A Rapid Code Acquisition Scheme in OOC-Based CDMA Systems

Keunhong Chae, Seokho Yoon

Abstract—We propose a code acquisition scheme called improved multiple-shift (IMS) for optical code division multiple access systems, where the optical orthogonal code is used instead of the pseudo noise code. Although the IMS algorithm has a similar process to that of the conventional MS algorithm, it has a better code acquisition performance than the conventional MS algorithm. We analyze the code acquisition performance of the IMS algorithm and compare the code acquisition performances of the MS and the IMS algorithms in single-user and multi-user environments.

Keywords—Code acquisition, optical CDMA, optical orthogonal code, serial algorithm.

I. INTRODUCTION

CODE acquisition is one of the most important technical steps in optical code division multiple access (CDMA) systems, since data demodulation in optical CDMA is possible only after the code acquisition is completed [1].

Among obstacles such as noise, multipath, and multiple access interference (MAI) that affect code acquisition performance in optical CDMA systems, the influence of noise and multipath on the system can be easily alleviated by using fiber-optic media [1]; however, the MAI still remains one of the severe impairments, and thus, rapid and correct synchronization in multiple access environments has attracted much research interest [2]-[7]. Conventionally, the serial-search algorithm using optical orthogonal code (OOC) is used for code acquisition due to its simplicity; however, the associated mean acquisition time rapidly increases as the length of the OOC increases.

In [8], the multiple-shift (MS) algorithm using the OOC was proposed for code acquisition in optical CDMA systems. This algorithm was shown to reduce the mean acquisition time of the simple serial-search algorithm by a half. In this paper, we propose an improved multiple-shift (IMS) algorithm in which the mean acquisition time is reduced by $\frac{1}{\sqrt{2}}$ compared with that of the MS algorithm.

II. SYSTEM MODEL

For an optical CDMA system with N subscribers, we can express the received signal $r(t)$ in the absence of noise and interference as

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$$r(t) = \sum_{n=1}^{N} s_n(t - \tau_n), \qquad (1)$$

where $s_n(t)$ is the signal of the nth subscriber, N is the number of subscribers, $\tau_n \in [0, T_b)$ denotes the time offset of the nth subscriber signal, and T_b denotes one bit duration. We consider the on-off-keying (OOK) modulation and assume that the bit rate is equal for all subscribers. Then, $s_n(t)$ can be expressed as

$$s_n(t) = \sum_{i=-\infty}^{\infty} b_i^{(n)} c_n(t - iT_b), \qquad (2)$$

where $b_i^{(n)} \in \{0, 1\}$ is the ith bit of the nth subscriber and $c_n(t)$ is the OOC of the nth subscriber and can be written as

$$c_n(t) = \sum_{j=0}^{F-1} a_j^{(n)} P_{T_c}(t - jT_c), \qquad (3)$$

where T_c is the chip duration of the OOC, $a_j^{(n)} \in \{0, 1\}$ is a binary sequence with a length of F and a weight of K (i.e., $\sum_{j=0}^{F-1} a_j^{(n)} = K$)), and P_{T_c} is the rectangular pulse defined as

$$P_{T_c} = \begin{cases} 1, & 0 \le t < T_c, \\ 0, & \text{otherwise} \end{cases} \qquad (4)$$

with a length of T_c. Some examples of OOC patterns at $K = 3$ are described in Fig. 1.

III. IMPROVED MULTIPLE-SHIFT (IMS) ALGORITHM

A. Multiple-Shift (MS) Algorithm

In the multiple-shift (MS) algorithm, total F cells in the search space are divided into Q groups, each of which contains M cells. In this paper, the relative time between the cells is set to T_c. The relation of Q and M is given by

$$Q = \left\lceil \frac{F}{M} \right\rceil, \qquad (5)$$

where $\lceil \ \rceil$ denotes the ceiling operation.

The MS algorithm consists of two stages. In the first stage, the received signal $r(t)$ is correlated with a group template shown in Fig. 2. The correlation is repeated on a group-by-group basis. If the correlation value corresponding to a group exceeds a given threshold, the time offset τ_n is decided to be in the group (called the correct group) and the process
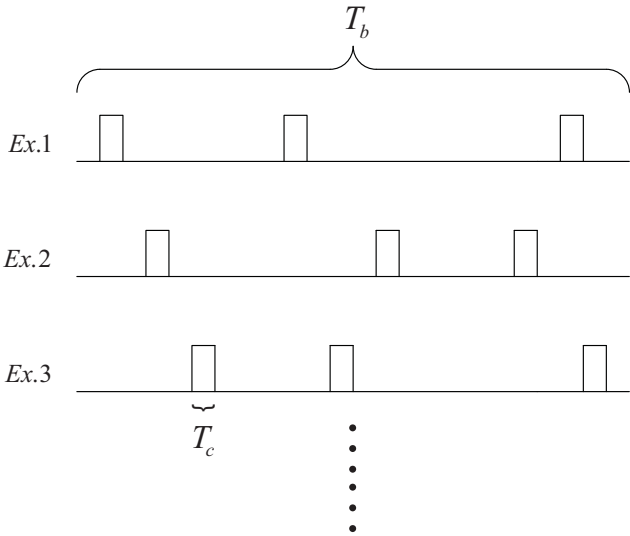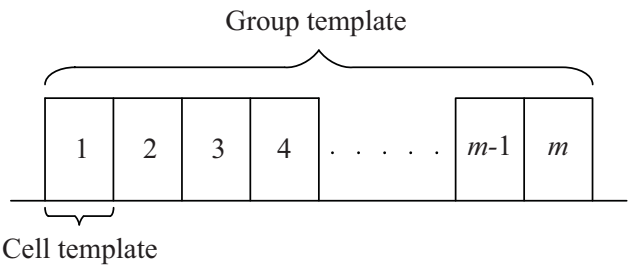
Fig. 1: Examples of OOC patterns at $k = 3$.

Fig. 2: The group and cell templates of the MS algorithm when $M = m$.

Fig. 3: The group and cell templates of the IMS algorithm when $M = m$.

is transferred to the second stage, where the correlation-based search is performed again with a cell template on a cell-by-cell basis over M cells in the correct group. As in the first stage, if the correlation value corresponding to a cell exceeds a given threshold, the cell is decided to be an estimate of the time offset τ_n.

Thus, the mean acquisition time (T_{MS}) of the MS algorithm is given by

$$T_{MS} = \frac{Q+1}{2} + \frac{M+1}{2}. \tag{6}$$

From (5) and (6), we can see that the minimum value of T_{MS} is obtained as $\sqrt{F}$ when $M = \sqrt{F}$.

B. Improved Multiple-Shift (IMS) Algorithm

The IMS algorithm proposed in this paper also consists of two stages as the MS algorithm. In the first stage, a group template as shown in Fig. 3 is used. The first half of the group template is positive and the second half of the group template is negative. If the correlation value of the correct group is positive, we search only the first half of M cells of the correct group in the second stage; otherwise, the search is performed over the second half of the M cells. Then, the
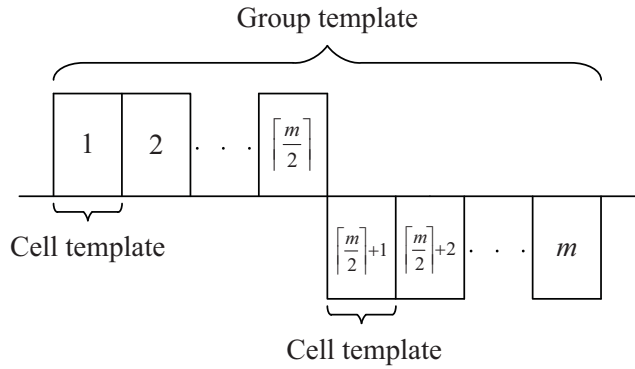
mean acquisition time (T_{IMS}) of the IMS algorithm is given by

$$T_{IMS} = \frac{Q+1}{2} + \frac{M+1}{4}. \tag{7}$$

As we can see in (7), the acquisition time in the second stage is reduced by a half and the minimum value of T_{IMS} is $\sqrt{\frac{F}{2}}$ when $M = \sqrt{2F}$.

IV. MARKOV CHAIN MODEL AND PERFORMANCE ANALYSIS

To perform the mathematical analysis of the IMS algorithm, we model it with Markov chain model. First, we define four probabilities: P_{FA} and P_{fa} represent the false alarm probabilities in the first and second stages, respectively, and P_D and P_d represent the detection probabilities in the first and second stages, respectively.

The Markov chain model has a main loop consisting of Q different nodes and each node is one of the groups in the first stage. Each node in the main loop has a sub loop consisting of two-sided $M/2$ different nodes, each node in the sub loop is one of the cells in the second stage, and each sided sub loop is a search space of the second stage decided by the result of the first stage. $H(z)$ represents the transfer function from the current node to the next node in the main loop, $h(z)$ is the transfer function from the current node to the next node in the sub loop, and L is defined to be the penalty factor caused by a false alarm.

Thus, we write $h(z)$ and $H(z)$ as

$$h(z) = (1 - p_{fa})z + p_{fa}z^{L+1} \tag{8}$$

and

$$H(z) = (1 - P_{FA})z + P_{FA}zh^{M/2}(z), \tag{9}$$

respectively.

The transfer function from Q to ACQ, denoted by $H_{det}(z)$, represents the detection of the correct state, and the transfer function from Q to node 1, denoted by $H_{miss}(z)$, represents

the missing of the correct state. Thus, $H_{det}(z)$ and $H_{miss}(z)$ can be written as

$$H_{det}^{(v)}(z) = p_d P_D z^2 h^{v-1}(z) \tag{10}$$

and

$$H_{miss}^{(v)}(z) = (1 - P_D)z + P_D(1 - p_d)z^2 h^{(M/2)-1}(z), \tag{11}$$

respectively, where $v \in \{1, 2, ..., M/2\}$ is a uniformly distributed random variable.

Considering that the first stage starts at the ith node, then the transfer function between the ith node and ACQ node is written as

$$U_i^{(v)}(z) = \frac{H^{Q-i}(z)H_{det}^{(v)}(z)}{1 - H_{miss}^{(v)}(z)H^{Q-1}(z)}, \tag{12}$$

where i is a uniformly distributed random variable in $\{1, 2, ..., Q\}$, and thus,

$$U(z) = \frac{H_{det}(z)}{1 - H_{miss}(z)H^{Q-1}(z)} \frac{1}{Q} \sum_{i=1}^{Q} H^{Q-i}(z), \tag{13}$$

where $H_{miss}(z) = H_{miss}^{(v)}$ and $H_{det}(z)$ with

$$H_{det}(z) = \frac{2}{M} \sum_{v=1}^{M/2} H_{det}^{(v)}(z). \tag{14}$$

We define that T_{IMS} is the number of dwell times required to find the correct state and model it as a random variable. Using the moment generating function, thus, $U(z)$ can be represented as

$$U(z) = E(z^{T_{IMS}}) \tag{15}$$

in terms of T_{IMS}, where

$$E(T_{IMS}) = \frac{dU(z)}{dz}\Big|_{z=1} = U'(1). \tag{16}$$

Then,

$$E(T_{IMS}) = \begin{array}{l} \frac{H'_{det}(1) + H'_{miss}(1)}{p_d P_D} \\ + (Q-1)H'(1)\frac{2 - p_d P_D}{2 p_d P_D}, \end{array} \tag{17}$$

where

$$H'(1) = 1 + \frac{M}{2}P_{FA}(1 + L_{p_{fa}}), \tag{18}$$

$$H'_{det}(1) = 2p_d P_D + \frac{(M/2) - 1}{2}p_d P_D(1 + L_{p_{fa}}), \tag{19}$$

and

$$H'_{miss}(1) = \begin{array}{l} (1 - P_D) + P_D(1 - p_d) \\ \times [(M/2) + 1 + \{(M/2) - 1\}L_{p_{fa}}]. \end{array} \tag{20}$$

In an ideal case (i.e., $p_d = P_D = 1$ and $p_{fa} = P_{FA} = 0$),

$$E(T_{IMS}) = \frac{Q+1}{2} + \frac{M+1}{4} \tag{21}$$

and so, by differentiating (21) as shown in

$$\frac{\partial E(T_{IMS})}{\partial M} = \frac{1}{4} + \frac{F}{2M^2} = 0, \tag{22}$$

we obtain the minimum value of T_{IMS} as $\sqrt{\frac{F}{2}} + \frac{3}{4}$ when $M = \sqrt{2F}$, and it is approximated as $\sqrt{\frac{F}{2}}$ when $F \gg 1$.
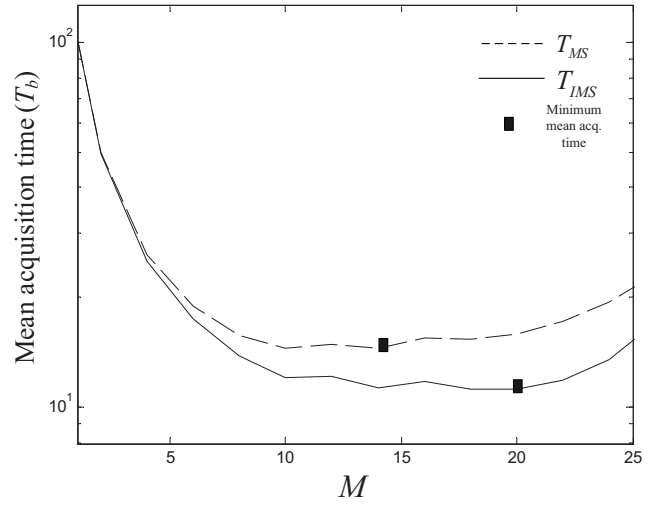
V. NUMERICAL RESULTS

Fig. 4: The mean acquisition time performance of the MS and IMS algorithms as a function of parameter M when $F = 200$, $K = 5$, and $TH = th = 5$.
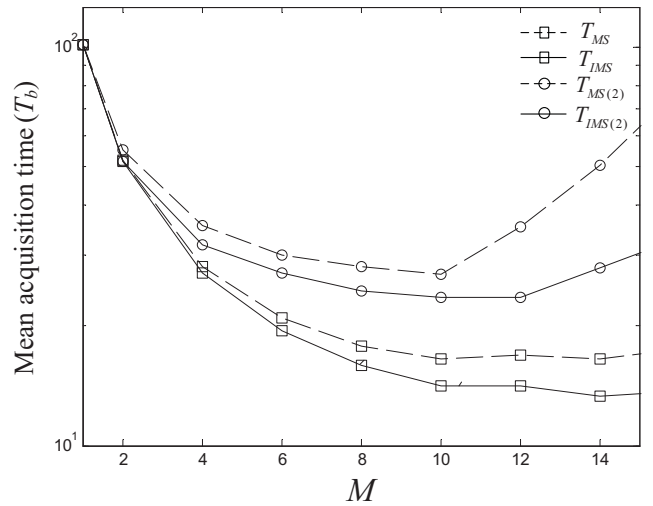

Fig. 5: The mean acquisition time performance of the MS and IMS algorithms as a function of parameter M in the case of two subscribers when $F = 200$, $K = 5$, and $TH = th = 5$.

Fig. 4 shows the mean acquisition time performance of the MS and IMS algorithms as a function of M, where TH and th are thresholds in the first and second stages, respectively. From Fig. 4, we can see that the minimum mean acquisition time of the IMS algorithm is reduced compared with that of the MS algorithm.

Fig. 5 shows the mean acquisition time performance of the MS and IMS algorithms as a function of M in the case of two subscribers, where $T_{MS(2)}$ and $T_{IMS(2)}$ represent the mean acquisition time of the MS and the IMS algorithm, respectively. From Fig. 5, we can see that the minimum mean acquisition time of the IMS algorithm is reduced compared with that of the MS algorithm in the case of two subscribers.

In addition, it is seen that the difference in the mean acquisition time increases as the value of M increases.

VI. Conclusion

In this paper, we have proposed a novel code acquisition algorithm called the IMS for optical CDMA systems. We have first formed an efficient template for the second stage, and then, have proposed the IMS algorithm using the template. The mathematical performance of the proposed IMS algorithm has been analyzed using the Markov chain model and from numerical results, it has been confirmed that the IMS algorithm provides a significant improvement over the MS algorithm in terms of the mean acquisition time.

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