

# A novel Iterative Approach for Phase Noise Cancellation in Multi-Carrier Code Division Multiple Access (MC-CDMA) Systems

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**Abstract**— The aim of this paper is to emphasize and alleviate the effect of phase noise due to imperfect local oscillators on the performances of a Multi-Carrier CDMA system. After the cancellation of Common Phase Error (CPE), an iterative approach is introduced which iteratively estimates Inter-Carrier Interference (ICI) components in the frequency domain and cancels their contribution in the time domain. Simulation are conducted in order to investigate the achievable performances for several parameters, such as the spreading factor, the modulation order, the phase noise power and the transmission Signal-to-Noise Ratio.

**Keywords**— Inter-carrier Interference, Multi-Carrier Code Division Multiple Access, Orthogonal Frequency Division Multiplexing, Phase noise.

## I. INTRODUCTION

MC-CDMA and OFDM techniques have attracted a great deal of attention in the last few years in the areas of broadcasting and wireless communications ([1],[2]). They are currently under investigation in order to be introduced in the 4<sup>th</sup> generation of mobile communication systems. The aim of the latter is to address several challenges arising in the field of Software Digital Radio (SDR): supporting different air interface standards, operating in multiple environments and adapting to several radio access techniques. Therefore, a high degree of flexibility is required for the analog radio frequency (RF) front-end, which should support a wide range of carrier frequencies and a large tuning range in the Voltage Controlled Oscillator (VCO). However, due to the difficulty of integrating the RF front-end and the digital-oriented back-end on a single chip, it is necessary to relax the specifications of the VCO phase noise and correct the RF imperfections in the baseband part.

One of the main advantages of OFDM and MC-CDMA techniques resides in their ability to combat multi-path fading without the need for complex equalization techniques. Another advantage is the high spectral efficiency achieved by mapping the modulated data onto several orthogonal carriers, with the conjunction of high-order modulations like M-QAM.

Therefore, the resulting signal becomes greatly sensitive to additive channel noise and carrier phase noise. Fortunately, this major drawback can be mitigated by an appropriate receiver structure. Several studies were conducted in order to model and alleviate the effect of phase noise in an OFDM receiver ([3]-[6]). In this paper, we propose a novel iterative procedure for the correction of the phase noise induced by imperfect oscillators on the MC-CDMA signal, based on a low-complexity estimation of frequency components. The paper is organized as follows: in section II, we describe the simulated MC-CDMA baseband transceiver and characterize the phase noise induced on the transmitter side. We then describe the proposed iterative method to alleviate its effect in the receiver. Simulation results of phase noise correction are shown and discussed in sections III and IV.

## II. MC-CDMA SYSTEM DESCRIPTION

Fig. 1 represents the general structure of a baseband MC-CDMA transmitter and receiver, in the presence of phase noise and additive channel noise. After the spreading of M-QAM mapped symbols, pseudo-random interleaving is applied to randomize the correlated errors due to the bursty nature of phase noise. The resulting symbols are then assigned to different sub-carriers by means of the IFFT. At the receiver side, phase-error correction is applied at the output of the M-QAM demodulator.

As for the spreading operation, it can be realized through two possible configurations:

A – After M-QAM mapping, the N symbols are spread by N orthogonal Hadamard codes. After combination of the resulting spread sequences, each resulting chip is assigned a sub-carrier frequency between the available N sub-carriers. In such a configuration, no pilots are sent among the transmitted symbols.

B – The N symbols are divided into groups of  $n_c$  M-QAM symbols (Fig. 2). Thus,  $n_c$  Hadamard codes are used to spread the symbols inside each of the  $n_g$  group, leading to  $n_g$  combined sequences of length  $n_c$  chips each and a set of inserted (non-spread) pilots.

The following notations are used for the rest of the paper:

- $\{c_k\}$ ,  $k = 0, \dots, n_c-1$  is a set of orthogonal Hadamard codes of length  $n_c$ .
- N is the number of sub-carriers.
- M is the number of constellation points in the M-ary QAM modulation.
- S is the set of binary transmitted data (of length  $N \cdot \log_2(M)$ ).

Manuscript received November 5, 2004.

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- $Y$  is the output of the M-QAM modulator (vector of length  $N$ ).
- $X_i$  is the interleaver input.
- $x$  is the Inverse Fourier Transform of vector  $X$ .
- $\phi$  is the phase noise.
- $r$  is the received signal vector, distorted by phase noise and additive channel noise.

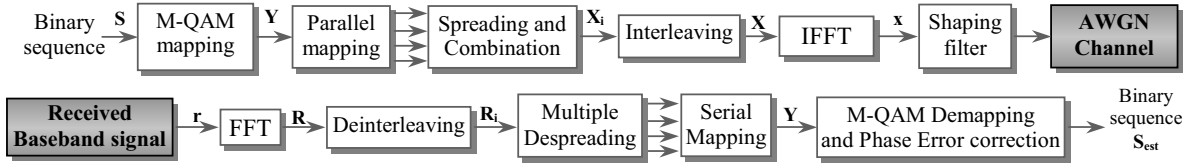


Fig. 1. Baseband part of the MC-CDMA conventional transceiver.

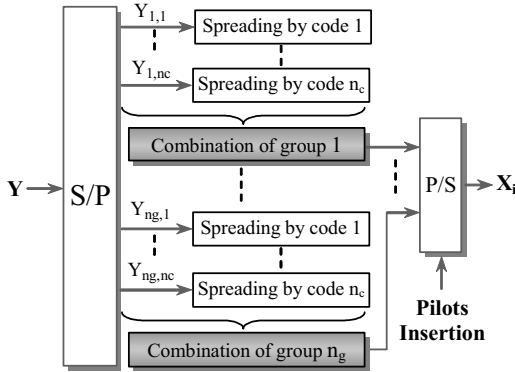


Fig. 2. Spreading and combination of the M-QAM symbols before interleaving and orthogonal frequency multiplexing.

The spread signal can be written as:  $X_{i,n} = \sum_{k=0}^{n_c-1} X_k \cdot c_{k,n}$ , (1)

where  $n$  is chip index (except for pilot symbols). As for the despreading, it can be realized by:

$$y_k = \frac{1}{N} \sum_{n=0}^{n_c-1} R_{i,n} \cdot c_{k,n}^*, k = 0, \dots, n_c - 1, \quad (2)$$

$c_{k,n}^*$  is the complex conjugate of code  $c_{k,n}$ .

The IFFT output is:  $x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j \frac{2\pi nk}{N}}$ ,  $n = 0, \dots, N-1$ . (3)

Assuming a frequency non-selective channel and perfect channel equalization, the received signal can be expressed, in the absence of non-linear distortion, by:

$$r_n = x_n \cdot e^{j\phi_n} + w_n, n = 0, \dots, N-1, \quad (4)$$

where  $w_n$  is the discrete additive white Gaussian noise (AWGN). The input to the deinterleaver is:

$$R_k = \sum_{n=0}^{N-1} r_n e^{-j \frac{2\pi nk}{N}}, k = 0, \dots, N-1. \quad (5)$$

Assuming small phase error samples [3] and using

that  $\sum_{n=0}^{N-1} e^{j \frac{2\pi(m-k)n}{N}} = \begin{cases} 0, m \neq k \\ N, m = k \end{cases}$ , we can write:

$$R_k = X_k + \frac{j}{N} X_k \sum_{n=0}^{N-1} \phi_n + \frac{j}{N} \sum_{m=0}^{N-1} X_m \sum_{n=0}^{N-1} \phi_n e^{j \frac{2\pi(m-k)n}{N}} + \sum_{n=0}^{N-1} w_n e^{-j \frac{2\pi nk}{N}}, \quad (6)$$

where the last term represents the FFT of the AWGN channel. Let  $I$  be the FFT of phase noise  $\phi$ . The FFT output

$$\text{is: } R_k = X_k (1 + jI_0) + j \sum_{m=0}^{N-1} X_m I_{k-m} + \sum_{n=0}^{N-1} w_n e^{-j \frac{2\pi nk}{N}}. \quad (7)$$

The first term shows a constant phase rotation for all sub-carriers and is called "Common Phase Error" or CPE. The second term contains the contributions, on each sub-carrier  $k$ , of all sub-carriers  $m \neq k$  (no longer orthogonal due to the phase noise) and models the "Inter-Carrier Interference" or ICI. On the other hand, the phase noise in the receiver's mixer can be modeled by a Wiener process with a linearly increasing

$$\text{variance through time [5]: } \theta(t) = \int_0^t n(\tau) d\tau, \quad (8)$$

$n(t)$  being a centered white Gaussian noise with variance  $\sigma^2$ . The power spectral density (PSD) of the process  $e^{j\theta(t)}$

$$\text{exhibits a Lorentzian shape [7]: } S(f) = k \cdot \frac{B}{B^2 + f^2}, \quad (9)$$

where  $k$  is a constant and  $B$  is the phase noise bandwidth. Due to its decreasing spectrum, only a limited number of phase noise components are significant and need to be estimated for phase noise cancellation. Let  $d$  be the approximation order for this estimation. Thus, the number of estimated coefficients is  $2d+1$ :  $\{I_0, I_1, I_2, \dots, I_d, I_{N-d}, \dots, I_{N-2}, I_{N-1}\}$ .

The novel approach introduced in our work is represented in Fig. 3. Phase error correction is performed in two different steps: CPE correction followed by iterative ICI correction. Since CPE is constant for all sub-carriers, it can be corrected by a proper phase shift of the signal constellation, which can be estimated using pilots inserted within a certain number of

$$\text{sub-carriers: } 1 + j\tilde{I}_0 = \frac{E[R_k X_k]}{E[X_k^2]} = \exp(j\bar{\phi}), \quad (10)$$

where  $E[R_k X_k]$  is the cross-correlation between the transmitted (known) pilot  $X_k$  and its received version  $R_k$ .  $E[X_k^2]$  is the pilots auto-correlation.

$S_{est}$  is the estimated binary data at the output of the M-QAM demapper,  $Y_{est}$  its corresponding constellation point, and  $X_{est}$  the resulting despread signal. By omitting channel noise, the  $m^{th}$  received and corrected symbol can be written as:

$$R_{est,m} = X_{est,m} + j \sum_{n=0}^{N-1} X_{est,n} \cdot I_{m-n}. \quad (11)$$

Therefore, the estimated phase noise components can be obtained by:  $I = -j \cdot \text{pinv}(X_{\text{est}}^M) \cdot (R_{\text{est}} - X_{\text{est}})$ , (12)

$$\text{where } X_{\text{est}}^M = \begin{pmatrix} X_{\text{est},0} & X_{\text{est},1} & \dots & X_{\text{est},d} & X_{\text{est},N-d} & \dots & X_{\text{est},N-1} \\ X_{\text{est},1} & X_{\text{est},2} & \dots & X_{\text{est},d+1} & X_{\text{est},N-d+1} & \dots & X_{\text{est},0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ X_{\text{est},N-1} & X_{\text{est},0} & \dots & X_{\text{est},d-1} & X_{\text{est},N-d-1} & \dots & X_{\text{est},N-2} \end{pmatrix},$$

and  $\text{pinv}(A)$  designates the pseudo-inverse matrix of matrix  $A$ :  $\text{pinv}(A) = (A^T \cdot A)^{-1} \cdot A^T$ , where  $A^T$  is the transpose of matrix  $A$ .

ICI cancellation is then performed as:

$$r_{\text{est}} = r_{\text{cpe}} \cdot \exp(-j\varphi_{\text{est}}), \quad (13)$$

$\varphi_{\text{est}}$  being the  $N$ -point IFFT of vector  $I$ . The iterative process is ended whenever the estimation on  $S_{\text{est}}$  no longer varies from one iteration to the next, as long as we don't reach a certain maximum number of iterations.

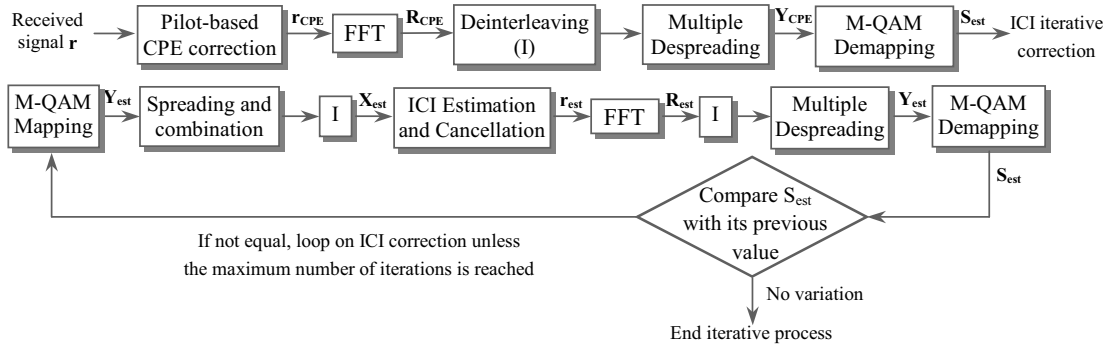


Fig. 3. Iterative algorithm for the correction of phase noise in MC-CDMA receiver.

### III. SIMULATION RESULTS FOR CPE AND ICI CORRECTION

The performances of the phase-noise suppression algorithm are evaluated for the case of a white Gaussian noise channel. Simulations results are represented by the system Symbol Error Rate (SER) in terms of the received symbol energy to channel noise power spectral density ratio ( $E_s/N_0$ ). Four pilots were inserted in a scattered manner within four different sub-carriers. As for the estimation order  $2d+1$ , our simulations showed that an insufficient value of  $d$  does not fit well the phase noise power spectral density and thus can cause deterioration in the receiver performances. Whereas taking into account a great number of phase components increases the sensitivity of the phase noise suppression algorithm to the channel noise. An optimum order of  $2d+1=7$  was determined by simulation. In Fig. 4, we show the results obtained for the case where no spreading is realized ( $n_c=1$ ), which corresponds to the case of OFDM mapping. A 64-QAM modulation was used with an FFT order of 64, and a standard deviation  $\sigma = 0.01$  for the Gaussian variable  $n(t)$ .

The adopted corrective algorithm with a variable number of iterations permits to recover a reception quality very close to that obtained in the absence of phase noise (AWGN only). Moreover, it presents a considerable gain in performance towards the case where only CPE correction is performed (perfectly or using pilot estimation).

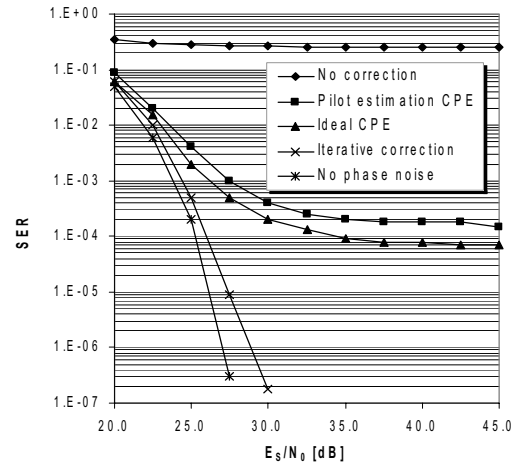


Fig. 4. Simulation results for the case of 64-QAM with  $n_c = 1$ ,  $N = 64$  sub-carriers and  $\sigma = 0.01$ .

Fig. 5 shows the results obtained for the case where the spreading factor is 4 ( $n_g=15$ ) or 64 ( $n_g=1$ ). In the last case, no pilots are inserted and CPE is supposed to be perfectly realized by a blind estimation technique. Although the system performances, in the absence of phase noise, seem to be better for  $n_c=64$ , the iterative algorithm permits a better convergence for moderate values of  $n_c$ . Indeed, the impact of wrong estimations on the algorithm performances is higher when  $n_c$  is maximum: an erroneous estimation will spread over a greater number of symbols. However, in all cases, the results obtained show a great deal of improvement in the system SER due to iterative phase noise correction. For values of  $\sigma$  greater than 0.02 (Fig. 6), we notice a saturation in the algorithm performances due to the predominance of the phase noise component.

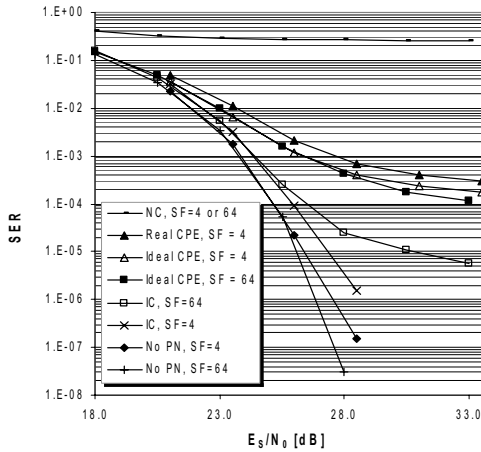


Fig. 5. Simulation results for the case of 64-QAM with  $n_c = 4$  and 64,  $N = 64$  sub-carriers and  $\sigma = 0.01$  (NC = No Correction, IC = Iterative Correction, PN = Phase noise).

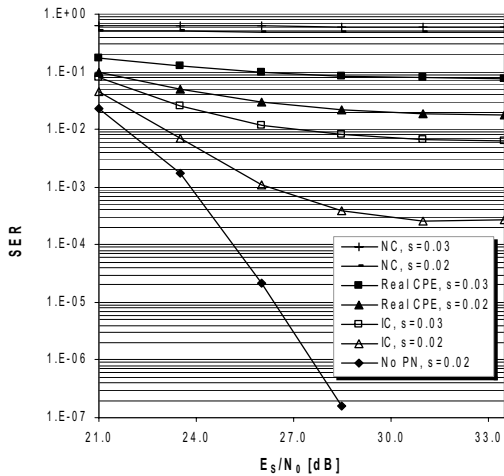


Fig. 6. Simulation results for the case of 64-QAM with  $n_c = 4$ ,  $N = 64$  sub-carriers,  $\sigma = 0.02$  and  $0.03$ .

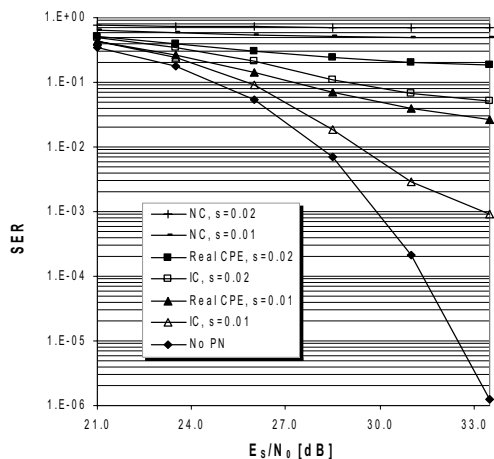


Fig. 7. Simulation results for the case of 256-QAM with  $n_c = 4$ ,  $N = 64$  sub-carriers and  $\sigma = 0.01$  and  $0.02$ .

Fig. 7 shows the system performances in the case of an increase in the modulation order (256-QAM). The loss in

performances is higher than in the case of 64-QAM. In fact, when high order modulations are used, the phase noise must be maintained very weak. Indeed, the performances decrease quickly for high values of parameter  $\sigma$ .

#### IV. CONCLUSION

In this paper, we introduced a novel approach for canceling the effect of phase noise in OFDM or MC-CDMA receivers. After CPE correction, we iteratively estimate and cancel the ICI components. The estimation is performed in the frequency domain in order to simplify the number of coefficients needed for time-domain cancellation. The system performances are presented for the case of an AWGN channel in order to compare the influence of several parameters like the spreading factor, the modulation order, the phase noise standard deviation, the transmission signal-to-noise ratio, etc. Our simulation results show a great deal of improvement towards the performances with no correction or with CPE cancellation only. We also demonstrate that the additional frequency diversity introduced by spreading the modulated symbols prior to OFDM mapping enhances the reception quality. However, the spreading factor should have a moderate value in order to alleviate error propagation through the iterative procedure on one hand, and insure an affordable complexity for the corrective algorithm. An optimal value for this parameter can be determined by simulation for each value of the sub-carriers number.

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