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A Posterior Predictive Model-Based Control Chart for Monitoring Healthcare

Yi-Fan Lin, Peter P. Howley, Frank A. Tuyl

***Abstract*—**Quality measurement and reporting systems are used in healthcare internationally. In Australia, the Australian Council on Healthcare Standards records and reports hundreds of clinical indicators (CIs) nationally across the healthcare system. These CIs are measures of performance in the clinical setting, and are used as a screening tool to help assess whether a standard of care is being met. Existing analysis and reporting of these CIs incorporate Bayesian methods to address sampling variation; however, such assessments are retrospective in nature, reporting upon the previous six or twelve months of data. The use of Bayesian methods within statistical process control for monitoring systems is an important pursuit to support more timely decision-making. Our research has developed and assessed a new graphical monitoring tool, similar to a control chart, based on the beta-binomial posterior predictive (BBPP) distribution to facilitate the real-time assessment of health care organizational performance via CIs. The BBPP charts have been compared with the traditional Bernoulli CUSUM (BC) chart by simulation. The more traditional "central" and "highest posterior density" (HPD) interval approaches were each considered to define the limits, and the multiple charts were compared via in-control and out-of-control average run lengths (ARLs), assuming that the parameter representing the underlying CI rate (proportion of cases with an event of interest) required estimation. Preliminary results have identified that the BBPP chart with HPD-based control limits provides better out-of-control run length performance than the central interval-based and BC charts. Further, the BC chart's performance may be improved by using Bayesian parameter estimation of the underlying CI rate.

***Keywords*—**Average run length, Bernoulli CUSUM chart, beta binomial posterior predictive distribution, clinical indicator, health care organization, highest posterior density interval.

I. Introduction

QUALITY measurement and reporting systems, which incorporate CIs, have become more visible in improving health care and organizational outcomes [1], [2]. CIs measure performance in a clinical setting, including the clinical management and outcome of patient care. When analyzed and reported correctly, CIs can highlight problems within a structure, process or outcomes, allowing for further investigation into any apparent issue to achieve quality improvement. The appropriateness of CIs is demonstrated through their application across the entire health care system, from administration to treatment processes to patient responses [3], across a broad range of clinical fields [4]. CIs are used to assess, compare, and determine the potential to improve care which will bring the long-term benefits of established prevention programs, improved health status and satisfaction among the general population, better trained professionals, and greater accountability within the industry [5].

In Australia, the Australian Council on Healthcare Standards (ACHS) has developed a set of CIs which are reviewed via collaboration with health care organizations (HCOs) and other government bodies. Currently, there are over 300 CIs with more than 800 HCOs participating in the ACHS CI program. There is growing support for quality improvement through CI analysis in Australia, as indicated by the increasing number of HCOs providing data to the ACHS. It is noted that supplying CI data to the ACHS for key services is a requirement of the accreditation process for health care providers in Australia as part of the Evaluation and Quality Improvement Program [6]. Moreover, it provides a national clinical benchmarking service and is comprised of comparative information on the processes and outcomes of health care. Participating HCOs are able to submit indicator data for inclusion in an extensive indicator database. Data are aggregated and analyzed six-monthly, and results are provided in the form of comparative reports. These reports compare results across all contributing HCOs as well as providing a comparison with "peer" HCOs based on a number of variables.

An annual report is also released by the ACHS detailing industry trends for the previous years, where significant differences between public and private HCOs exist, as well as the clinical areas that have excessive event of interest frequencies compared to the expected proportion of occurrences. Expected proportions are obtained from documentation of national standards, internal comparison within the health care provider and external comparison across analogous HCOs and are used to quantify the potential to improve [7], [8]. These reports involve retrospective analysis which provides insight into where quality improvement is required [9]. However, control charts enable HCOs to monitor their performance during the 6-months rather than simply waiting for the retrospective reports.

II. Method

There is large variation in the number of admissions at risk of the event of interest occurring per HCO, with differences in the populations of each HCO depending on demographics and clinical specializations, giving rise to both systematic and sampling variation.

For the Beta-Binomial two-stage hierarchical model, the

Yi-Fan Lin, Peter P Howley, and Frank A Tuyl are with the School of Mathematical and Physical Sciences/Statistics, The University of Newcastle, Callaghan 2308, Australia (e-mail: yi-fan.lin@uon.edu.au, peter.howley@newcastle.edu.au, frank.tuyl@newcastle.edu.au).

proportions for all HCOs in the system are distributed according to $P_i \sim Beta(\mu M,(1-\mu)M)$, where μ is the overall mean CI proportion, and M, $M = (\mu(1-\mu)/\sigma^2)-1$, indicates the spread of proportions among the HCOs and is inversely related to the variance of the proportions between HCOs and σ^2. The observed number of occurrences within each HCO, O_i, is distributed according to $O_i \sim Binomial(D_i, P_i)$, where D_i is the number of admissions at risk of the event of interest occurring in the i[th] HCO, from which the posterior distribution of the proportion for each HCO in the system is obtained (see Fig. 1).

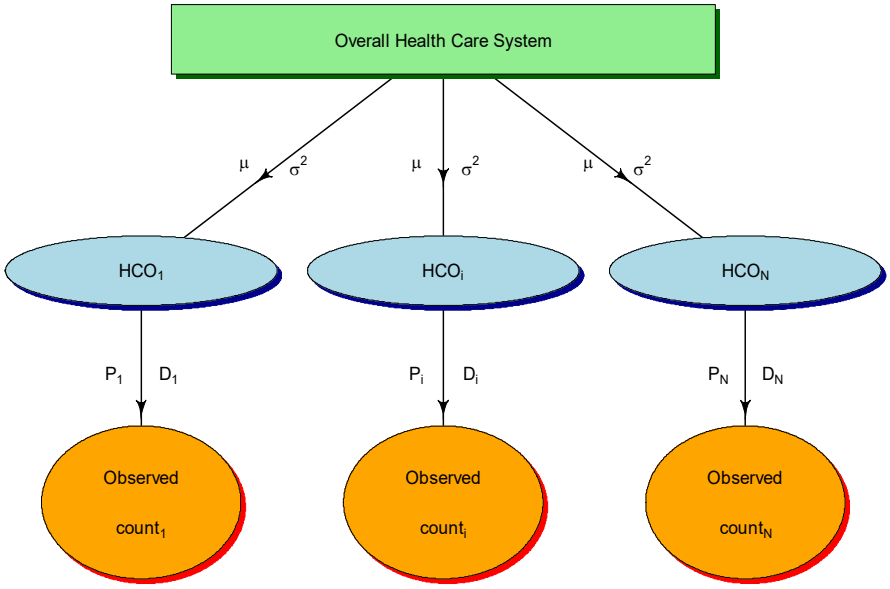

Fig. 1 Two-stage hierarchical model in the health care system

The posterior predictive distribution gives the probability of observing a future occurrence, O_i^F, from the total number of future admissions, D_i^F, and is obtained by combining the posterior distribution with the sampling distribution for the future number of occurrences of the event of interest at the i[th] HCO, $O_i^F \sim Binomial(D_i^F, P_i^*)$, where P_i^* is the posterior distribution. The posterior predictive distribution is given as in (1).

$$P_i^{**} \triangleq P(O_i^F | D_i^F, O_i, D_i, \mu, M) = \frac{D_i+M-1}{D_i^F+D_i+M-1} \frac{\binom{O_i^F+O_i+M\mu-1}{O_i^F}\binom{D_i^F+D_i-O_i^F-O_i+M(1-\mu)-1}{D_i^F-O_i^F}}{\binom{D_i^F+D_i+M-2}{D_i^F}} \quad (1)$$

A. BBPP Control Chart

A Bayesian control chart has been proposed for real-time monitoring of CI data in the health care system based upon the BBPP distribution, and tested for the Australian environment. Fig. 2 demonstrates construction of the BBPP chart's limits. Using (1), the control intervals, or limits, are obtained for a given D_i^F.

This research investigates two different interval estimates, the traditional "central" interval and the HPD interval, for the given Type I error rate, α. For a given D_i^F, the BBPP probabilities of $O_i^F \subset [0, \ldots, D_i^F]$ are obtained via (1). The control limits are obtained according to (2)-(6) for a given type I error, α (5% was applied in this research).

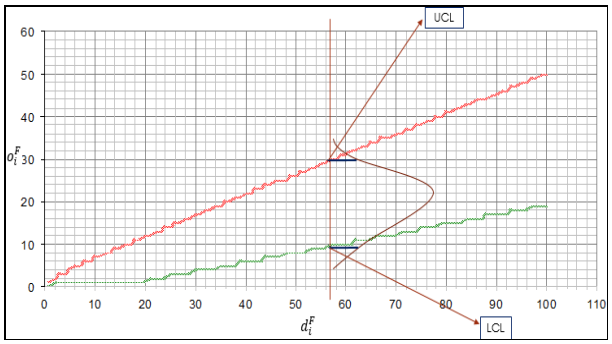

Fig. 2 BBPP-based limits for control chart

Central intervals (based on equal probability in each of the two tails of the BBPP distribution): for each D_i^F, the lower limit, $LCL_{D_i^F}$, is determined as in (2) and (3).

$$\sum_{o_i^F=0}^{LCL_{D_i^F}} p^{**} < \frac{\alpha}{2} \quad (2)$$

and

$$\sum_{o_i^F=0}^{(LCL_{D_i^F})+1} p^{**} \geq \frac{\alpha}{2} \quad (3)$$

For each D_i^F, the upper limit, $UCL_{D_i^F}$, is determined as in (4) and (5).

$$\sum_{o_i^F=0}^{(UCL_{D_i^F}-1)} p^{**} < 1-\frac{\alpha}{2} \quad (4)$$

and

$$\sum_{o_i^F=0}^{UCL_{D_i^F}} p^{**} \geq 1-\frac{\alpha}{2} \quad (5)$$

HPD intervals (allowing for unequal probability in the two tails of the BBPP distribution, aiming for *shortness*). For each D_i^F, find $LCL_{D_i^F}$ and $UCL_{D_i^F}$ such as in (6).

$$\min_{0\leq LCL_{D_i^F}\leq UCL_{D_i^F}\leq D_i^F}\left(UCL_{D_i^F}-LCL_{D_i^F}\middle|\sum_{o_i^F=LCL_{D_i^F}}^{UCL_{D_i^F}} p^{**}\geq 1-\alpha\right) \quad (6)$$

By repeating this procedure for each D_i^F, limits for the chart are obtained. The control limits provide a range within which the HCO is expected to perform in the future if the proportion of admissions having the event of interest remains unchanged.

The control chart provided to an individual HCO for a given CI can exhibit both sets of control limits. In order to monitor their performance, HCOs will plot the cumulative sum (run) of the events of interest for each additional admission in a chart with these limits as given in (7).

$$O_{d_i^F,i} = O_{d_i^F-1,i} + X_{d_i^F,i} \text{ where } O_{0,i} = 0 \quad (7)$$

For every additional admission, the run is extended horizontally by one unit and will rise by one value when an event of interest occurs. If the run falls on or outside, the control limits the HCO should investigate to determine if this was due to natural variation or due to a shift in the underlying proportion.

B. Parameter Space Considered

In the field of ACHS CIs data, although the BBPP charts are widely applicable outside of the health care setting, a factorial design experiment was conducted to test the two charts over the following set of parameter values,

1. Overall mean proportion of admissions having the event of interest:

$$\mu \in \{0.01, 0.05, 0.1, 0.15, \dots, 0.5\}$$

2. Overall HCO system variance:

$$\sigma^2 \in \{0.01, 0.05, 0.1, 0.15, \dots, 0.5\}$$

3. Number of admissions at risk of the event of interest occurring for the i[th] HCO:

$$D_i \in \{10, 20, 30, 50, 100, 200\}$$

4. The proportion of admissions having the event of interestfor the i[th] HCO:

$$P_i \in \{0.01, 0.03, 0.05, \dots, 0.49\}$$

5. Observed number of occurrences of the event of interest in the i[th] HCO:

$$O_i \sim Bin(D_i, P_i)$$

6. Number of future admissions at risk of the event of interest occurring for the i[th] HCO:

$$D_i^F \in \{1,2, \dots, 1000\}$$

7. Change in the underlying proportion of admissions having the event of interest:

$$\%_{change} \in \{1\%, 5\%, 10\%, 15\%, \dots, 50\%\}$$

C. Applying Simulated Data

Based on the assumption of Bernoulli trials with underlying proportion P_i, the in-control and out-of-control data can be generated as follows.

1. Process always in-control: The in-control data are based on in-control proportion P_{0i}.

In control data:

$$p(X_{t,i}=1) = P_{0i};\ p(X_{t,i}=0) = 1-P_{0i} \text{ where } t = 1, 2, 3, \dots, D_i^F$$

where t= 1, 2, 3,…, D_i^F.

2. Process immediately out-of-control: The process is changed immediately, from the 1[st] point, and $X_{t,i}$ follows the Bernoulli distribution with out-of-control proportion, P_{1i}.

Out of control data (proportion changed immediately):

$$p(X_{t,i}=1) = P_{1i};\ p(X_{t,i}=0) = 1-P_{1i} \text{ where } t = 1, 2, 3, \dots, D_i^F$$

where t= 1, 2, 3,…, D_i^F.

We simulated 1000 samples with 1000 Bernoulli trails on the same condition. The Bernoulli CUSUM and posterior predictive-based control charts were applied to these simulated samples in order to obtain run lengths (RLs). From RLs, average RLs (ARLs) were obtained.

D. Evaluation of the Methods

In order to correctly and quickly detect changes in the underlying CI proportion when a change exists, and not detect a non-existing change, both in-control and out-of-control ARLs were calculated.

For each combination, there are 1000 RLs (1000 replications) for each scheme, namely in-control RLs (inRLs) and out-of-control immediately RLs (outRLs). Instead of regular means, it seemed to be appropriate to use trimmed means (5% trimmed) to obtain ARLs (or TARLs).

Multiple linear and logistic regression models were applied to test the effects of the simulated parameters on the paired differences in in-control expected ARLs. A saturated model was first constructed and stepwise model selection was applied. Furthermore, tree based models, including regression and classification trees, were applied to explore the performance of the charts throughout the parameter space.

An effective prospective monitoring tool for CI data should

operate so as to not be excessively demanding on the resources either required to chase a false alarm or being used while a true change in the parameters remains undetected. Therefore, in judging the performance of charts, a high in-control ARL and low out-of-control ARL are desired.

E. Charts in Application

This study considered 1000 replications (for a given set of parameters) to support reliable estimates of the ARLs for each chart. In context, practically significant differences are more important than statistical significance since with a large enough number of replications, even the smallest of differences may be identified as statistically significant.

If one considers a day as the maximum time within which a true change should be detected, the equivalent number of patients per day for the BBPP chart needs to be identified. This will depend upon the number of patients at risk of the event of interest at i^{th} hospital. Assuming that admissions are uniformly distributed across the six-month period, daily admissions would range from 0.05 ($10/182.5$) to 1.10 ($200/182.5$) due to the range of D_i for the CIs. Given the relatively small value for even the upper value of this range, it was determined that the practical significance threshold should be set at zero for this research.

III. Results

A. In-Control ARLs

Table I shows the in-control expected difference ARLs, $eARLs^{in}_{BC-BBPP_{CI}}$ and $eARLs^{in}_{BC-BBPP_{HPD}}$, indicating the BC having far greater ARLs than the BBPP charts. However, Table II shows the $eARLs^{in}_{BBPP_{CI}}$ and $eARL^{in}_{BBPP_{HPD}}$, which reflect good performance by each.

All five main effects and almost all two-ways interaction terms were important according to the final (logistic) regression models.

The most important factors are the proportion of admissions having the event of interest for the ith HCO (P_i), the change in underlying proportion of admissions having the event of interest ($\%_{change}$) and the number of admissions at risk of the event of interest occurring for the ith HCO (D_i).

TABLE I
A Summary of the Difference in In-Control Expected ARLs

	Mean	Median	SD	Min	Max	IQR
$eARLs^{in}_{BC-BBPP_{CI}}$	238.5	207.8	351.7	-612.2	993.0	516.2
$eARLs^{in}_{BC-BBPP_{HPD}}$	302.2	276.5	324.3	-516.4	993.0	485.4

TABLE II
A Summary of the Individual In-Control Expected ARLs

	Mean	Median	SD	Min	Max	IQR
$eARLs^{in}_{BBPP_{CI}}$	452.6	523.4	215.8	7.0	795.9	284.7
$eARLs^{in}_{BBPP_{HPD}}$	388.5	434.3	183.7	7.0	718.3	235.1
$eARLs^{in}_{BC}$	691.8	681.7	203.8	153.6	1000.0	301.7

In summary, classification tree analysis showed that the BBPP charts have a longer in-control ARL within the following parameter space:

- Medium or larger change in underlying proportion of admissions having the event of interest ($\%_{change} > 25\%$).
- Medium or higher HCO proportion of admissions having the event of interest on i^{th} hospital (P_i >0.24).
- Medium or smaller the number of admissions at risk of the event of interest occurring (D_i <75).
- Medium or larger overall mean proportion of admissions having the event of interest (μ>0.3).

B. Out-Of-Control ARLs

Table III shows that the BBPP charts are superior to BC chart in the out-of-control expected difference ARLs, $eARLs^{out}_{BC-BBPP_{CI}}$ and $eARLs^{out}_{BC-BBPP_{HPD}}$.

TABLE III
A Summary of the Difference in Out-Of-Control Expected ARLs

	Mean	Median	SD	Min	Max	IQR
$eARLs^{out}_{BC-BBPP_{CI}}$	167.9	137.4	380.4	-727.1	992.9	550.2
$eARLs^{out}_{BC-BBPP_{HPD}}$	239.3	218.0	352.5	-614.9	993.1	520.9

In summary, classification tree analysis showed that the BBPP charts have a shorter out-of-control ARLs within the parameter space defined by the following:

- Medium or smaller change in underlying proportion of admissions having the event of interest ($\%_{change}$ < 38%).
- Medium or smaller HCO proportion of admissions having the event of interest on ith HCO (P_i<0.3).
- Medium or larger the number of admissions at risk of the event of interest occurring (D_i>40).
- Medium or smaller overall mean proportion of admissions having the event of interest (μ<0.32).

IV. Discussion

In monitoring of health care data, the non-Bayesian CUSUM charts have been widely applied. However, the health care system may benefit from utilizing the inherent hierarchical nature of the data and thus Bayesian based models and charts which account for such.

Given that neither chart consistently outperforms the other at detecting changes in the underlying proportion across the entire parameter space explored, it may be feasible to consider using a particular chart for a given CI in given the existing knowledge about CIs (μ and σ^2) and their likely values (D_i, P_i and $\%_{change}$). In this research, we have identified a parameter space in which the BBPP control chart detected changes in the underlying proportion more quickly than the CUSUM alternative (see section B of results). Moreover, it is acknowledged that the HCOs' future performance is measured relative to this previous performance (underlying proportion). If the HCO had been performing poorly last period, then performing within these limits would indicates a repeated poor performance. This implies that the HCO may not realize that they were performing poorly, relative to the health care system when they fall within the limits. In order to address this issue, limits can also be calculated based on the expected level of performance for the size of HCO. This enables the HCO to consider its performance relative to the average HCO of its size.

Finally, some CIs may not require such a highly-sensitive practical significance threshold. In that case, detecting changes within a one-week period may suffice, and the practical significance threshold could be set accordingly. In practice, HCOs are encouraged to set the practical significance threshold according to the targets and timeframes within their own organization.

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