International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:8, No:7, 2014

A Parametric Study on Lateral Torsional Buckling of European IPN and IPE Cantilevers

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***Abstract*—**IPN and IPE sections, which are commonly used European I shapes, are widely used in steel structures as cantilever beams to support overhangs. A considerable number of studies exist on calculating lateral torsional buckling load of I sections. However, most of them provide series solutions or complex closed-form equations. In this paper, a simple equation is presented to calculate lateral torsional buckling load of IPN and IPE section cantilever beams. First, differential equation of lateral torsional buckling is solved numerically for various loading cases. Then a parametric study is conducted on results to present an equation for lateral torsional buckling load of European IPN and IPE beams. Finally, results obtained by presented equation are compared to differential equation solutions and finite element model results. ABAQUS software is utilized to generate finite element models of beams. It is seen that the results obtained from presented equation coincide with differential equation solutions and ABAQUS software results. It can be suggested that presented formula can be safely used to calculate critical lateral torsional buckling load of European IPN and IPE section cantilevers.

***Keywords*—**Cantilever, IPN, IPE, lateral torsional buckling.

I. Introduction

LATERAL torsional buckling is one of the important failure modes for slender beams. When the magnitude of the load acting on the beam reaches to a critical level, the beam experiences global buckling in which the beam is twisted and laterally buckled. This case is called lateral torsional buckling, which is abbreviated as LTB (Fig. 1).

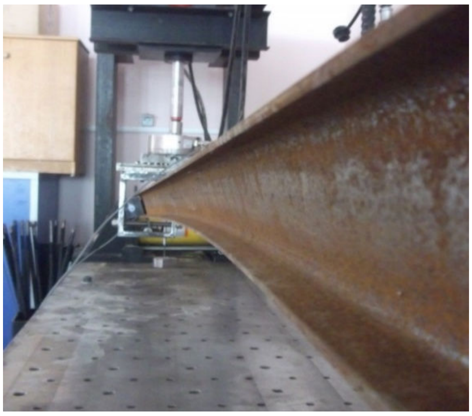

Fig. 1 Lateral torsional buckling (LTB)

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LTB is effective on laterally unrestrained beams which are loaded so as to be under bending about their strong axis. Calculating the smallest load causing LTB of the beam, which is known as critical LTB load, is a hard problem to solve. Especially for the sections which are under greater warping moments during torsion such as I sections, presentation of a closed form solution is not practical. Many studies are conducted in search of this solution. Challamel and Wang presented exact stability criteria for the lateral torsional buckling of cantilever strip beam under combined intermediate and end transverse point loads. In the study, the two-dimensional stability criteria are expressed in closed-form solutions using Bessel functions [1]. Goncalves presented a geometrically exact beam formulation to calculate lateral torsional buckling loads of Euler–Bernoulli/Vlasov thin-walled beams with deformable cross-section [2]. Benyamina et al. investigated the lateral torsional buckling behavior of doubly symmetric web tapered thin-walled beams. In the study, Ritz's method is deployed in order to derive the algebraic equilibrium equations and an analytical formula is proposed for the lateral buckling strength of web tapered beams [3]. Hodges and Peters investigated cantilever strip and I beams. By using energy method and a comparison function for the twisting angle of the beam, an approximate closed-form expression for the lateral torsional buckling load is presented [4]. Andrade et al. extended the domain of application of well-known 3-factor formula to I section cantilevers. Cantilever I beams with equal or unequal flanges and fully built-in or free to warp at the support are considered. Concentrated load at free end and uniformly distributed load cases are taken into account [5]. Zhang and Tong presented a comparative study on the flexural-torsional buckling of thin-walled cantilevers. Authors presented explicit solutions for predicting critical loads of doubly symmetric cantilevers [6]. As for well-known steel structure codes; AISC provides a recommendation for critical LTB load of cantilevers to stay on the safe side [7]. However, EC3 doesn't provide an equation to calculate critical LTB load [8]. Since LTB is related to many parameters including material and section properties, searching for another general closed-form solution for cantilever I beams will probably lead to involve with complex integrals and series. For this reason, scope of the study is narrowed down to widely used I shapes to present a practical equation.

In this paper, a simple parametric equation to calculate critical LTB load of European IPN and IPE section cantilevers which are subjected to loads acting at shear center are presented. First, differential equation of LTB is solved numerically for IPN and IPE cantilevers with various lengths. Then simple expressions for section properties of IPN and IPE sections are obtained. Finally, a slenderness ratio and a parametric equation which is related to mentioned slenderness is presented by using the results of numerical solutions. Parameters are calculated for concentrated load at free end, uniformly distributed load, combination of concentrated load at free end uniformly distributed load and constant moment cases which are given in Figs. 2 (a), (b), (c) and (d), respectively.

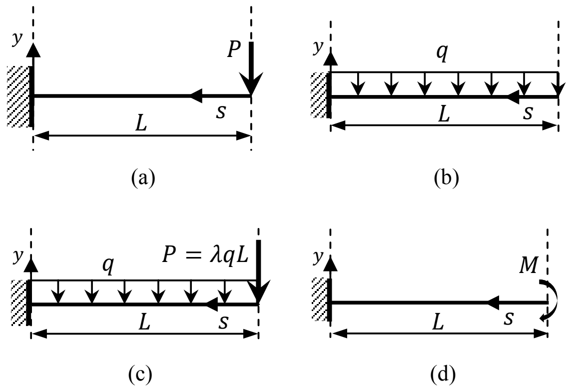


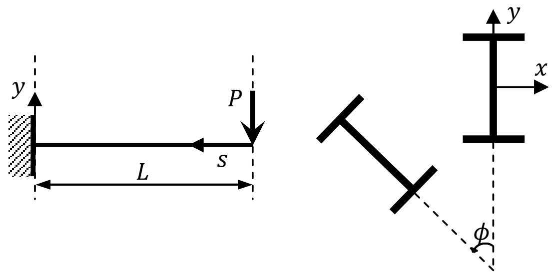
Fig. 2 Considered loading cases (a) Concentrated load at free end, (b) uniformly distributed load, (c) Combination of concentrated load at free end and uniformly distributed load, (d) Constant moment

In Fig. 2 (a), L is cantilever length, s is the distance from free end and P is the magnitude of the concentrated load acting at free end. In Fig. 2 (b), q is the magnitude of uniformly distributed load. In Fig. 2 (c), magnitude of concentrated load at free end is calculated by multiplying qL by coefficient λ. Finally, in Fig. 2d, M is the magnitude of bending moment. In the study, P, q, $q+\lambda qL$ and Mindicates the loading cases introduced in Figs. 2 (a), (b), (c) and (d), respectively. As stated before, parameters are calculated for mentioned loading cases. However, parameters can be calculated for any loading case and application domain of the formula can be extended.

II. Lateral Torsional Buckling of Cantilever I Beams

LTB of cantilever I beams is more complex than LTB of simply supported I beams. Free end of the cantilever beam, is laterally unrestrained and free to twist, causes variation of the twisting angle function with respect to loading case, section properties and cantilever length. With this drawback, in addition to subtle differential equation solution, it is also hard to apply energy methods to obtain an equation for critical LTB load of cantilever I beams as a result of complex twisting angle form.

For easy statement of the problem, a cantilever I beam which is loaded by a concentrated load acting at shear center is given in Fig. 3.



(a) (b)

Fig. 3 Cantilever I beam

Fig. 3 (b) is the cross section of stable and buckled beam at point s which is given in Fig. 3 (a). In Fig. 3 (b),x and y are strong and weak axis of the section, respectively andϕ is twisting angle. Differential equation of LTB is presented by Timoshenko for the loading case given in Fig. 3, which is introduced as P case in the study, as follows[9].

$$\frac{d^4\phi}{ds^4}-\frac{C}{C_1}\frac{d^2\phi}{ds^2}-\frac{P^2s^2}{EI_yC_1}\phi=0 \tag{1}$$

In (1), E is elasticity modulus, C is torsional rigidity calculated by multiplying shear modulus (G) by torsional constant (I_t), C_1 is warping rigidity calculated by multiplying elasticity modulus by warping coefficient (I_w) andI_y is moment of inertia about weak axis. Similarly, differential equation of LTB for other loading cases can be written by neglecting small terms. Equations (2)–(4) are differential equations of LTB for q, $q+\lambda qL$ and M cases, respectively [10].

$$\frac{d^4\phi}{ds^4}-\frac{C}{C_1}\frac{d^2\phi}{ds^2}-\frac{q^2s^4}{4EI_yC_1}\phi=0 \tag{2}$$

$$\frac{d^4\phi}{ds^4}-\frac{C}{C_1}\frac{d^2\phi}{ds^2}-\frac{q^2s^2(2\lambda L+s)^2}{4EI_yC_1}\phi=0 \tag{3}$$

$$\frac{d^4\phi}{ds^4}-\frac{C}{C_1}\frac{d^2\phi}{ds^2}-\frac{M^2\phi}{EI_yC_1}=0 \tag{4}$$

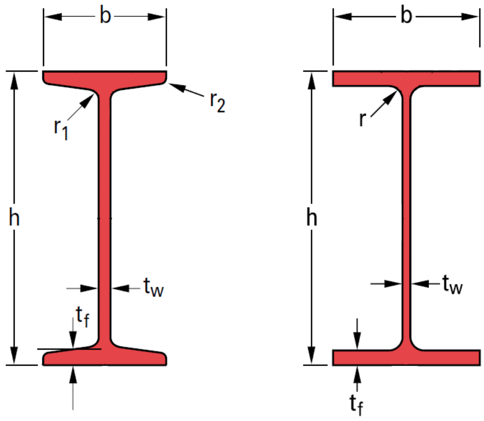
It can be seen from (1)–(4) that in addition to loading case, lateral torsional buckling of I sections are related to cantilever length, elasticity modulus, shear modulus, torsional constant, warping coefficient and moment of inertia about weak axis. To determine LTB load, differential equation of considered loading case should be solved for critical load. It is easier to apply numerical methods to solve these differential equations instead of searching for a closed form solution. Even numerical solution of mentioned equations is hard and not practical. In this study, finite differences method [11] is used for numerical solution of the mentioned equations. Calculations are made by dividing each beam to 500 finite elements. It is seen from the calculations that, dividing beams

International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:8, No:7, 2014

into more finite elements does not cause significant change in results. Result obtained by numerical evaluation of differential equation of LTB is used to calculate parameters which are required to calculate critical LTB loads of IPN and IPE cantilever beams with presented equation.

III. Section Properties

Because of small geometry differences, section properties of IPN and IPE sections should be considered separately. Flange thickness of IPN sections is not constant and corners of flanges have fillets. IPE sections have flanges thickness of which is constant and flange corners of IPE sections are sharp (Fig. 4) [12].



(a) (b)

Fig. 4 (a) IPN section geometry, (b) IPE section geometry

In Fig. 4, h is section height, b is flange width, t_f is flange thickness and t_w is web thickness. In Fig. 4 (a), r_1 is flange-web connection fillet radius and r_2 is flange corner fillet radius. In Fig. 4 (b), r shows flange-web connection fillet radius. Section properties which are effective on LTB can be expressed in terms of section height, flange width and flange thickness with a good approximation (see Table I).

TABLE I
Parametric Section Properties

Section Property	IPN[a]	IPE[b]
Moment of inertia about weak axis (I_y)	$0.143t_fb^3$	$0.167t_fb^3$
Torsional constant (I_t)	$0.874bt_f^3$	$0.850bt_f^3$
Warping constant (I_w)	$0.032t_fh^2b^3$	$0.038t_fh^2b^3$

[a]Considered IPN sections are IPN100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300, 320, 340, 360, 380, 400, 450, 500, 550 and 600.
[b]Considered IPE sections are IPE100, 120, 140, 160, 180, 200, 220, 240, 270, 300, 330, 360, 400, 450, 500, 550 and 600.

In Table I, reference sections which are used to obtain presented relations are given. By assuming Poisson's Ratio as $\nu = 0.3$ for structural steel, shear modulus can be written as given below as a common expression.

$$G = \frac{E}{2(1+\nu)} = \frac{E}{2(1+0.3)} = 0.385E \tag{5}$$

In order to present an equation for critical LTB load of cantilever IPN and IPE sections, section properties are calculated by the equations given in Table I and shear modulus is taken $0.385E$(5).

IV. Parametric Study

Parametric study is conducted by following a similar procedure which is presented for single angle sections by Aydin et al. [13] and Aydin [14]. For IPN and IPE sections, a simple approximate equation which doesn't require the use of section properties included in (1)–(4) can be written by using the simplifications introduced in Chapter II. To calculate critical LTB load with the equation given below, only required parameters are cantilever length (L), elasticity modulus (E), section height (h), flange width (b) and flange thickness (t_f).

$$P_{cr} = q_{cr}L = M_{cr}/L = \frac{K_b}{C_k}E\left(\frac{t_f^2}{h}\right)^2 \tag{6}$$

In (6),P_{cr}, q_{cr} and M_{cr}represents critical concentrated load, uniformly distributed load and moment, respectively. Critical LTB load type (P_{cr}, q_{cr}orM_{cr}) obtained from (6) varies due to considered loading case. For $q+\lambda qL$case, obtained critical LTB load is in terms of q_{cr}, which is the critical value of uniformly distributed load. Critical value of the concentrated load at free end can be calculated by $\lambda q_{cr}L$ where λ is the concentrated load multiplier.

In (6), only K_b parameter changes according to loading case. Other parameters are material and section properties. $K_b$is a dimensionless coefficient which depends on loading case and section properties. For each loading case a different K_b curve should be used. Also, for $q+\lambda qL$case, form of the K_b curve varies by value of λ. Therefore, for different combinations of concentrated load at free end and uniformly distributed load, separate K_b curves should be given. $K_b$can be calculated approximately by a slenderness ratio C_k given below.

$$C_k = \left(\frac{Lt_f}{hb}\right)^2 \tag{7}$$

In this study,K_b parameters are calculated for P, q, $q+0.5qL$, $q+1.0qL$ and M cases. However, K_b parameter for any loading case can be calculated with respect to C_k. In Fig. 5, C_k-K_b graph is given for IPN sections.

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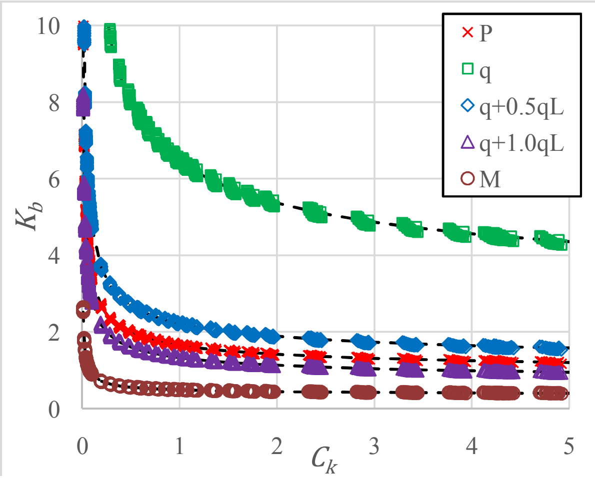

Fig. 5 C_k-$K_b$relation for IPN sections

Similarly, Fig. 6 presents the C_k-K_b relation for IPE sections.

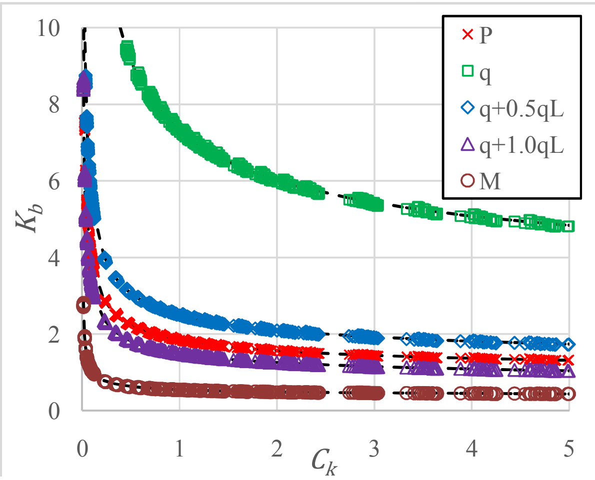

Fig. 6 C_k-$K_b$relation for IPE sections

In graphs presented in Figs. 5 and 6, series show K_b parametersfor cantilever beams with various C_k values. These curves are obtained by numerical solution of differential equation of LTB for considered loading cases. Instead of reading K_b value from plots, it is more accurate to calculate by an approximate function. Curves drawn with dashed lines over series are fitted functions. These fitted functions which include C_k as parameter are given in Table II.

At the end of the parametric study, calculating critical LTB load of IPN or IPE beams can be summarized in 3 steps. First, C_k value should be calculated. Then $K_b$parameter can be found for the considered loading case by the approximate functions given in Table II. Finally, critical LTB load can be found by substituting calculated parameters in (6).

TABLE II
K_b PARAMETERS

Loading Case	IPN	IPE
P	$0.61C_k^{-0.56} + 1.04C_k^{-0.05}$	$0.73C_k^{-0.56} + 1.11C_k^{-0.05}$
q	$2.92C_k^{-0.55} + 3.53C_k^{-0.07}$	$3.50C_k^{-0.55} + 3.80C_k^{-0.07}$
q+0.5qL	$0.87C_k^{-0.56} + 1.35C_k^{-0.06}$	$1.05C_k^{-0.56} + 1.45C_k^{-0.06}$
q+1.0qL	$0.51C_k^{-0.56} + 0.82C_k^{-0.06}$	$0.61C_k^{-0.56} + 0.88C_k^{-0.06}$
M	$0.12C_k^{-0.62} + 0.37C_k^{-0.02}$	$0.15C_k^{-0.62} + 0.40C_k^{-0.02}$

V. COMPARISON

Results obtained by presented equation are compared to differential equation solutions and ABAQUS software results. Differential equations are solved numerically by finite differences method. ABAQUS models are generated with S8R5 shell elements. This rectangular element has 8 nodes and 5 degrees of freedom at every node. For easy modeling, flange corner fillets of IPN beams and flange-web connection fillets of both IPN and IPE sections are neglected. Also, flange thickness of IPN sections is assumed to be constant. Solid finite element models with exact section geometries are generated and results of mentioned solid and shell finite element models are compared. It is seen from the results that, above stated assumptions don't have a significant effect on results. Therefore, it is proper to use simplified shell finite element models for comparison. Buckled form of an ABAQUS shell model is given in Fig. 7.

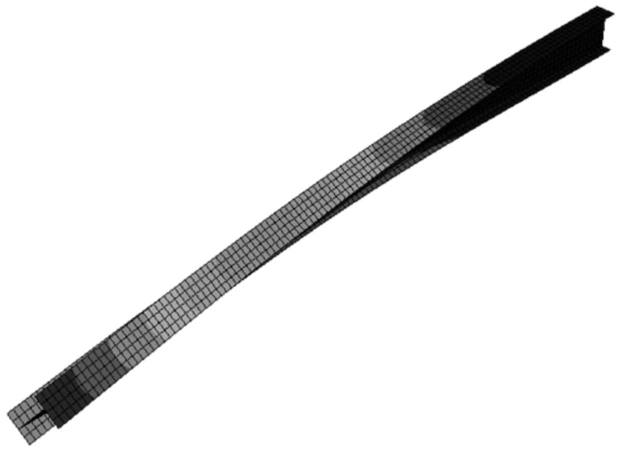

Fig. 7 ABAQUS shell model

Four sections are selected to compare the results. These sections are IPN100, IPE200, IPN300 and IPE400. Comparison of IPN100 section is made for M case. IPE200, IPN300 and IPE400 sections are compared for$P$, $q$and $q + 0.5qL$ cases, respectively. By this comparison, it is possible to see the accuracy of the presented equation on small, medium and large sections and under different loading cases.

C_k-critical LTB load relation of IPN100 cantilever beam for M case is given in Fig. 8.

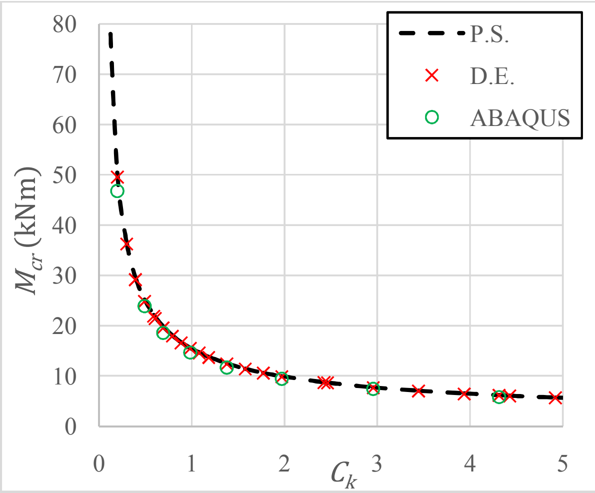

Fig. 8 C_k-critical LTB load graph of IPN100 cantilever beam for M case

In Fig. 8, P.S. series indicate the results obtained by presented formula. D.E. indicates the differential equation solutions which are introduced in Chapter II. Finally, ABAQUS indicate the results obtained by ABAQUS software.

C_k-critical LTB load relation of IPE200 cantilever beam for P case is given in Fig. 9.

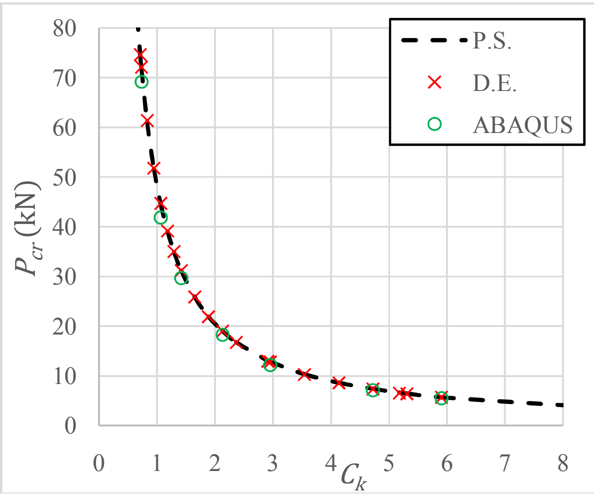

Fig. 9 C_k-critical LTB load graph of IPE200 cantilever beam for P case

C_k-critical LTB load relation of IPN300 cantilever beam for q case is given in Fig. 10.

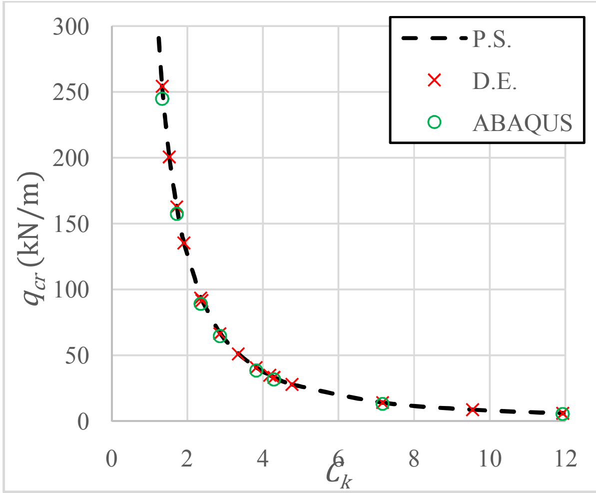

Fig. 10 C_k-critical LTB load graph of IPN300 cantilever beam for q case

Finally, C_k-critical LTB load relation of IPE400 cantilever beam for $q + 0.5qL$ case is given in Fig. 11.

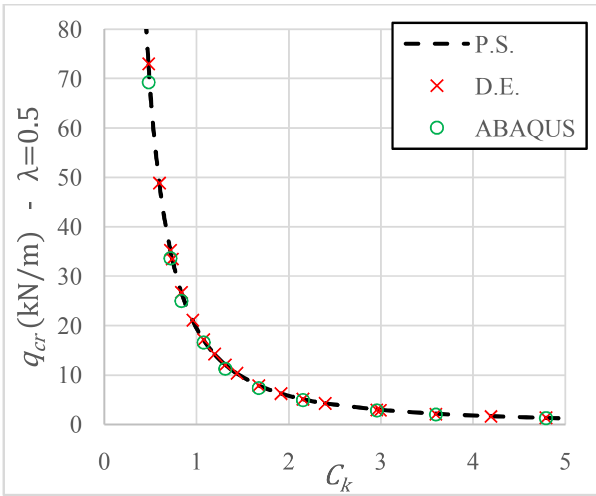

Fig. 11 C_k-critical LTB load graph of IPE400 cantilever beam for $q + 0.5qL$ case

It can be seen from Figs. 8-11 that the results obtained by presented formula are in accordance with differential equation solutions and ABAQUS results. It is concluded that critical LTB loads of European IPN and IPE cantilever beams can be safely calculated by the presented equation.

VI. Conclusion

In this paper, a parametric equation to calculate LTB load of cantilever IPN and IPE sections are presented. Section properties of mentioned sections are simplified and LTB load parameter is calculated for concentrated load at free end, uniformly distributed load, combination of concentrated load at free end and uniformly distributed load and constant moment cases. However, K_b parameter of presented equation can be calculated for any loading case. The results obtained by presented equation, differential equation solutions and ABAQUS software models are compared. It is seen that results obtained by presented equation perfectly coincide with differential equation and ABAQUS solutions. Lateral torsional buckling loads of IPN and IPE section cantilevers can be determined by presented formula and can be safely used in design procedures.

ACKNOWLEDGMENT

Author thanks to Prof. M. Ruhi Aydin for his valuable papers which are inspirations of this study.

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