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A Numerical Model for Studying Convectional Lifting Processes in the Tropics

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***Abstract*—**A simple model for studying convectional lifting processes in the tropics is described in this paper with some tests of the model in dry air. The model consists of the density equation, the wind equation, the vertical velocity equation, and the temperature equation. The model domain is two-dimensional with length 100 km and height 17.5 km. Plan for experiments to investigate the effects of the heating surface, the deep convection approximation and the treatment of velocities at the boundaries are discussed. Equations for the simplified treatment of moisture in the atmosphere in future numerical experiments are also given.

***Keywords*—**Numerical weather prediction, Finite differences, Convection lifting.

I. Introduction

CONVECTIONAL lifting is associated with surface heating of the air at the ground surface. If enough heating occurs, the mass of air becomes warmer and lighter than the air in the surrounding environment. A simplified numerical model for studying the behavior of air over a heated surface in the troposphere in a tropical climate is described in this paper. The model equations are derived from the fundamental system of partial differential equations of computational of fluid dynamics [1]. The air pressure is eliminated using the ideal gas equation.

II. Model Description

There is one horizontal dimension x, the vertical dimension z, and the time dimension t. The variables are located on a staggered grid with stretched grid spacing in the vertical dimension z and constant grid spacing in the horizontal dimension x. The model is designed for two dimensions. The variables are located on a staggered grid with stretched grid spacing in the vertical dimension z and constant grid spacing in the horizontal dimension x. vertical dimension z and constant grid spacing in the horizontal dimension x. In this model the molecular viscosity terms are omitted, the body forces are friction at the Earth's surface in the horizontal momentum equation and gravity in the vertical momentum equation; the Coriolis force is omitted; heating and cooling of the air occur at the Earth's surface; kinetic energy and potential energy in the temperature equation are omitted, and the deep convection approximation is used.

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The deep convection approximation [2] is

TABLE I
LIST OF SYMBOLS

Symbol	Units	Description
x	m	Horizontal distance
z	m	Vertical height
t	s	Time
g	$m/s2$	Acceleration of gravity
R	J/kgK	Gas constant for air
c_v	J/kgK	Specific heat of air
ρ	$kg/m3$	Air density
u	m/s	Horizontal velocity
w	m/s	Vertical velocity
T	K	Temperature
z_0	m	Roughness length of the surface
q_s	$W/m2$	Surface heating rate per unit area
i, k		Horizontal and vertical cell indices
ρ_v	$kg/m3$	Water vapor density
m_c	$kg/m3$	Condensed cloud water per unit volume
L	J/kg	Latent heat of condensation of water

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=-\frac{w}{\rho^0}\frac{\partial \rho^0}{\partial z}, \tag{1}$$

where $\rho^0(z)$ is calculated from a steady background temperature profile $T^0(z)$ and the assumption of hydrostatic equilibrium in the undisturbed atmosphere.

The steady background temperature profile is an approximation to the annual mean upper air temperatures at Bangkok [3] represented by the formula

$$T^0(z)=302-0.00675z, \tag{2}$$

where $T^0(z)$ is in kelvins and z is in meters.

A. Governing Equations

1) The density equation

The density forecast equation derived from the continuity equation [4]:

The density equation without the deep convection approximation

$$\frac{\partial \rho}{\partial t} = -u\frac{\partial \rho}{\partial x} - w\frac{\partial \rho}{\partial z} - \rho\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right), \tag{3}$$

and the density equation with the deep convection approximation is

$$\frac{\partial \rho}{\partial t} = -u\frac{\partial \rho}{\partial x} - w\frac{\partial \rho}{\partial z} - \rho\frac{w}{T^0}\left(\frac{\partial T^0}{\partial z} + \frac{g}{R}\right). \tag{4}$$

2) The wind equation

The wind forecast equation derived from the horizontal momentum equation [4] is

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - w\frac{\partial u}{\partial z} - \frac{RT}{\rho}\frac{\partial \rho}{\partial x} - R\frac{\partial T}{\partial x} - \frac{0.16u|u|}{\left[\ln\left(\frac{0.5\Delta z}{z_0}\right)\right]^2 \Delta z}. \tag{5}$$

The last term is horizontal friction, which is applied only in the layer of air of thickness Δz at the Earth's surface where the roughness length is z_0.

3) The vertical velocity equation

The vertical velocity equation derived from the vertical momentum equation [4] is

$$\frac{\partial w}{\partial t} = -u\frac{\partial w}{\partial x} - w\frac{\partial w}{\partial z} - \frac{RT}{\rho}\frac{\partial \rho}{\partial z} - R\frac{\partial T}{\partial z} - g. \tag{6}$$

4) The temperature equation

The temperature forecast equation derived from the energy equation [4] is

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - w\frac{\partial T}{\partial z} - \frac{RT}{c_v}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + \frac{q_s}{c_v \rho \Delta z}. \tag{7}$$

The surface heating term q_s is applied only in the layer of air at the Earth's surface.

B. Finite difference set up

The domain of the model is divided into a 25×100 array of cells. The horizontal width of each cell is $\Delta x = 1$ km. A vertical coordinate s will be used in accordance with the transformation $z = 75s + 25s^2$ in order to give thin layers at the Earth's surface and thick layers at the top of the troposphere [5].

The model variables are evaluated at points on an Arakawa-C grid. The model variables ρ and T are in the center of each cell, the horizontal velocity is on the left side of the cell, and the vertical velocity is on the bottom of the cell as shown in Fig. 1.

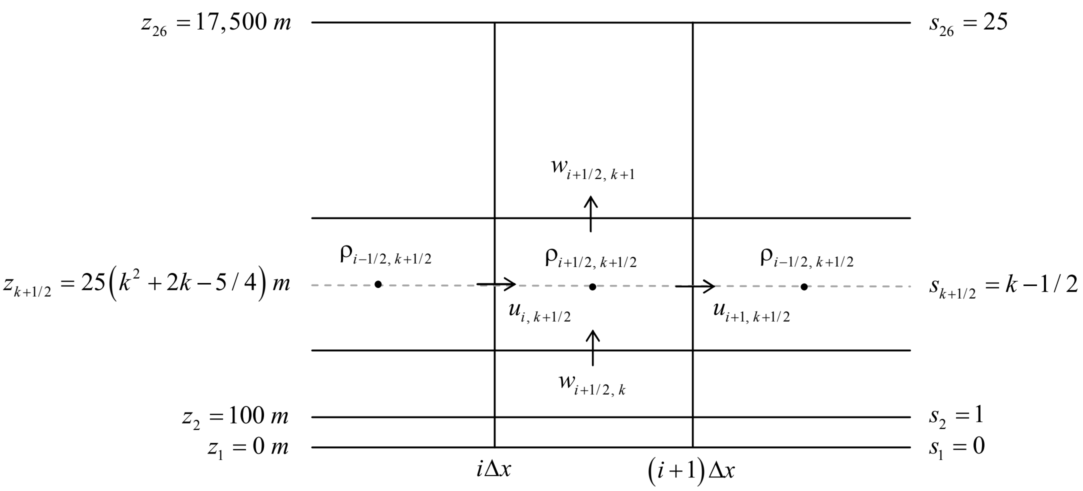

Fig. 1 The grid of the simple model with the stretched grid

The leapfrog method [6] is used to calculate the model variables at time $n+1$ from the values at time n. The Euler method is used for the first time step. First and second order finite difference approximations are used in the modeling of space derivatives.

The initial values of the model variables in each cell are functions of the height of the cell above the Earth's surface, but are constant along the horizontal rows of cells. The temperature equation [5] is given by

$$T^{0}_{k+0.5} = 302 - 0.00675\left(25k^2 + 50k - 31.25\right), \qquad (8)$$

where $k = 1, 2, \ldots, 25$ and $w = u = 0$ everywhere.

We assume w is zero everywhere in the initial state, and that the initial values of ρ and T satisfy the hydrostatic equation.

III. Numerical Experiments

To represent surface heating as in a city heat island, in the middle of the domain we put

$$q_s = 500\exp\left(-\frac{(i-50.5)^2}{50}\right). \qquad (9)$$

Preliminary results showed that $\Delta t = 0.3$ second is too large a time step to give stable results for one hour without the deep convection approximation, and $\Delta t = 0.5$ second is too large with the deep convection approximation [7].

Two different experiments model were done to study the effects of the heated surface:

Case1. Without the deep convection approximation.
Case2. With the deep convection approximation.

Boundary conditions in both cases:
The value of ρ and T constant in the cells $i = 1, i = 100$ and $k = 25$.
The value of u constant in the cells $i = 1, i = 101$ and $k = 25$.
The value of w constant in the cells $i = 1$, $i = 100$, $k = 1$ and $k = 26$.

The object of these experiments will be to find the largest value of Δt that gives stable results for model times of the order one hour, and study the wind that is produced.

IV. Results and Discussion

Preliminary results have shown that the model without the deep convection approximation is not suitable while the deep convection approximation gives reasonable results in numerical experiments on the vertical movement processes in the tropics as shown in Fig. 2. Moreover they have shown that the Δt value can be up to 0.4 seconds

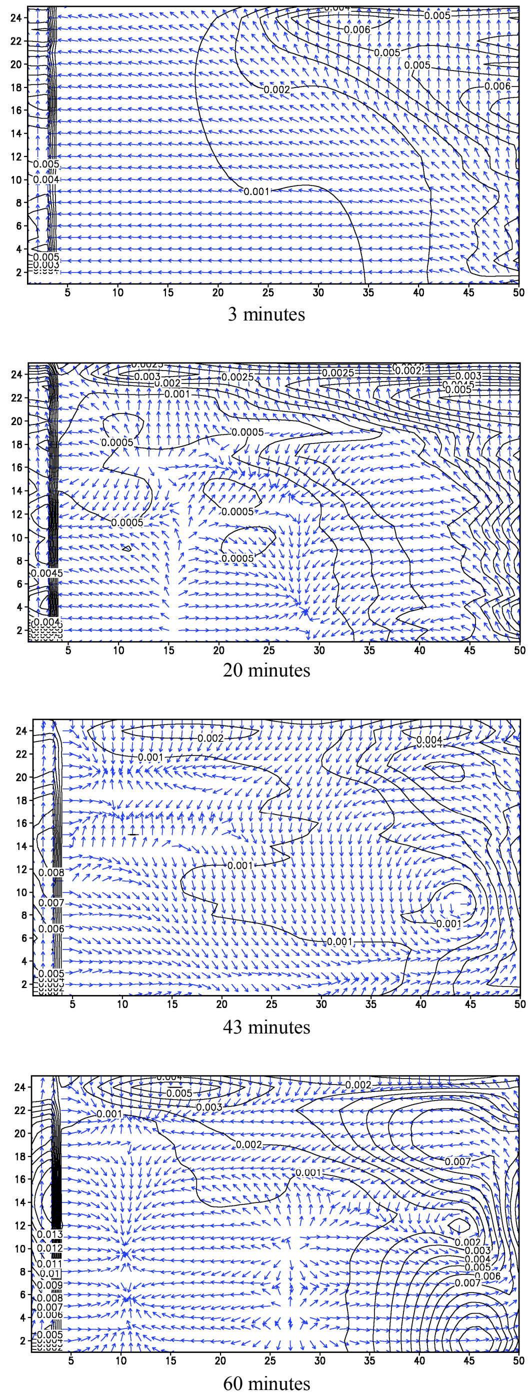

Fig. 2 The half results are shown lifting wind is produced by the heating at the middle surface for $\Delta t = 0.4$ seconds

In addition, relaxation at the boundary, reducing and expanding the length of the domain, increasing and decreasing the heating at the surface and smoothing in time have little effect on the model without the deep convection approximation.

V. Future Work

Future developments of this model will include water vapor and condensed cloud water. The equations to be use are as follows. They are based on the simplifying assumptions that the thermal properties of moist air are equal to those of dry air with deep convection approximation. The water vapor density ρ_s is obtained as a function of temperature by integrating the Clausius-Clapeyron equation [8] with a constant latent heat of condensation to give

$$\rho_s = \frac{A}{R_v T} e^{-B/T}, \tag{10}$$

where $R_v = 461.5\ Jkg^{-1}K^{-1}$, $A = 2.53 \times 10^8\ kpa$, and $B = 5.42 \times 10^3\ K$.

A. Unsaturated air

The temperature equation

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - w\frac{\partial T}{\partial z} - \frac{RT}{c_v}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + \frac{q_s}{c_v \rho \Delta z} \tag{11}$$

The water vapor equation

$$\frac{\partial \rho_v}{\partial t} = -u\frac{\partial \rho_v}{\partial x} - w\frac{\partial \rho_v}{\partial z} - \frac{\rho_v w}{T^0}\left(\frac{\partial T^0}{\partial z} + \frac{g}{R}\right) \tag{12}$$

The condensed cloud water equation

$$m_c = 0 \tag{13}$$

B. Saturated air

The temperature equation

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - w\frac{\partial T}{\partial z} + \frac{W}{1+EQ}, \tag{14}$$

where $W = -\frac{RT}{c_v}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + \frac{q_s}{c_v \rho \Delta z}$,

$$E = \frac{A(B-T)}{R_v T^3} e^{-B/T},$$

$$Q = \frac{L}{c_v \rho}.$$

The water vapor equation

$$\frac{\partial \rho_v}{\partial t} = -u\frac{\partial \rho_v}{\partial x} - w\frac{\partial \rho_v}{\partial z} + \frac{EW}{1+EQ} \tag{15}$$

The condensed cloud water equatio

$$\frac{\partial m_c}{\partial t} = -u\frac{\partial m_c}{\partial x} - w\frac{\partial m_c}{\partial z} - (\rho_v + m_c)\frac{w}{T^0}\left(\frac{\partial T^0}{\partial z} + \frac{g}{R}\right) - \frac{EW}{1+EQ}$$

(16)

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