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A Novel Pareto-Based Meta-Heuristic Algorithm to Optimize Multi-Facility Location-Allocation Problem

Vahid Hajipour, Samira V. Noshafagh, and Reza Tavakkoli-Moghaddam

***Abstract*—**This article proposes a novel Pareto-based multi-objective meta-heuristic algorithm named non-dominated ranking genetic algorithm (NRGA) to solve multi-facility location-allocation problem. In NRGA, a fitness value representing rank is assigned to each individual of the population. Moreover, two features ranked based roulette wheel selection including select the fronts and choose solutions from the fronts, are utilized. The proposed solving methodology is validated using several examples taken from the specialized literature. The performance of our approach shows that NRGA algorithm is able to generate true and well distributed Pareto optimal solutions.

***Keywords*—**Non-dominated ranking genetic algorithm, Pareto solutions, Multi-facility location-allocation problem.

I. Introduction

NOWADAYS, among different classifications of optimization methodologies, developing multi-objective evolutionary algorithms to optimize the problems with conflicting objectives has found considerable attention. Among different classifications of discrete location–allocation models, this article attempts to find Pareto-solution of a specific location-allocation problem (LAP) within queuing framework. Current et al. [1] introduced eight basic facility location models namely p-median, p-center, p-dispersion, set covering, maximal covering, fixed charge, hub, and minisum. Moreover, several models and different solving methodologies have been proposed in [2]-[4]. As a main purpose of manufactures and service providers, customer satisfaction is mirrored as customer-desired characteristics [5]-[7].

The LAP combined with other aspects of industrial and operational management such as queuing theory has received considerable attentions in the literature. Wang et al. [8] proposed a facility location model within the M/M/1 queuing system. Berman and Drezner [9] developed facility location model within M/M/m queuing framework in which more than one server can be located at each facility. Berman et al. [10] introduced a similar model with more constraints on the lost demand in which the number of facilities is minimized. Pasandideh and Niaki [11] proposed a bi-objective facility location problem within M/M/1 queuing framework on the p-median problem. They modeled the bi-objective problem using the desirability approach and solved the model employing a genetic algorithm. Hajipour and Pasandideh [12] proposed a multi--objective facility location problem within $M^{[x]}/M/1$ queuing framework. They also presented a genetic algorithm which integrated by Lp-metric approach to find efficient solutions. Pasandideh et al. [13] proposed two parameter-tuned meta-heuristic algorithms to solve the multi-objective facility location-allocation problem.

Besides, several evolutionary algorithms have since been developed which combine rules and randomness mimicking natural phenomena. These phenomena include biological evolutionary processes for example evolutionary algorithm [14], [15], genetic algorithm (GA) [16], [17], animal behavior [18], [19], the physical annealing process [20], and the musical process of searching for a perfect state of harmony [21]. Many researchers have recently studied these meta-heuristic algorithms to solve various optimization problems.

Recently, non-dominated ranked genetic algorithm (NRGA) as another multi-objective evolutionary algorithm is proposed by Al Jaddan et al. [22] to solve multi-objectives optimization models. While the implementation of NRGA is limited in the literature, therefore, in this paper, we presented NRGA to solve multi-objective facility location problems with competing objectives which presented by Hajipour and Pasandideh [12]. Computational results show the robustness of the NRGA method to obtain well-distributed optimal solutions.

The structure of the remainder of the paper is as follows. In the next section, the concept and definitions of the multi-objective optimization problem is illustrated. Then, next section analyzed the results and comparisons. Finally, in Section V, conclusions are made and possible future research works are suggested.

II. Concept and Definition

Many real-world problems involve simultaneous optimization of several objectives. In this type of optimization problems, there is usually no single optimal solution. Hence, all objectives are considered when a set of alternative solutions are optimal in the wider sense, which no other solutions in the search space are superior to them. These solutions are known as Pareto-optimal.

In order to clarify the point, some basic multi-objective concepts are required to be reviewed [23]. Consider a multi-objective model with a set of conflict objectives as follow:

Vahid Hajipour is with Young Researchers Club ,Qazvin Branch, Islamic Azad University, Qazvin, Iran (phone: +989123829766; fax: +982813358026; E-mail: v.hajipour@qiau.ac.ir).

$$\begin{aligned} & f(\vec{x}) = \left[f_1(\vec{x}), ..., f_m(\vec{x}) \right] \\ & S.t. \\ & g_i(\vec{x}) \le 0 \ , \ \ i = 1, 2, ..., I \ , \ \vec{x} \in X \end{aligned} \tag{1}$$

where $\vec{x}$ denotes m-dimensional vectors that can get real, integer, or even Boolean values and X is the feasible region. Then, for a minimization model, we say solution $\vec{a}$ dominates solution $\vec{b}\,(\vec{a}, \vec{b} \in X)$ if:

(I) $f_i(\vec{a}) \le f_i(\vec{b}), \ \ \forall i = 1, 2, ..., m$

(II) $\exists i \in \{1, 2, ..., m\} : f_i(\vec{a}) < f_i(\vec{b})$

Moreover, a set of solutions that cannot dominate each other is called Pareto solutions set or Pareto front. The main goal of multi-objective problems are stated as: (I) appropriate convergence and (II) appropriate diversity; which formed a good Pareto front. Accordingly, Pareto-based algorithms aim to achieve the Pareto optimal front during the evolution process. The Pareto optimal front is called to the front of the last iteration of the algorithms.

III. The Proposed NRGA

Solving the proposed non-linear integer programming model is difficult, so the use of meta-heuristic methods is justified. The steps involved in proposed NRGA of this research are as follows.

A. Initialization

The parameters of the proposed NRGA are: (1) Probability of crossover (P_c); (2) Population size $(nPop)$ that is the number of solutions for sustaining in each generation; (3) Number of iteration in each temperature (nIt); and (4) Probability of mutation (P_m). In this research, to generate initial population, the random generation policy has been utilized.

B. The Coding Process

In order to increase the feasibility of the generated solutions, a coding scheme is proposed. In encoding scheme, numbers of required facilities associated with allocation of the customers to the facilities are decision variables that must be considered in the solution representation. In order to satisfy some constraints, representation is formed in three parts: (I) number of customer nodes (M) is indicated by first part of the representation as a $1 \times M$ vector. Each member is assigned a random number between zero and one; (II) number of facility nodes (N) is indicated by second part of the representation as a $1 \times N$ vector. Similarly, each member is assigned a random number between zero and one; and (III) the third part is consisted a random number between one and the maximum member of on-duty servers (V). After representation, the decoding process of the representation is considered in a backward order. When a random number is generated, the first three genes of the second part of the representation is selected and after sorting, each costumer is allocated to one of the active facilities.

C. Main Loop of NRGA

NRGA is a new multi-objective genetic algorithm to find feasible Pareto front solutions. NRGA is similar to NSGA-II with the difference that in the selection operation the roulette wheel strategy is employed [24]. In NRGA, a fitness value representing rank is assigned to each individual of the population. In this regard, two features ranked based roulette wheel selection including: (I) select the fronts and (II) choose solutions from the fronts, are used. The selection probability of fronts, P_f , and the selection probability of solutions, P_{fs} , are obtained using (2) and (3).

$$P_f = \frac{2 \times Rank_f}{NF \times (NF + 1)} \quad ; \ f = 1, ..., NF \tag{2}$$

$$P_{fs} = \frac{2 \times Rank_{fs}}{NS_f \times (NS_f + 1)} \quad ; \ f = 1, ..., NF \quad s = 1, ..., NS \tag{3}$$

where NF and NS_f are the number of fronts and the number of solutions in front f, respectively. Equation (22) ensures that a front with highest rank has the highest probability to be selected. Similarly, based on (23), solutions with more crowding distance are assigned higher selection probability. In this respect, sort population according to fast non domination sorting operator and then best solutions from first ranked of population are chosen. Following this, individuals of each front are ranked based on their crowding distance operator. Therefore, each individual in population has a two tiers ranked that the first one shows the index of the front of that individual and second one shows rank of the individual among the selected front. As mentioned, two tiers ranked based on roulette wheel selection are applied in which first tier to select the front and the other one to select solution from the front.

D. The Operators of NRGA

In this paper, using a user-specified crossover probability the continuous uniform crossover is used [24]. This crossover method guarantees the legality of the offspring.

The mutation operator like crossover operator selects parts of the chromosome to mutate. The swap mutation operator was used here [24]. In swap operator two positions are selected randomly and their contents are swapped.

Initialize Population P
Generate random population − size N
Evaluate objectives values and constraints
Calculate the rank of objectives values, $R_{f_i}, (i = 1, 2, ..., k)$ for each solution, $R_{f_i} \in [1 - N]$
Calculate the rank of the sum of the constraints violation R_g for each solution $R_g \in [(N+1) - (2N)]$
Convert the constrained problem to unconstrained one using the equation (8) for each objective function, for each solution in P
Assign Rank (level) Based on Pareto dominance – sort
Calculate the crowding distance between members on each front
Generate offspring Population Q from P
{Ranked based Roulette Wheel Selection
Recombination and Mutation
Evaluate objectives values and constraints}
for $g = 1$ to G **do**
for each member of the combined population ($P \cup Q$)
Calculate the rank of objectives values, $R_{f_i}, (i = 1, 2, ..., k)$ for each solution in the combined population $P \cup Q, R_{f_i} \in [1 - 2N]$
Calculate the rank of the sum of the constraints violation R_g for each solution in the combined population $P \cup Q, R_g \in [(2N+1) - (4N)]$
Convert the constrained problem to unconstrained one using the equation (8) for each objective function, for each solution in P
Assign Rank (level) based on Pareto – sort
Calculate the crowding distance between members on each front
end for
(elitist) Select the members of the combined population based on least dominated N solution to make the population P of the next generation. Ties are resolved by taking the less crowding distance.
Calculate the rank of objectives values, $R_{f_i}, (i = 1, 2, ..., k)$ for each solution, $R_{f_i} \in [1 - N]$.
Calculate the rank of the sum of the constraints violation R_g for each solution $R_g \in [(N+1) - (2N)]$.
Convert the constrained problem to unconstrained one using the equation (8) for each objective function, for each solution in P.
Assign Rank (level) Based on Pareto dominance sort.
Calculate the crowding distance between members on
Q = Create next generation from P { .
| { Ranked based Roulette Wheel Selection.
Recombination and Mutation.
Evaluate objective values and constraints } | .
end for

Fig. 1 Pseudo code of proposed NRGA [22]

E. Stopping Criteria

The algorithm is stopped after a predetermined number of iterations.

At end, to clarify the trend of the proposed NRGA, Pseudo code of NRGA is represented in Fig. 1.

IV. Results

To evaluate the performances of the proposed NRGA, four standard metrics of multi-objective algorithms including diversity (D), spacing (S), and number of Pareto solution (NOS) are applied [25].

As mentioned above, the proposed multi-objective algorithm is applied to solve the multi objective facility location problems in the literature [12]. The experiments are implemented on 20 test problems. The demand rate of service requests from customer batch node i follows a uniform distribution, i.e., $\lambda_i \approx Uni[2,15]$. The service rate for server j follows a uniform distribution in [65, 95] as well, i.e., $\mu_i \approx Uni[65,95]$. The travelling time t_{ij} is calculated as a proportion of the Euclidean distance among customer batch i and potential facility j and follows a uniform distribution in the interval [65, 95]. The batch size is random variable following a geometric distribution with parameter 0.5, i.e., $S \approx Geometric(0.5)$. The fixed costs of locating and cost of adding one unit to system's capacity are related to service rate for each size of problem. Fixed cost of establishing facility j at potential node j follows a uniform distribution in the interval [100,500], i.e., $C_j \approx Uni(100,500)$. The other parameters are $\alpha = 0.5$, $\beta = 0.95$. The input parameters of the NRGA including P_c, P_m, *nPop*, and *nIt* are set on 0.8, 0.2, 25, and 100, respectively.

The algorithm compare with non-dominated sorting genetic algorithm (NSGA-II) to demonstrate capability of the proposed algorithm to solve the multi-objective optimization problems. The result analysis show that in NOS metric both algorithms work same; while, in spacing and diversity metrics, the proposed NRGA perform better performance. To clarify the results, Figs. 2-4 represent the provided results, graphically.

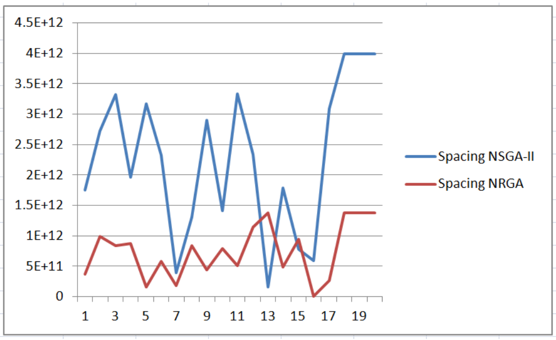

Fig. 2 Comparisons of NRGA and NSGA-II on Spacing

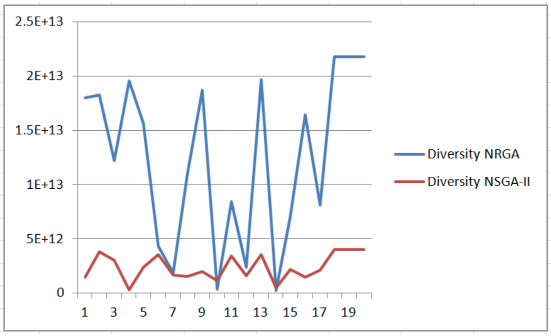

Fig. 3 Comparisons of NRGA and NSGA-II on Diversity

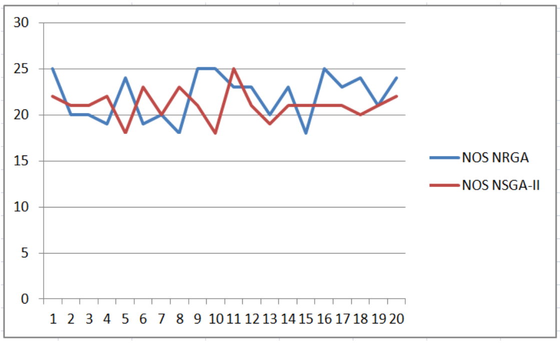

Fig. 4 Comparisons of NRGA and NSGA-II on NOS

V. Conclusion and Future Research

In this paper, a Pareto-based multi-objective meta-heuristic algorithm named NRGA is proposed to solve multi-facility location-allocation problem. The proposed NRGA is justified using several numerical illustrations taken from the specialized literature. The performance of our approach shows that NRGA algorithm is able to work better than NSGA-II especially in terms of diversity and spacing as two common standard metrics for comparing multi-objective optimization problems. As future research, one can develop other Pareto-based multi-objective meta-heuristic algorithm meta-heuristic and compared it with our proposed NRGA according to standard metrics.

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