers. Chakravarthy and Babu [9] employed a fuzzy-genetic approach for selection of process parameters in maintaining the desired depth of cut with a fixed size of orifice and focusing nozzle. More recently, the same approach has been employed by Srinivasu and Babu [10] to model and optimize the varying conditions of focusing nozzle in AWJ. Their study aims at selecting suitable process parameters that can control the depth of cut within the desired limits; taking into account varying nozzle diameter due to wear.

Although there are few studies in modeling and optimization of process parameters in AWJ, most of them are limited to the particular circumstances and are computationally complex. The present study attempts to make use of available experimental data to relate important process parameters to process output variables, through developing

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empirical regression models for various target parameters. In the next stage, the proposed model is implanted into a simulated annealing (SA) optimization procedure to identify a proper set of process parameters that can produce the desired depth of cut.

II. Modeling Development

The important controlling process parameters in AWJ cutting include water pressure (P), jet traverse rate (V), abrasive flow rate (M_f) and diameter of focusing nozzle (d_f). In this study, depth of cut has been chosen as the main process response characteristics to investigate the influence of the above parameters. We first develop a mathematical model to relate the process control parameters to the process response characteristics. The empirical model for the prediction of depth of cut in terms of the controlling parameters will be established by means of piecewise linear regression analysis. The experimental results were obtained using design of experiment (DOE) technique. For illustrative purposes, the data presented by Srinivasu et al. [13] is used here. Table I shows some of the experiment settings obtained by Taguchi DOE matrix.

TABLE I
DOE Matrix and results for the AWJ

No.	d_f (mm)	P (MPa)	M_f (kg/min)	V (mm/min)	h (mm)
1	0.8	206	0.035	125	11.37
2	1.0	168	0.056	119	10.32
3	1.1	165	0.059	110	10.35
4	1.2	165	0.071	123	10.00
5	1.3	177	0.065	114	10.45
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
30	1.0	234	0.081	38	47.38
31	1.1	232	0.084	37	48.05
32	1.2	237	0.085	37	48.94
33	1.3	232	0.091	38	48.61
34	1.4	232	0.094	37	47.78
35	1.5	239	0.095	37	46.59

As shown a total of 35 experiments were performed to gather the required data. In this table, the first four columns show the process parameters settings given by Taguchi DOE matrix. The last column (h) is the measured process output resulted from different experiments.

The general form of a regression mathematical model is as follows:

$$Y= a_0 + a_1.P + a_2.V + a_3.M_f + a_4.d_f + a_{11}.P^2 + a_{22}.V^2 + a_{33}.M_f^2 + a_{44}.d_f^2 + a_{12}.P.V + a_{13}.P.M_f + a_{14}.P.d_f + a_{23}.V.M_f + a_{24}.V.d_f + a_{34}.M_f.d_f \quad (3)$$

Different regression functions (linear, curvilinear, logarithmic, etc.) are fitted to the above data and the coefficients values (a_i) are calculated using regression analysis. The best model is the most fitted function to the experimental data. Such a model can accurately represent the actual AWJ process. Therefore, in this research, the adequacies of various functions have been evaluated using analysis of variance (ANOVA) technique.

The model adequacy checking includes test for significance of the regression model and test for significance on model coefficients [11]. ANOVA results recommend that the quadratic model is statistically the best fit in this case. Statistical analysis show that the associated P-value for the model is lower than 0.05; i.e. α=0.05, or 95% confidence. This illustrates that the model is statistically significant. Based on ANOVA, the values of R^2 and adjusted R^2 are over 99% for h. This means that regression model provides an excellent explanation of the relationship between the independent variables and h response.

Table II shows the values of "T-value"' and "P-Value" for each term on the performances of h. In the case of depth of cut (h) the d_f , d_f^2 and P.V can be regarded as significant term due to their "P-value" being less than 0.05. The backward elimination process removes the rest of insignificant terms to adjust the fitted quadratic model. The final proposed curvilinear model is presented below:

$$h = -61.1 + 73.7\, d_f + 0.365\, P - 29.3\, d_f^2 + 3031\, M_f^2 + 0.00173\, V^2 - 0.0781\, d_f . P - 0.00190\, P. V - 2.61\, M_f. V \quad (4)$$

For illustrative purposes, the distributions of real data around regression lines for these models are illustrated in Fig. 1. This figure demonstrates a good conformability of the developed models to the real process and hence is used to represent the actual process.

TABLE II
DOE Matrix and results for the AWJ

symbol	Degree of freedom	h	
		T-valu	P-valu
P	1	0.80	0.434
V	1	-0.01	0.992
M_f	1	-0.73	0.476
d_f	1	3.78	0.001*
P*P	1	-0.11	0.910
V*V	1	2.10	0.048
M_f*M_f	1	1.95	0.065
d_f*d_f	1	-4.89	0.000*
P*V	1	-3.49	0.002*
P*M_f	1	1.03	0.315
P*d_f	1	-1.62	0.120
V*M_f	1	-0.97	0.345
V*d_f	1	0.03	0.978
M_f*d_f	1	0.65	0.522
Residual	20		
Total	34		

*significant

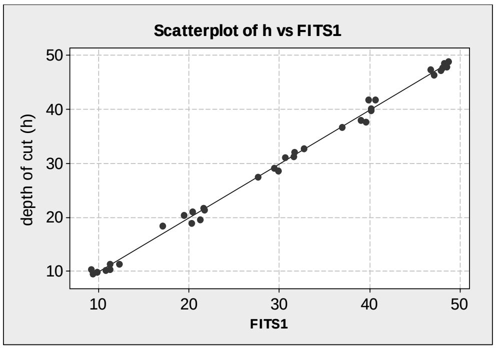

Fig. 1 Predicted *h* versus actual values

III. The Optimization Procedure

The mathematical models furnished above provide one to one relationships between process parameters and AWJ cutting response characteristic, depth of cut. They can be used in two ways:

1) Predicting AWJ cutting response characteristic (h) for any given set of input parameters.

2) Determining a set of process parameters values for a desired AWJ characteristic specification (h).

In many practical situations, one needs to set the process parameters in such a way that a desired output is obtained (in this case depth of cut, h).Since finding the optimal set of input parameters for a given h is the problem of combination explosion, evolutionary algorithms can be employed as the optimizing procedure. These techniques would make the combination converge to solutions that are globally optimal or nearly so. Evolutionary algorithms are powerful optimization techniques widely used for solving combinatorial problems. As a new and promising approach, one of these algorithms, called SA, is implemented for optimization purposes in this research.

Simulated Annealing (SA) is one of the novel algorithms initially proposed by Kirkpatrick [12]. SA is an approach to simulate the thermodynamic process of annealing (cooling a molten metal slowly to the solid state). It is a powerful optimization technique, which can theoretically converge asymptotically to the global optimum solution with probability '1' when the initial temperature is high enough and the cooling rate is infinitely low. The process of this evolutionary operation adopts the new values based on Boltzmann rule: when an optimized value is adopted, a non-improving value is also adopted with a certain probability. This rule is expressed as the following form:

$$e^{-\Delta E/T_0} \geq \text{ran. } (0, 1) \qquad (5)$$

For optimization process, we first define the prediction function as follow:

$$\text{Error} = \frac{\text{Target }(h) - \text{Predict }(h_d)}{\text{Predict }(h_d)} \times 100 \qquad (6)$$

This function is used as the fitness function in the optimization process. In the above function, *h* is depth of cut, and h_d is the desired output value for the cutting operation. The objective is to set the process parameters at such levels that these values are achieved. In other words, we want to minimize the difference between the desired output and the output given by the SA algorithm. This is done by minimizing the error function given by equation (6). By doing so, the process parameters are calculated in such way that the AWJ cutting parameters approach their desired values.

IV. An Illustrative Example

In this section a numerical example is presented to illustrate the performance of the proposed model and the solution procedure. The error function given in (6), along with AWJ cutting model (4) is embedded into SA algorithm. The objective is to determine the values of control parameters (P, V, M_f, d_f) in such a way that the process output response (h) converges towards its target value.

The algorithm was coded in MATLAB 7.0 software and executed on a Pentium 4 computer. The best set of search parameters, found through several trial runs, is as follow: initial temperature (T_0) = 250; cooling rate (α) =0.98; and termination criteria = 500 iterations or error less than 0.01.

The comparison between predicted and desired values of process responses is shown in Table III As shown, all the parameters deviate from their desired values by less than 0.5%. These results illustrate that the proposed procedure can be efficiently used to determine optimal process parameters for any desired output values of AWJ cutting process.

TABLE III
Comparison between Target and Calculated Values

No.	Target	Predicted values for process parameters and output					Error
	h_d	d_f	P	M_f	V	h	(%)
1	11.37	0.8	100	0.098	98	11.3981	- 0.24
2	11.44	1.5	168	0.04	57	11.4851	- 0.39
3	18.54	0.8	166	0.092	92	18.5162	0.13
4	21.15	1.5	208	0.052	56	21.0526	0.46
5	28.69	0.8	206	0.091	76	28.7221	- 0.11
6	32.83	1.5	205	0.092	50	32.8329	- 0.01
7	36.83	0.8	185	0.095	50	36.7814	0.13
8	38.15	1.5	197	0.090	43	38.0513	0.25
9	46.59	1.5	239	0.094	36	46.6053	- 0.03

V. Conclusion

In the present work, a second order curvilinear regression model is developed to represent relationship between input process parameters and output cutting characteristic. The adequacy of the proposed model has been investigated using ANOVA technique. The results of ANOVA indicate that the proposed model has very good conformability to the real process.

The purpose of developing the mathematical model relating the responses and their process parameters is to facilitate the optimization of AWJ cutting parameters.

Therefore, a Simulated Annealing based procedure has been developed to predict the best process parameters values for any desired cutting characteristic. Computational results have proven that the proposed SA method can efficiently and accurately determine cutting parameters so as a desired depth of cut is obtained. From the above analyses, we conclude that the SA algorithm provides an effective and speedy optimization technique for the specific parameter estimation problem we have defined in this study.

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