

# A Rapid Code Acquisition Scheme in OOC-Based CDMA Systems

Keunhong Chae, Seokho Yoon

**Abstract**—We propose a code acquisition scheme called improved multiple-shift (IMS) for optical code division multiple access systems, where the optical orthogonal code is used instead of the pseudo noise code. Although the IMS algorithm has a similar process to that of the conventional MS algorithm, it has a better code acquisition performance than the conventional MS algorithm. We analyze the code acquisition performance of the IMS algorithm and compare the code acquisition performances of the MS and the IMS algorithms in single-user and multi-user environments.

**Keywords**—Code acquisition, optical CDMA, optical orthogonal code, serial algorithm.

## I. INTRODUCTION

CODE acquisition is one of the most important technical steps in optical code division multiple access (CDMA) systems, since data demodulation in optical CDMA is possible only after the code acquisition is completed [1].

Among obstacles such as noise, multipath, and multiple access interference (MAI) that affect code acquisition performance in optical CDMA systems, the influence of noise and multipath on the system can be easily alleviated by using fiber-optic media [1]; however, the MAI still remains one of the severe impairments, and thus, rapid and correct synchronization in multiple access environments has attracted much research interest [2]-[7]. Conventionally, the serial-search algorithm using optical orthogonal code (OOC) is used for code acquisition due to its simplicity; however, the associated mean acquisition time rapidly increases as the length of the OOC increases.

In [8], the multiple-shift (MS) algorithm using the OOC was proposed for code acquisition in optical CDMA systems. This algorithm was shown to reduce the mean acquisition time of the simple serial-search algorithm by a half. In this paper, we propose an improved multiple-shift (IMS) algorithm in which the mean acquisition time is reduced by  $\frac{1}{\sqrt{2}}$  compared with that of the MS algorithm.

## II. SYSTEM MODEL

For an optical CDMA system with  $N$  subscribers, we can express the received signal  $r(t)$  in the absence of noise and interference as

K. Chae is with the College of Information and Communication Engineering, Sungkyunkwan University, Suwon, 440746, Korea (chae0820@skku.edu).

S. Yoon is with the College of Information and Communication Engineering, Sungkyunkwan University, Suwon, 440746, Korea (corresponding author to provide phone: +82-31-290-7973; fax: +82-31-290-7231; e-mail: syoon@skku.edu)

$$r(t) = \sum_{n=1}^N s_n(t - \tau_n), \quad (1)$$

where  $s_n(t)$  is the signal of the  $n$ th subscriber,  $N$  is the number of subscribers,  $\tau_n \in [0, T_b)$  denotes the time offset of the  $n$ th subscriber signal, and  $T_b$  denotes one bit duration. We consider the on-off-keying (OOK) modulation and assume that the bit rate is equal for all subscribers. Then,  $s_n(t)$  can be expressed as

$$s_n(t) = \sum_{i=-\infty}^{\infty} b_i^{(n)} c_n(t - iT_b), \quad (2)$$

where  $b_i^{(n)} \in \{0, 1\}$  is the  $i$ th bit of the  $n$ th subscriber and  $c_n(t)$  is the OOC of the  $n$ th subscriber and can be written as

$$c_n(t) = \sum_{j=0}^{F-1} a_j^{(n)} P_{T_c}(t - jT_c), \quad (3)$$

where  $T_c$  is the chip duration of the OOC,  $a_j^{(n)} \in \{0, 1\}$  is a binary sequence with a length of  $F$  and a weight of  $K$  (i.e.,  $\sum_{j=0}^{F-1} a_j^{(n)} = K$ ), and  $P_{T_c}$  is the rectangular pulse defined as

$$P_{T_c} = \begin{cases} 1, & 0 \leq t < T_c, \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

with a length of  $T_c$ . Some examples of OOC patterns at  $K = 3$  are described in Fig. 1.

## III. IMPROVED MULTIPLE-SHIFT (IMS) ALGORITHM

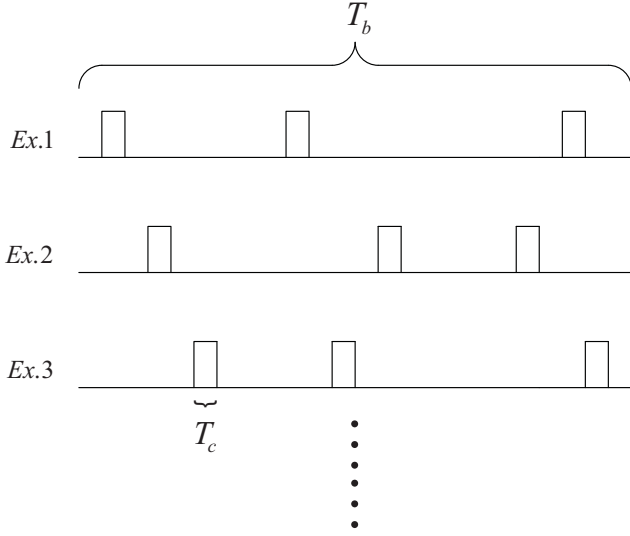
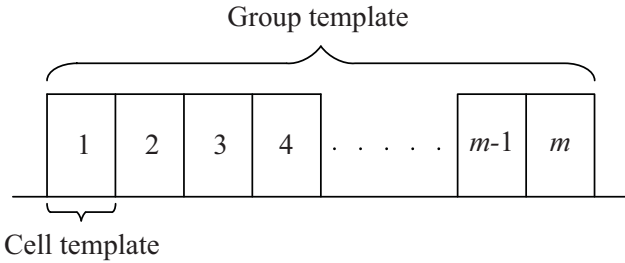
### A. Multiple-Shift (MS) Algorithm

In the multiple-shift (MS) algorithm, total  $F$  cells in the search space are divided into  $Q$  groups, each of which contains  $M$  cells. In this paper, the relative time between the cells is set to  $T_c$ . The relation of  $Q$  and  $M$  is given by

$$Q = \left\lceil \frac{F}{M} \right\rceil, \quad (5)$$

where  $\lceil \cdot \rceil$  denotes the ceiling operation.

The MS algorithm consists of two stages. In the first stage, the received signal  $r(t)$  is correlated with a group template shown in Fig. 2. The correlation is repeated on a group-by-group basis. If the correlation value corresponding to a group exceeds a given threshold, the time offset  $\tau_n$  is decided to be in the group (called the correct group) and the process


 Fig. 1: Examples of OOC patterns at  $k = 3$ .

 Fig. 2: The group and cell templates of the MS algorithm when  $M = m$ .

is transferred to the second stage, where the correlation-based search is performed again with a cell template on a cell-by-cell basis over  $M$  cells in the correct group. As in the first stage, if the correlation value corresponding to a cell exceeds a given threshold, the cell is decided to be an estimate of the time offset  $\tau_n$ .

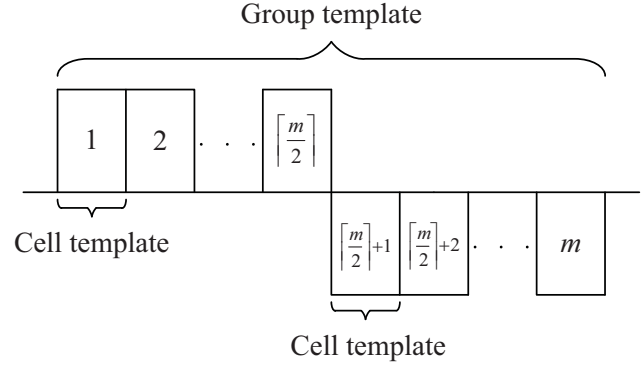
Thus, the mean acquisition time ( $T_{MS}$ ) of the MS algorithm is given by

$$T_{MS} = \frac{Q+1}{2} + \frac{M+1}{2}. \quad (6)$$

From (5) and (6), we can see that the minimum value of  $T_{MS}$  is obtained as  $\sqrt{F}$  when  $M = \sqrt{F}$ .

### B. Improved Multiple-Shift (IMS) Algorithm

The IMS algorithm proposed in this paper also consists of two stages as the MS algorithm. In the first stage, a group template as shown in Fig. 3 is used. The first half of the group template is positive and the second half of the group template is negative. If the correlation value of the correct group is positive, we search only the first half of  $M$  cells of the correct group in the second stage; otherwise, the search is performed over the second half of the  $M$  cells. Then, the


 Fig. 3: The group and cell templates of the IMS algorithm when  $M = m$ .

mean acquisition time ( $T_{IMS}$ ) of the IMS algorithm is given by

$$T_{IMS} = \frac{Q+1}{2} + \frac{M+1}{4}. \quad (7)$$

As we can see in (7), the acquisition time in the second stage is reduced by a half and the minimum value of  $T_{IMS}$  is  $\sqrt{\frac{F}{2}}$  when  $M = \sqrt{2F}$ .

## IV. MARKOV CHAIN MODEL AND PERFORMANCE ANALYSIS

To perform the mathematical analysis of the IMS algorithm, we model it with Markov chain model. First, we define four probabilities:  $P_{FA}$  and  $P_{fa}$  represent the false alarm probabilities in the first and second stages, respectively, and  $P_D$  and  $P_d$  represent the detection probabilities in the first and second stages, respectively.

The Markov chain model has a main loop consisting of  $Q$  different nodes and each node is one of the groups in the first stage. Each node in the main loop has a sub loop consisting of two-sided  $M/2$  different nodes, each node in the sub loop is one of the cells in the second stage, and each sided sub loop is a search space of the second stage decided by the result of the first stage.  $H(z)$  represents the transfer function from the current node to the next node in the main loop,  $h(z)$  is the transfer function from the current node to the next node in the sub loop, and  $L$  is defined to be the penalty factor caused by a false alarm.

Thus, we write  $h(z)$  and  $H(z)$  as

$$h(z) = (1 - p_{fa})z + p_{fa}z^{L+1} \quad (8)$$

and

$$H(z) = (1 - P_{FA})z + P_{FA}z^{M/2}(z), \quad (9)$$

respectively.

The transfer function from  $Q$  to  $ACQ$ , denoted by  $H_{det}(z)$ , represents the detection of the correct state, and the transfer function from  $Q$  to node 1, denoted by  $H_{miss}(z)$ , represents

the missing of the correct state. Thus,  $H_{det}(z)$  and  $H_{miss}(z)$  can be written as

$$H_{det}^{(v)}(z) = p_d P_D z^2 h^{v-1}(z) \quad (10)$$

and

$$H_{miss}^{(v)}(z) = (1 - P_D)z + P_D(1 - p_d)z^2 h^{(M/2)-1}(z), \quad (11)$$

respectively, where  $v \in \{1, 2, \dots, M/2\}$  is a uniformly distributed random variable.

Considering that the first stage starts at the  $i$ th node, then the transfer function between the  $i$ th node and  $ACQ$  node is written as

$$U_i^{(v)}(z) = \frac{H^{Q-i}(z)H_{det}^{(v)}(z)}{1 - H_{miss}^{(v)}(z)H^{Q-1}(z)}, \quad (12)$$

where  $i$  is a uniformly distributed random variable in  $\{1, 2, \dots, Q\}$ , and thus,

$$U(z) = \frac{H_{det}(z)}{1 - H_{miss}(z)H^{Q-1}(z)} \frac{1}{Q} \sum_{i=1}^Q H^{Q-i}(z), \quad (13)$$

where  $H_{miss}(z) = H_{miss}^{(v)}$  and  $H_{det}(z)$  with

$$H_{det}(z) = \frac{2}{M} \sum_{v=1}^{M/2} H_{det}^{(v)}(z). \quad (14)$$

We define that  $T_{IMS}$  is the number of dwell times required to find the correct state and model it as a random variable. Using the moment generating function, thus,  $U(z)$  can be represented as

$$U(z) = E(z^{T_{IMS}}) \quad (15)$$

in terms of  $T_{IMS}$ , where

$$E(T_{IMS}) = \left. \frac{dU(z)}{dz} \right|_{z=1} = U'(1). \quad (16)$$

Then,

$$E(T_{IMS}) = \frac{H'_{det}(1) + H'_{miss}(1)}{p_d P_D} + (Q-1)H'(1) \frac{2-p_d P_D}{2p_d P_D}, \quad (17)$$

where

$$H'(1) = 1 + \frac{M}{2} P_{FA}(1 + L_{p_{fa}}), \quad (18)$$

$$H'_{det}(1) = 2p_d P_D + \frac{(M/2) - 1}{2} p_d P_D(1 + L_{p_{fa}}), \quad (19)$$

and

$$H'_{miss}(1) = (1 - P_D) + P_D(1 - p_d) \times [(M/2) + 1 + \{(M/2) - 1\}L_{p_{fa}}]. \quad (20)$$

In an ideal case (i.e.,  $p_d = P_D = 1$  and  $p_{fa} = P_{FA} = 0$ ),

$$E(T_{IMS}) = \frac{Q+1}{2} + \frac{M+1}{4} \quad (21)$$

and so, by differentiating (21) as shown in

$$\frac{\partial E(T_{IMS})}{\partial M} = \frac{1}{4} + \frac{F}{2M^2} = 0, \quad (22)$$

we obtain the minimum value of  $T_{IMS}$  as  $\sqrt{\frac{F}{2}} + \frac{3}{4}$  when  $M = \sqrt{2F}$ , and it is approximated as  $\sqrt{\frac{F}{2}}$  when  $F \gg 1$ .

## V. NUMERICAL RESULTS

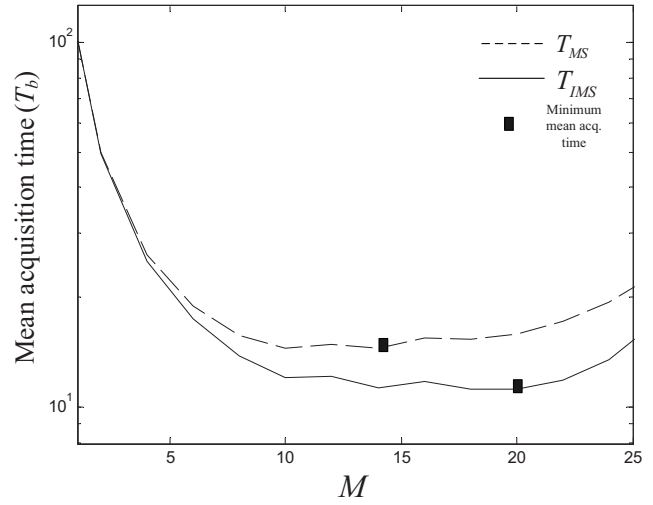


Fig. 4: The mean acquisition time performance of the MS and IMS algorithms as a function of parameter  $M$  when  $F = 200$ ,  $K = 5$ , and  $TH = th = 5$ .

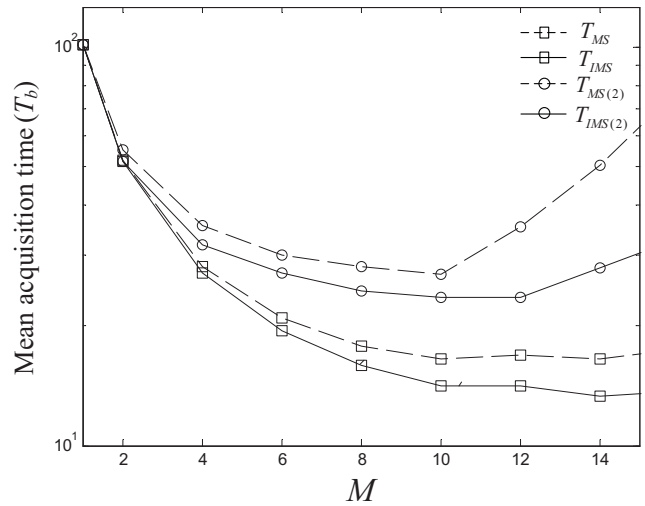


Fig. 5: The mean acquisition time performance of the MS and IMS algorithms as a function of parameter  $M$  in the case of two subscribers when  $F = 200$ ,  $K = 5$ , and  $TH = th = 5$ .

Fig. 4 shows the mean acquisition time performance of the MS and IMS algorithms as a function of  $M$ , where  $TH$  and  $th$  are thresholds in the first and second stages, respectively. From Fig. 4, we can see that the minimum mean acquisition time of the IMS algorithm is reduced compared with that of the MS algorithm.

Fig. 5 shows the mean acquisition time performance of the MS and IMS algorithms as a function of  $M$  in the case of two subscribers, where  $T_{MS(2)}$  and  $T_{IMS(2)}$  represent the mean acquisition time of the MS and the IMS algorithm, respectively. From Fig. 5, we can see that the minimum mean acquisition time of the IMS algorithm is reduced compared with that of the MS algorithm in the case of two subscribers.

In addition, it is seen that the difference in the mean acquisition time increases as the value of  $M$  increases.

## VI. CONCLUSION

In this paper, we have proposed a novel code acquisition algorithm called the IMS for optical CDMA systems. We have first formed an efficient template for the second stage, and then, have proposed the IMS algorithm using the template. The mathematical performance of the proposed IMS algorithm has been analyzed using the Markov chain model and from numerical results, it has been confirmed that the IMS algorithm provides a significant improvement over the MS algorithm in terms of the mean acquisition time.

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**Keunhong Chae** received the B.S.E. degree in information and communication engineering from Sungkyunkwan University, Suwon, Korea, in 2014. He is currently working toward the Ph.D. degree in the College of Information and Communication Engineering, Sungkyunkwan University. His research interests include binary offset carrier modulation and cognitive radio.

**Seokho Yoon** received the B.S.E. (summa cum laude), M.S.E., and Ph.D. degrees in electrical engineering from Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 1997, 1999, and 2002, respectively. From April 2002 to June 2002, he was with the Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology (MIT), Cambridge. From July 2002 to February 2003, he was with the Department of Electrical Engineering, Harvard University, Cambridge, as a Postdoctoral Research Fellow. In March 2003, he joined the College of Information and Communication Engineering, Sungkyunkwan University, Suwon, Korea, where he is currently an Associate Professor. His research interests include spread spectrum systems, mobile communications, detection and estimation theory, and statistical signal processing.