International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:4, No:12, 2010

A Quality Optimization Approach: An Application on Next Generation Networks

Gülfem I. Alptekin, S. Emre Alptekin

Abstract**—**The next generation wireless systems, especially the cognitive radio networks aim at utilizing network resources more efficiently. They share a wide range of available spectrum in an opportunistic manner. In this paper, we propose a quality management model for short-term sub-lease of unutilized spectrum bands to different service providers. We built our model on competitive secondary market architecture. To establish the necessary conditions for convergent behavior, we utilize techniques from game theory. Our proposed model is based on potential game approach that is suitable for systems with dynamic decision making. The Nash equilibrium point tells the spectrum holders the ideal price values where profit is maximized at the highest level of customer satisfaction. Our numerical results show that the price decisions of the network providers depend on the price and QoS of their own bands as well as the prices and QoS levels of their opponents' bands.

Keywords**—**cognitive radio networks, game theory, next generation wireless networks, spectrum management.

I. Introduction

DEMAND for frequency spectrum use is growing rapidly with growing trends in the use of information technology and the increased mobility of societies. Despite its scarcity, many measurement campaigns have shown, most of the time spectrum is underutilized at a given location [1][2]. The results force many regulators to consider alternative approaches for more efficient use of spectrum resources. In 2004, International Telecommunication Union, in Geneva, found that "many TV channels are unused over significant geographical areas" and concluded that "cognitive radio techniques appear to be a promising approach" for using spectrum more efficiently. The statement of ITU has not escaped the attention of the FCC, and in 2004, FCC legalized secondary markets for spectrum and issued a request for industry comment on sharing of the unused TV bands. [3][4].

UK regulator Ofcom has so far taken the decision of deregulate the airwaves in such a way that the licensee can relicense (sub-lease) some of its rights to other parties, instead of filling underutilized bands with smarter and smarter radios [5]. In such a way, Ofcom plans to have exclusive use of certain frequencies. Ofcom expects to convert more than 70% of the UK's spectrum to the new regime by 2010 [1]. These different approaches can provide significant economic and social benefits only if they become widely available and utilized, i.e. if they are commercially successful.

There are valuable research on spectrum management that are based on strategic games. Niyato and Hossain proposed an oligopoly market environment where multiple primary service providers compete with each other to offer access opportunities to the secondary users [6]. In another work, they investigated three different pricing models for spectrum trading in a cognitive environment [7]. They showed performance analysis of these models where primary service providers have different behaviors. Bloem et al., suggests a Stackelberg game model that allows cognitive radio pairs to update their transmission powers and frequencies simultaneously. Then, they define the virtual prices for communicating over a licensed channel [8]. The convergence conditions for various game models in cognitive radio networks were investigated by Neel et al. [9].

In this paper, we focused on the competition during the sub-lease process of the underutilized bands of spectrum license holders, under the control of a regulator. We aim at calculating the optimum band prices that maximize the net profit of license holders while simultaneously satisfying buyers. In such an architecture, the license holders decisions depend not only on thier own strategy but also on those of the other license holders'. Therefore, the secondary market architecture is a natural context in which to apply game theory. To do so, we come up with a potential game, where the players are the spectrum holders and their strategy is the choice of the unit price of the offered band subject to QoS constraints.

II. Game Theory And Potential Games

Game theory has been recognized as a cornerstone of micro-economics that can be applied to analyze problems with conflicting objectives and interactive decision makers [10]. A game consists of three fundamental components: A finite set of players (decision makers) $N = \{1, 2, \ldots, n\}$, a set of strategies, $S = \{S_1, S_2, \ldots, S_n\}$ available to those players, and the set of payoffs (utilities), $\{u_i\} = \{u_1, u_2, .., u_n\}$, for each combination of strategies that the players wish to maximize. Each player's utility function, ui, is a function of the particular strategy chosen by player i, si, and the particular strategies

Gülfem I. Alptekin is with the Department of Computer Engineering at Galatasaray University, Istanbul, 34357 TURKEY (phone: 090-212-2274480; fax: 090-212-2595557; e-mail: gisiklar@gsu.edu.tr).

S. Emre Alptekin is with the Department of Industrial Engineering at Galatasaray University, Istanbul, 34357 TURKEY (e-mail: ealptekin@gsu.edu.tr).

chosen by all of the other players in the game, s_{-i} Player i prefers the strategy s_i over s_i', if $u_i(s_i) \geq u_i(s_i')$.

A fundamental concept for normal form games is the Nash Equilibrium (NE) [11].

Definition 1: A strategy profile si is said to be a Nash Equilibrium, iff $\forall i \in N$, $u_i(s_i) \geq u_i(s_i', s_{-i})$, $\forall s_i' \in S_i$.

A potential game is a normal form game which has the property that there exists a function known as the potential function V: $S \rightarrow \Re$, that reflects the change in utility value accrued by unilaterally deviating player [12]. In potential games, each player's utility function is replaced by the potential function, and the incentive of all players to change their strategy can be expressed in this global function.

Definition 2: A game $\Gamma = \{N, S, \{U_i\}\}$ is an exact potential game if there exists a function V: $S \rightarrow \Re$ such that for all $i \in [1, N]$, $s_i \in S$, and all $s' \in S$,

$$V(s_i, s_{-i}) - V(s_i', s_{-i}) = U_i(s_i, s_{-i}) - U_i(s_i', s_{-i})$$

The function V is called an exact potential function for the game Γ. The most important result of potential games for us is their convergence and stability properties. Any potential game in which players take actions sequentially converges to a pure strategy Nash Equilibrium that maximizes the potential function [12]. This is stated in Theorem 1 [10]:

Theorem 1. If V is an ordinal potential function of the game Γ and $s^* \in \{\arg\max_{s' \in S} V(s')\}$ is a maximizer of the potential function, then s* is the Nash Equilibrium of the game.

In an exact potential game, only one player acts at each time step and the acting player maximizes its utility, given the most recent actions of the other players. With an accurate update algorithm, it is possible to converge to Nash equilibrium regardless of the order of play and the initial condition of the game [8].

III. Proposed Approach

We built our approach on the secondary market architecture proposed by Ofcom [5]. At the top of this hierarchy, the regulatory body (e.g. FCC or Ofcom) issues relatively long-term spectrum leases, say for a 10 year period, on contiguous blocks to spectrum holding network providers (NPs), for large geographical regions. This market is referred as the primary marketplace. The marketplace that we concentrate on is the second one, where the long-term license holders (NPs) sub-lease previously bought spectrum to potential buyers (ASPs). These offered spectrum bands could be from any frequency band interval. The trading is realized under the control of a spectrum exchange regulator, a governmental agency or a private one. This could be the same regulatory body as the one at the top of the hierarchy. If the ASP accepts the spectrum offer, the required portion is allocated to it for the duration of the time period. At the lowest level, we have the customers of the ASPs; but we will focus mainly on the relationships among spectrum holders and buyers for this model.

A. Our Proposed Game Model

It is assumed that we are working with a normal form game defined as $\Gamma = \{N, S, \{U_i\}\}$, whose properties are: Players: The network providers (NPs). The spectrum exchange game that we consider consists of a set of N network providers, denoted by $i \in [1, N]$, that hold long term spectrum licenses. They compete with each other to sub-lease maximum part of their unutilized bands. Strategies: The choice of the price of the offered frequency band subject to endogenously determined QoS constraints and capacity constraints. Commodity of the spectrum exchange market: The frequency spectrum band.

Our objective is to find the price of the offered spectrum bands such that the NPs achieve as high of a utility value, and the ASPs as high of a service quality as possible. In analyzing the outcome of the game, since the players make decisions independently and are influenced by the other players' decisions, we are interested to determine if there exists a convergence point, from which no player would deviate anymore, i.e. Nash equilibrium.

We assume that each provider's spectrum band has two service parameters: Price and quality of service (QoS) level. $p = \{p_{1k}, \ldots, p_{Nk}\}$ is the price vector where p_{ik} is the price that NP_i charges ASP_k, and $q = \{q_{1k}, \ldots, q_{Nk}\}$ where q_{ik} is the quality measure of the spectrum band offered by NP_i to ASP_k with k [1, M].

We assume that the price of the offered band that is given by $_{NPi}$ consists of two components: A base price ($\underline{p_i}$) which depends on the demand to NP_i and a quality-related price ($\overline{p_{ik}}$) which depends on the QoS level of NP_i's network, (i.e. the QoS level of the offered band) for ASP_k. The higher the NP_i's demand, the higher its base price is, and the higher the QoS level of the offered band, the higher the quality-related price is. As the base price is a function of its own demand, an NP offers the same base price to each ASP:

$$\underline{p_i} = c_i + k_i \left(\sum_{k \in \{1,\ldots,M\}} D_{ik} \right) \tag{1}$$

where c_i represents the fixed costs that the NP faces in order to offer these bands, D_{ik} represents the demand of ASP_k to NP_i, and k_i is a positive constant that represents to what extent the NP's base price is influenced from its total demand. Besides, in a competitive environment, the price of a band should also be influenced from the QoS levels of other NPs' bands, i.e. the quality-related component, $\overline{p_{ik}}$, depends on the entire QoS vector, q:

$$\overline{p_{ik}} = w_{ik}.q_{ik} - \sum_{\substack{j=1 \\ j \neq i}}^{n} w_{jk}.q_{jk} \tag{2}$$

where w_{ik} is a positive constant that represents the importance that NP_i attaches to its QoS level when setting the price of the band, q_{ik} is the QoS level of NP_i's bands offered to ASP_k, and w_{jk} is a positive constant that represents the importance that NP_i attaches to its opponent's QoS levels when setting its price. Hence, the price of an offered band to ASP_k by NP_i is calculated by adding the quality-related component to the base price:

$$p_{ik} = \underline{p_i} + \overline{p_{ik}} \quad (3)$$

In the proposed approach, the demand of ASP_k is assumed to be linearly affected by the price of the band. In a competitive telecommunication market, demand should be a function of the NP's own price decision as well as its opponents' price decisions. A positive coefficient b_k represents to what extent its price variations influence NP_i's demand, while the coefficients t_{jk} represent to what extent the price variations of NP_i's opponents influence NP_i's demand. Assuming the base demand of ASP_k is a_k, the demand of ASP_k from NP_i can be written as:

$$D_{ik}(\mathrm{p}) = a_k - b_k . p_{ik} + \sum_{\substack{j=1 \\ i \neq j}}^{n} t_{jk} . p_{jk} \quad (4)$$

We define the spectrum band requirement of an ASP as its base price (a_k). The spectrum bands that ASP_k sub-leases from different NPs should not exceed its base demand:

$$\sum_i D_{ik} = a_k \quad (5)$$

When we integrate (1), (2) and (4) into the equation (3), we obtain the price expression as:

$$\begin{aligned} p_{ik} = c_i + k_i \left(\sum_k \left[a_k - b_k . p_{ik} + \sum_{\substack{j=1 \\ i \neq j}}^{n} t_{jk} . p_{jk} \right] \right) + \\ + w_{ik} . q_{ik} - \sum_{\substack{j=1 \\ j \neq i}}^{n} w_{jk} . q_{jk} \end{aligned} \quad (6)$$

We define $\mathrm{S} = \times s_{ik}, i \in N, k \in M$ as the strategy space of NP_i. The upper and lower bound constraints are given by:

$$s_{ik} = \left\{ (p_{ik}, q_{ik}) : 0 \leq c_i \leq p_{ik} \leq p_{ik}^{\max} ; 0 \leq q_{ik}^{\min} \leq q_{ik} \leq q_{ik}^{\max} \right\} \quad (7)$$

Beyond some price, demand will be zero whatever the prices and QoS levels of opponents are. Accordingly, the NP itself or the central regulator defines an upper bound on price. The lower bound is set so as to keep the net profit of the NP positive.

1) Quality of the spectrum band

The selection and design of performance metrics is an important, but relatively less addressed issue in the design of cognitive radio networks [13]. In the context of this paper, ASPs need the performance metrics in order to differentiate spectrum bands in the market according to their QoS levels. In this study, we will present some of the QoS parameters that could be considered for spectrum bands.

According to their marketing preferences, each ASP should determine which QoS parameters to consider and what their importance weights should be. First of all, the spectrum band which is offered to ASP_k should be in its operating intervals. If the ASP requires an UHF band for mobile services, any VHF band will not serve it. An interference related metric (i.e. interference temperature) could be another quality metric, since the main idea of the spectrum management is to detect the unused spectrum holes that will not interfere the existing users. Spectrum utilization in terms of throughput and goodput, SINR, INR or BER degradation in the network, network access time, vulnerability to denial-of-service attack, response time for interactive data applications are the examples that are grouped in the network-level metrics. Another quality parameter could be a factor that depends on which type of network that ASP will operate. For instance, the interval of the spectrum band can be taken as a quality parameter for cellular networks; since the signal propagate farther and penetrate buildings better in lower frequency bands.

We assume that the QoS parameters that ASP_i considers are added to form a total quality parameter, q_{ik}, that is defined in the range of [0, 1].

2) Opportunity cost of the radio spectrum

In the secondary market architecture, NPs sub-lease their unused or underused resources (the spectrum bands or bandwidths) to the ASPs. Hence, NPs should also consider their opportunity costs when setting the prices. The opportunity cost is defined as the value of an asset or resource in the next best alternative that is foregone by virtue of its actual use [14]. In our context, it is the value of the unused spectrum bands to the network provider that derives the highest benefit from being able to use it. Spectrum has a non-zero opportunity cost if there is excess demand for it now or in the future from current and potential alternative uses. Although the chosen cost parameters and their importance weights may depend on NPs, we have formulated the opportunity cost of NP_i received from ASP_k as:

$$OC_{ik}(D_{ik}) = \left[t_{1ik} . BF_k + t_{2ik} . LF_k \right] . D_{ik} . p_{ik} \quad (8)$$

with BF_k, the band factor, and LF_k the location factor. They are both defined in the [0-1] range. The location factor increases proportional to the congestion of the region that the spectrum portion will be in use. The band factor increases with the number of technologies that can operate on this band; since NPs will have the opportunity to reach more ASPs. The importance weights, t_{1ik} and t_{2ik}, are used to adjust the cost value according to the marketing preferences of NPs, where t_{1ik} and t_{2ik} are positives and t_{1ik}+t_{2ik}=1.

3) Utility Model

We define U_i: $S \rightarrow \Re$ as the set of utility functions that the players associate with their price and QoS level strategies. The utility function of the NP_i from ASP_k is represented by U_{ik} (p_{ik}, q_{ik}). The net profit (net revenue) of an NP is considered as its utility, and it is given by the sum of the differences of its opportunity cost (OC_{ik}) from its revenues from all the ASPs:

$$U_i(\mathrm{p,q}) = \sum_{k\in\{1,..M\}} \left[p_{ik}.D_{ik} - OC_{ik}(D_{ik}) \right] \tag{9}$$

We assume that $U_i(p, q)$ is continuous in p and concave in p_{ik} for all $i \in [1, N]$ and $k \in [1, M]$. The NP can use this utility expression while making the decision of whether it should sub-lease the spectrum band or not. The sub-lease process is thought to have business value when its net profit is positive, and to be unprofitable when it is negative. Among a set of spectrum alternatives, the one with the highest $U_i(p, q)$ generates the most value, and should be favored over the others. Hence, our result space consists of the values which make the net profit positive.

4) Nash Equilibrium

Single-parameter Nash equilibrium: Let $U_i(p, q)$ be the net revenue of NP_i, when the vector of prices set by all network providers, p, and the vector of QoS parameters, q, of all providers is fixed at values $q_{ik}, q_{1k}, q_{2k}, \ldots, q_{Nk}$. Then, a single-parameter Nash equilibrium in p at q is the vector p^* that solves for all i:

$$U_i(\mathrm{p^*,q}) = \max_{(p_{ik},q)\in\Re_i} U_i \begin{pmatrix} p^*_{1k},\ldots,p^*_{(i-1)k},p^*_{ik},p^*_{(i+1)k},\ldots,p^*_{Nk}, \\ q_{1k},\ldots,q_{(i-1)k},q_{ik},q_{(i+1)k},\ldots,q_{Nk} \end{pmatrix} \tag{10}$$

If the equilibrium strategy profile in (10) is deterministic, a pure strategy Nash equilibrium exists. For finite games, even if a pure strategy Nash equilibrium does not exist, a mixed strategy Nash equilibrium can be found [15].

5) The Potential Game Formulation

The following utility expression is written as factorized as possible to better distinguish the terms of coordination game, self-motivated game, dummy game or bilateral symmetric game, if it exists. In our proposed model, t_{1ik}, t_{2ik}, BF_k and LF_k have the same values at each time step in the game. Therefore, we have not integrated the term [t_{1ik}.BFk + t_{2ik}. LF_k] in the utility function during the demonstrations.

$$U_i(\mathrm{p,q}) = \sum_{k=1}^{M}\left[p_{ik}.D_{ik}\right] = \sum_{k=1}^{M} \left[\begin{array}{l} a_k.c_i + a_k.k_i.\sum_{l=1}^{M} a_l - a_k.k_i.\sum_{l=1}^{M} b_l.p_{il} + a_k.k_i.\sum_{l=1}^{M}\sum_{j=1,i\neq j}^{N} t_{jl}.p_{jl} + a_k.w_{ik}.q_{ik} \\ -a_k.\sum_{j=1}^{N} w_{jk}.q_{jk} - c_i.b_k.p_{ik} - b_k.p_{ik}.k_i.\sum_{l=1}^{M} a_l + b_k.p_{ik}.k_i.\sum_{l=1}^{M} b_l.p_{il} \\ -b_k.p_{ik}.k_i.\sum_{l=1}^{M}\sum_{j=1,i\neq j}^{N} t_{jl}.p_{jl} - b_k.p_{ik}.w_{ik}.q_{ik} + b_k.p_{ik}.\sum_{j=1,i\neq j}^{N} w_{jk}.q_{jk} + c_i.\sum_{j=1,i\neq j}^{N} t_{jk}.p_{jk} \\ +k_i.\sum_{k} a_k.\sum_{j=1,i\neq j}^{N} t_{jk}.p_{jk} - k_i.\sum_{k} b_k.p_{ik}.\sum_{j=1,i\neq j}^{N} t_{jk}.p_{jk} + k_i.\sum_{k}\left(\sum_{j=1,i\neq j}^{N} t_{jk}.p_{jk}\right)^2 \\ +w_{ik}.q_{ik}.\sum_{j=1,i\neq j}^{N} t_{jk}.p_{jk} - \sum_{j=1,i\neq j}^{N} w_{jk}.q_{jk}.\sum_{j=1,i\neq j}^{N} t_{jk}.p_{jk} \end{array} \right. \tag{11}$$

$$\frac{\partial^2 U_i}{\partial p_{ik}\partial p_{jk}} = \sum_{j=1,i\neq j}^{N} t_{jk} \quad \text{and} \quad \frac{\partial^2 U_j}{\partial p_{ik}\partial p_{jk}} = \sum_{i=1,i\neq j}^{N} t_{ik} \tag{12}$$

A related exact potential function for our proposed game Γ is given as:

$$V(\mathrm{p,q}) = \sum_{i=1}^{N}\sum_{k=1}^{M} \left[\begin{array}{l} -a_k.k_i.\sum_{l=1}^{M} b_l.p_{il} - c_i.b_k.p_{ik} - b_k.p_{ik}.k_i.\sum_{l=1}^{M} a_l + b_k.p_{ik}.k_i.\sum_{l=1}^{M} b_l.p_{il} \\ -b_k.p_{ik}.k_i.\sum_{l=1}^{M}\sum_{j=1}^{N} t_{jl}.p_{jl} - b_k.p_{ik}.w_{ik}.q_{ik} + b_k.p_{ik}.\sum_{j=1}^{N} w_{jk}.q_{jk} - k_i.\sum_{l=1}^{M} b_l.p_{il}.\sum_{j=1}^{N} t_{jk}.p_{jk} \end{array} \right] \tag{13}$$

IV. Numerical Results And Discussion

In our solution algorithm, we assume that only one NP acts at each time step which is chosen in a round-robin fashion. In each time step, the algorithm finds the price value for an NP that maximizes its potential function. Doing so, it utilizes the most recent price decisions of other players, which are found in the previous time step.

In the scenario, we have two NPs and two ASPs, where NP_1 has 2 x 6 MHz of spectrum band (704-710 MHz and 734-740 MHz) that is owned from the FCC Auction 78, and NP_2 has 2 x 6 MHz of band (1856-1862 MHz and 1872-1878 MHz) that is too much for its customer pool and their utilization profiles. NP_2 wants to lease its extra bands that usually remain underutilized to different service providers for making money. All parameters for the second scenario are given in Table I.

TABLE I
PARAMETERS USED IN SCENARIO 2

	NP_1		NP_2				
	ASP_1	ASP_2	ASP_1	ASP_2		NP_1	NP_2
w_{ik}	0.4	0.2	0.15	0.1	c_i	1.4	1.4
w_{jk}	0.04	0.04	0.09	0.09	q_i	0.8	0.8
t_{jk}	0.01	0.08	0.08	0.14	k_i	0.08	0.05
t_{1ik}	0.5	0.5	0.5	0.5			
t_{2ik}	0.5	0.5	0.5	0.5			
b_k	0.1	0.2	0.1	0.2			
a_k	12	12	12	12			
BF_k	1	1	1	1			
LF_k	0.8	0.8	0.8	0.8			

Similar to ASPs, we differentiate the NPs according to their marketing preferences. w_{ik} parameters represents the sensitivity coefficient that reflects the attitude of an ASP towards QoS level variations. NP_1 represents an aggressive spectrum holder that attaches great importance to its QoS level (w_{1k}), compared to NP_2. It also pays much attention to the QoS levels of its opponent (w_{2k}) when defining its spectrum prices. As ASP_1 has a high profile, w_{i1} parameters are set higher than w_{i2} parameters. The two network providers are assumed to have same fixed costs (c_i). t_{1ik} and t_{2ik} parameters are all set equal to 0.5 for the sake of simplicity; but they can always be adjusted to reflect the marketing preferences of NPs.

TABLE II
RESULTS OF THE SCENARIO IN THE EQUILIBRIUM

	NP_1		NP_2	
	ASP_1	ASP_2	ASP_1	ASP_2
p*	3.532	3.372	2.664	2.624
Demand	11.860	11.693	11.769	11.745
Utility	81.323		62.164	

Table II summarizes the results. NP_1 will receive its maximum utility when it offers a spectrum band of 11.860 MHz to ASP_1 with 3.532 as the unit price and 11.693 MHz to ASP_2 with 3.372 as the unit price. Both NPs offer more bands to high profile ASP, ASP_1, with higher prices. As ASP_2 is assumed to be more sensitive to price, it is more reasonable that ASP_2 demands more band from NP_2 (11.745>11.693). Similarly, as ASP_1 is assumed to be more sensitive to quality rather than price, it is more reasonable that ASP_1 demands more band from NP_1 (11.860>11.769). In this scenario, in order to deal with NP_1 with higher prices and higher demand values; NP_2 should decrease its prices to 2.664 and 2.624 for ASP_1 and ASP_2, respectively, to reach its maximum utility value.

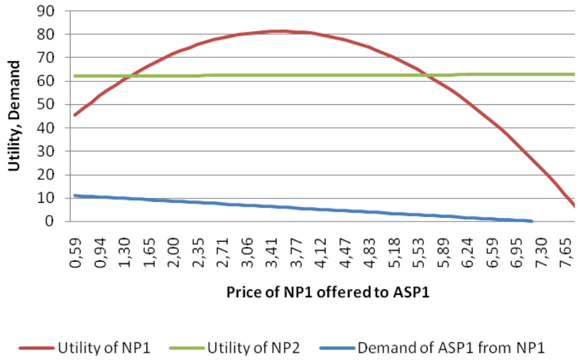

Fig.1. Utility and demand variations as a function of price

Fig. 1 represents the variations of utility function values of NP_1 and NP_2 in respect to NP_1's price offer to ASP_1. We can observe that the prices higher than the equilibrium value have a negative effect on its utility, when the opponents' offers remain the same. Furthermore, we show the negative effect of price increase on the demand in the same figure. The utility function curve verifies the important feature of our model: Through the equilibrium price (3.532), the utility of NP_1 increases with its price increases; however the utility value shows a decreasing trend for the price higher than 3.532.

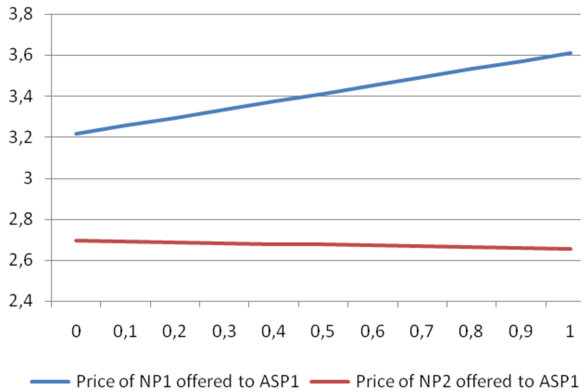

Fig. 2. Price variations as a function of quality

Fig. 2 represents the variations of price offered to ASP_1 from NP_1 and offered to ASP_2 from NP_1 in respect to the quality of NP_1. We assume that the quality level of NP_2 and all the other parameters are fixed. The results show that the price offered to ASP_1 increases with quality increases because the demand to NP_1 increases. The price offer of NP_2 shows a decreasing trend since the demand to NP_2 decreases and NP_2 has to decrease its price so as to keep its demand stable.

V. CONCLUSIONS

Next generation wireless networks are expected to use flexible spectrum sharing techniques for achieving more efficient and fair spectrum usage. In this paper, we consider the framework of short term sub-lease of unutilized spectrum bands to different service providers from the viewpoint of a potential game. In the proposed potential game, the network providers are thought as the players that optimizes a joint objective function, the potential function. Our model assumes that the price of the spectrum band depends both on its own QoS level and on the QoS levels of its opponents. Besides, the second component of our proposed price formulation reflects the demand variations to a network provider: If the demand to a network provider's bands increases, its bands become more expensive, and the band prices of the network providers that have low demand decrease. Furthermore, in our proposed model, we assume that the demand to a network provider decreases as the price of its bands increases, and the demand increases as the prices of its bands decrease. The demand formulation also considers the prices offered by other network providers in the market. As outcome of the game, we calculate the optimum prices of the offered frequency bands given their QoS levels. Our simulation results show that the prices offered to ASPs depend on the QoS level of the offered band as well as on the prices and QoS levels offered by the opponents.

We conclude that the demand models must be chosen with great care, since the choice of its parameters has profound implications for the market equilibrium. The empirical study of how firms in different market positions, with different marketing perspectives should determine its demand function will be one of our future works. The model in this paper captures the interaction among network providers and application service providers. A future work would be a more comprehensive model covering all three layers: Network providers, application service providers and end users.

ACKNOWLEDGMENT

The authors acknowledge the financial support of the Galatasaray University Research Fund.

REFERENCES

[1] R. Rubenstein, "Radios Get Smart", *IEEE Spectrum*, February 2007.
[2] Shared Spectrum Report, available online: http://www.sharedspectrum.com/measurements/download/Ireland_Spectrum_ Occupancy_Measurements_v2.pdf / .
[3] FCC, "The development of secondary markets – report and order and further notice of proposed rule making", *Federal Communications Commission Report*, pp. 103-113, 2003.
[4] FCC, "Promoting efficient use of spectrum through elimination of barriers to the development of secondary markets – third report and order", *Federal Communications Commission Report*, pp. 07-52, 2007.
[5] Qinetiq, "Cognitive Radio Technology, A Study for Ofcom – Vol. 1", available at: http://www.ofcom.org.uk/research/technology/research/emer_tech/cograd/cograd_main.pdf, 2007.
[6] D. Niyato and E. Hossain, "Competitive Pricing for Spectrum Sharing in Cognitive Radio Networks: Dynamic Game, Inefficiency of Nash Equilibrium, and Collusion", *IEEE Journal on Selected Areas in Communications* Vol. 26 No. 1, pp. 192-202, 2008-a.
[7] D. Niyato and E. Hossain, "Market-equilibrium, competitive, and cooperative pricing for spectrum sharing in cognitive radio networks: analysis and comparison", *IEEE Transactions on Wireless Communications* Vol. 7 No. 11, pp. 4273-4283, 2008-b.
[8] M. Bloem, T. Alpcan and T. Başar, "A Stackelberg Game for Power Control and Channel Allocation in Cognitive Radio Networks", in Proc: *2nd International Conference on Performance Evaluation Methodologies and Tools* (321), 2007.
[9] J.O. Neel, J.H. Reed and R.P. Gilles, "Convergence of Cognitive Radio Networks", in Proc: IEEE Wireless Communications and Networking Conference, Vol. 4, pp. 2250-2255, 2004.
[10] A.B. MacKenzie and L.A. DaSilva, *Game Theory for Wireless Engineers*, Morgan & Claypool Publishers, 2006.
[11] J. Nash, "Non-cooperative games", *The Annals of Mathematics*, Vol. 54 No. 2: pp. 286-295, 1951.
[12] D. Monderer and L.S. Shapley, "Potential Games", *Games and Economic Behavior*, Vol. 14, pp. 124-143, 1996.
[13] Y. Zhao, S. Mao, J.O. Neel and J.H. Reed, "Performance Evaluation of Cognitive Radios: Metrics, Utility Functions and Methodologies", *Accepted to IEEE JSAC on Cognitive Radio*, last edited 1/2/08, 2008.
[14] M. Cave, Review of Radio Spectrum Management, for Department of Trade and Industry and HM Treasury, 2002.
[15] T. Başar and G.J. Olsder, *Dynamic Noncooperative Game Theory*, Academic Press, second edition, 1995.