

A New Stability Analysis and Stabilization Of Discrete-Time Switched Linear Systems Using Vector Norms Approach

Marwen Kermani, Anis Sakly, and Faouzi M'sahli

Abstract—In this paper, we aim to investigate a new stability analysis for discrete-time switched linear systems based on the comparison, the overvaluing principle, the application of Borne-Gentina criterion and the Kotelyanski conditions. This stability conditions issued from vector norms correspond to a vector Lyapunov function. In fact, the switched system to be controlled will be represented in the Companion form. A comparison system relative to a regular vector norm is used in order to get the simple arrow form of the state matrix that yields to a suitable use of Borne-Gentina criterion for the establishment of sufficient conditions for global asymptotic stability. This proposed approach could be a constructive solution to the state and static output feedback stabilization problems.

Keywords—Discrete-time switched linear systems, Global asymptotic stability, Vector norms, Borne-Gentina criterion, Arrow form state matrix, Arbitrary switching, State feedback controller, Static output feedback controller.

I. INTRODUCTION

NOWADAYS, switched systems have attracted a growing interest. Such systems are common through a various range of application areas. In fact, this field is not only of practical importance, but also to provide an elegant framework for dealing with a large number of applications, for example, in chemical processes, flexible manufacturing systems, robotic systems, automotive industry, aircraft and air traffic control, large-scale power systems, computer-controlled systems and communication networks can be modeled as switched systems. In addition, switched systems find considerable applications in many other engineering fields [1] – [5].

The last decade has seen growing research activities in the field of discrete-time switched systems. In fact the stability and stabilization issues under arbitrary switching are fundamental in the design and analysis of such systems. This problem has been difficult and essential in researches, though it has attracted growing attention in the literature [6]–[11].

Focusing on this matter, frequently, we are required to find conditions that guarantee the stability of the switched systems under arbitrary switching law; though, it's well known that the

existence of a Common Lyapunov Function (CLF) is a sufficient condition for stability. Then, how to concept the CLF to study stability has a great importance. It is proving analytically in [12] that for switched systems which are stable, it wasn't found a CLF for this seek. This result has incited the scientist's community to look for other Lyapunov functions types which can be grouped in a general way under the name of Multiple Lyapunov Functions (MLF) [13]. Despite the development and diversity of approaches used in order to analyze the stability of discrete-time switched linear systems, whereas these letters are limited. This drawback has motivated us to recourse for another approach.

Based on the vector norms notion [14]–[16], the aggregation techniques and the application of the Borne-Gentina criterion in the discrete-time version [17], this paper present new stability conditions for discrete-time switched linear systems and under arbitrary switching.

The major motivation to recourse for this approach that it deals with a very large class of switched systems, since no restrictive assumption is made on the state matrix such as orthogonality in [11]. Whereas, the existence of a Lyapunov function has not got a constructive solution currently, even for a family of linear stationary systems [5].

The remainder of this paper is organized as follows: in the next section, we present the description and the problem formulation of the studied switched systems. In section III, sufficient stability conditions of these discrete-time switched linear systems based on vector norms approach are presented, a validation on examples is drawn, and finally, some concluding remarks are summarized in section IV.

II. SWITCHED SYSTEMS DESCRIPTION AND PROBLEM FORMULATION

The discrete-time switched linear system is formed by N subsystems described by the following state equation [18]:

$$x(k+1) = \sum_{i=1}^N \zeta_i(k) A_i(x(k), k) x(k) \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the state vector of the system at time k , $A_i (i = 1, \dots, N)$ is a matrix of appropriate dimensions denoting the subsystems, and $N \geq 1$ denotes the number of subsystems.

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The switching function ζ_i is an exogenous function which depends only on the time and not on the state, it is defined through:

$$\zeta_i = \begin{cases} 1 & \text{if } A_i \text{ is active} \\ 0 & \text{otherwise} \end{cases} \text{ and } \sum_{i=1}^N \zeta_i(t) = 1 \quad (2)$$

In this work, when the switched linear system is controlled, it will be considered in the controllable form, where the matrices A_i are given in the following form:

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \\ -a_i^n & \cdots & \cdots & -a_i^1 \end{bmatrix} \quad (3)$$

III. STABILITY CONDITIONS PRESENTATION

The switched systems has been considered all throughout this paper are represented in the canonical controllability base [19], described by the state matrix as Companion is transformed into a system characterized by state matrix in the arrow form [20], [21]. This particular form allows the application of the Borne-Gentina criterion [17].

In [16], a change of base for the system given by (1) under the arrow form gives:

$$z(k+1) = \sum_{i=1}^N \zeta_i(k) M_i z(k) \quad (4)$$

where $z = Px$, M_i is a matrix in the arrow form and P is the corresponding passage matrix:

$$M_i = P^{-1} A_i P = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 & \beta_1 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \alpha_{n-1} & \beta_{n-1} \\ \gamma_i^1 & \cdots & \cdots & \gamma_i^{n-1} & \gamma_i^n \end{bmatrix} \quad (5a)$$

$$P = \begin{bmatrix} 1 & 1 & \cdots & 1 & 0 \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} & 0 \\ (\alpha_1)^2 & (\alpha_2)^2 & \cdots & (\alpha_{n-1})^2 & \vdots \\ \vdots & \vdots & \cdots & \vdots & 0 \\ (\alpha_1)^{n-1} & (\alpha_2)^{n-1} & \cdots & (\alpha_{n-1})^{n-1} & 1 \end{bmatrix} \quad (5b)$$

$$\begin{aligned} \beta_j &= \prod_{\substack{q=1 \\ q \neq j}}^{n-1} (\alpha_j - \alpha_q)^{-1} \quad \forall j = 1, 2, \dots, n-1 \\ \gamma_i^j &= -P_i(\alpha_j) \quad \forall j = 1, 2, \dots, n-1 \\ P_i(\lambda) &= \lambda^n + \sum_{q=0}^{n-1} a_i^{n-q} \lambda^q \\ \gamma_i^n &= -a_i^1 - \sum_{j=1}^{n-1} \alpha_j \end{aligned} \quad (5c)$$

The discrete-time switched linear system can be represented by:

$$z(k+1) = Mz(k) \quad (6)$$

with:

$$M = \sum_{i=1}^N \zeta_i(k) M_i = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 & \beta_1 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \alpha_{n-1} & \beta_{n-1} \\ \sum_{i=1}^N \zeta_i(k) \gamma_i^1 & \cdots & \cdots & \sum_{i=1}^N \zeta_i(k) \gamma_i^{n-1} & \sum_{i=1}^N \zeta_i(k) \gamma_i^n \end{bmatrix} \quad (7)$$

In such conditions, if $p(y)$ denotes a vector norm of y , satisfying component to component the equality $p(y) = [|y_1|, |y_2|, \dots, |y_n|]$, it is possible by the use of the aggregation techniques [22] to define a comparison discrete-time system $z(k) \in \Re^n$ such that:

$$z(k+1) = M_D z(k) \quad (8)$$

In this expression, the comparison matrix M_D for discrete-time is deduced from the matrix M substituting all its elements by their absolute values; it can be written as:

$$M_D = \begin{bmatrix} |\alpha_1| & 0 & \cdots & 0 & |\beta_1| \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & |\alpha_{n-1}| & |\beta_{n-1}| \\ \max(|\gamma_i^1|) & \cdots & \cdots & \max(|\gamma_i^{n-1}|) & \max(|\gamma_i^n|) \end{bmatrix} \quad (9)$$

By applying the discrete-time version of Borne-Gentina criterion [22] to the previous system, we can state the following theorem.

Theorem 1. The discrete-time switched linear system described by (1) is globally asymptotically stable if there exist α_j ($j = 1, 2, \dots, n-1$), $\alpha_j \neq \alpha_q \forall j \neq q$, such as:

$$i) \quad 1 - |\alpha_j| > 0 \quad \forall j = 1, 2, \dots, n-1 \quad (10)$$

$$ii) \quad 1 - \max(|\gamma_i^n|) - \left(\sum_{j=1}^{n-1} \max(|\gamma_i^j|) |\beta_j| (1 - |\alpha_j|)^{-1} \right) > 0 \quad (11)$$

Remark 1.

We note that the stability conditions given are very useful in many switching control problems. Suppose that we have on hand an open-loop system:

$$x(k+1) = \sum_{i=1}^N \zeta_i(k) (A_i x(k) + B_i u(k)) \quad (12)$$

where $x(k)$ is the state, $u(k)$ is the control input at k , A_i , B_i are constant matrices of appropriate dimension and $\zeta_i(k)$ is the switched function. We also suppose that we can design a set of state feedback controllers $u(k) = -K_i x(k)$, $i = 1, 2, \dots, N$.

We suppose that the linear models of the switched system are set in the controllable form given by:

$$A_i = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 \\ -a_i^n & -a_i^{n-1} & \dots & -a_i^1 \end{bmatrix} \text{ and } B_i = B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (13)$$

So, the closed-loop switched discrete-time system is given by:

$$x(k+1) = \sum_{i=1}^N \zeta_i(k) (A_i - BK_i) x(k) \quad (14)$$

As an application of theorem 1, we will check these conditions through the following example.

Example 1. We consider the discrete-time switched linear system described by:

$$A_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2.226 & -5.329 & 4.104 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1.492 & -4.028 & 3.546 \end{pmatrix},$$

$$A_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2.226 & -5.472 & 4.21 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

We can analytically prove that the three subsystems are unstable; then we will stabilize the switched system with a state feedback controller, holding to the conditions given by theorem 1 with a particular choice of the parameters:

$$K_1 = [k_1^1 \ k_1^2 \ k_1^3], \ K_2 = [k_2^1 \ k_2^2 \ k_2^3] \text{ and } K_3 = [k_3^1 \ k_3^2 \ k_3^3].$$

So, the closed-loop system can be written as follows:

$$A_1^C = A_1 - BK_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2.226 - k_1^1 & -5.329 - k_1^2 & 4.104 - k_1^3 \end{pmatrix}$$

$$A_2^C = A_2 - BK_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1.492 - k_2^1 & -4.028 - k_2^2 & 3.546 - k_2^3 \end{pmatrix}$$

$$A_3^C = A_3 - BK_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2.226 - k_3^1 & -5.472 - k_3^2 & 4.21 - k_3^3 \end{pmatrix}$$

According to [21], the minimal overvaluing matrix relatively to the regular vector norm p :

$$p(x) = [|x_1|, |x_2|, |x_3|]^T \quad (15)$$

is such as:

$$M_1 = P^{-1} A_1^C P = \begin{pmatrix} \alpha_1 & 0 & \beta_1 \\ 0 & \alpha_2 & \beta_2 \\ \gamma_1^1 & \gamma_1^2 & \gamma_1^3 \end{pmatrix},$$

$$M_2 = P^{-1} A_2^C P = \begin{pmatrix} \alpha_1 & 0 & \beta_1 \\ 0 & \alpha_2 & \beta_2 \\ \gamma_2^1 & \gamma_2^2 & \gamma_2^3 \end{pmatrix}$$

and:

$$M_3 = P^{-1} A_3^C P = \begin{pmatrix} \alpha_1 & 0 & \beta_1 \\ 0 & \alpha_2 & \beta_2 \\ \gamma_3^1 & \gamma_3^2 & \gamma_3^3 \end{pmatrix}$$

with:

$$\begin{cases} \gamma_1^1 = -P_1(\alpha_1) = -\left[(\alpha_1)^3 + (-4.104 + k_1^3)(\alpha_1)^2 + (k_1^2 + 5.329)\alpha_1 + (-2.226 + k_1^1)\right] \\ \gamma_1^2 = -P_1(\alpha_2) = -\left[(\alpha_2)^3 + (-4.104 + k_1^3)(\alpha_2)^2 + (k_1^2 + 5.329)\alpha_2 + (-2.226 + k_1^1)\right] \\ \gamma_1^3 = -(-4.104 + k_1^3 + \alpha_1 + \alpha_2) \\ \gamma_2^1 = -P_2(\alpha_1) = -\left[(\alpha_1)^3 + (-3.546 + k_2^3)(\alpha_1)^2 + (k_2^2 + 4.028)\alpha_1 + (-1.492 + k_2^1)\right] \\ \gamma_2^2 = -P_2(\alpha_2) = -\left[(\alpha_2)^3 + (-3.546 + k_2^3)(\alpha_2)^2 + (k_2^2 + 4.028)\alpha_2 + (-1.492 + k_2^1)\right] \\ \gamma_2^3 = -(-3.546 + k_2^3 + \alpha_1 + \alpha_2) \end{cases}$$

and:

$$\begin{cases} \gamma_3^1 = -P_3(\alpha_1) = -\left[(\alpha_1)^3 + (-4.21 + k_3^3)(\alpha_1)^2 + (k_3^2 + 5.472)\alpha_1 + (-2.226 + k_3^1)\right] \\ \gamma_3^2 = -P_3(\alpha_2) = -\left[(\alpha_2)^3 + (-4.21 + k_3^3)(\alpha_2)^2 + (k_3^2 + 5.472)\alpha_2 + (-2.226 + k_3^1)\right] \\ \gamma_3^3 = -(-4.21 + k_3^3 + \alpha_1 + \alpha_2) \end{cases}$$

Then, the stability conditions deduced from theorem 1 are:

- i) $|\alpha_1|, |\alpha_2| < 1$
- ii) $-\max(|\gamma_1^1|, |\gamma_1^2|, |\gamma_1^3|)|\beta_1|(1-|\alpha_1|)^{-1} > 0$

When we take $\alpha_1 = 0.1$, $\alpha_2 = 0.2$ and we suppose that for particular constraints the choice of K_i is imposed such that the pole placement is different for the three subsystems by taking: $K_1 = [2.19 - 5 \ 3.8]$, $K_2 = [1.5 - 4 \ 3.2]$ and

$$K_3 = [2.225 - 5.4 \ 4]$$

Then, condition (ii) is verified numerically:

$$\begin{aligned} & 1 - \max(|0.004|, |0.046|, |-0.09|) \\ & - |10| \times 1.25 \times \max(|-0.02564|, |-0.013|, |-0.00776|) \\ & - |10| \times 1.11 \times \max(|-0.00514|, |-0.0051|, |-0.00834|) \\ & = 1 - 0.09 - 0.0250 \times 1.25 \times 10 - 0.0083 \times 1.11 \times 10 = 0.518 > 0 \end{aligned}$$

When $t_f = kT_e$ is fixed to 9s, the switched time $t_1 = k_1T_e = 3s$, $t_2 = k_2T_e = 6s$ the original state vector $x(0) = [1 \ 1 \ 1]^T$, the evolution of states with respect to time is given by Fig. 1.

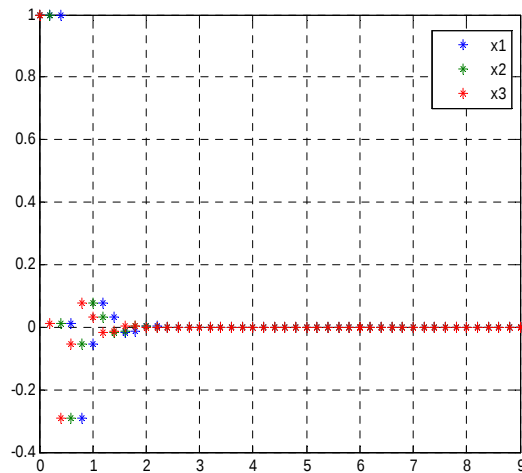


Fig. 1 Evolution of state vector for example 1

For second order discrete-time switched systems, when α and γ_i^1 are positive, theorem 2 can be simplified to the following corollary.

Corollary 2. The discrete-times switched linear system of second order is globally asymptotically stable if there exists $\alpha \in]0, 1[$ such as:

$$\text{i) } P_i(\alpha) < 0 \quad i = 1, 2, \dots, N \quad (16)$$

$$\text{ii) } P_i(1) > 0 \quad i = 1, 2, \dots, N \quad (17)$$

$$\text{iii) } \alpha + a_i^1 < 0 \quad i = 1, 2, \dots, N \quad (18)$$

■

Remark 2

The stability conditions proposed are very useful in many switching control problems. Suppose that we have on hand an open-loop system:

$$\begin{cases} x(k+1) = \sum_{i=1}^N \zeta_i(k) (A_i x(k) + B_i u(k)) \\ y(k) = \sum_{i=1}^N \zeta_i(k) C_i x(k) \end{cases} \quad (19)$$

where $x(k)$ is the state, $u(k)$ is the control input, A_i , B_i are constant matrices of appropriate dimension and $\zeta_i(k)$ is the switched function. We also suppose that we can design a set of state feedback controllers $u(k) = -K_i x(k)$, $i = 1, 2, \dots, N$.

We suppose that the linear models of the switched system are set in the controllable form given by:

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \\ -a_i^n & -a_i^{n-1} & \cdots & -a_i^1 \end{bmatrix}, B_i = B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$\text{and } C_i^T = \begin{bmatrix} C_i^1 \\ \vdots \\ C_i^{n-1} \\ C_i^n \end{bmatrix} \quad (20)$$

So, the discrete-time switched system in the closed-loop is given by:

$$x(k+1) = \sum_{i=1}^N \zeta_i(k) (A_i - BK_i C_i) x(k) \quad (21)$$

In the following, we will treat the next example by using this corollary 1.

Example 2. We consider the discrete-time switched linear system of second order described by:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.9048 & 1.905 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -0.8187 & 1.819 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_1 = [-0.3619 \ 0.4] \text{ and } C_2 = [-0.1725 \ 0.1906]$$

It is simple to see that the two subsystems are unstable; our task is to given a Stabilization domain for this switched system with a static output feedback controller characterized by the parameters K_1 and K_2 , holding to the conditions given by corollary 1.

So, the closed-loop system can be written as follows:

$$A_i^C = A_i - BK_i C_i = \begin{bmatrix} 0 & 1 \\ -0.9048 + 0.3619K_1 & 1.905 - 0.4K_1 \end{bmatrix}$$

and:

$$A_2^C = A_2 - BK_2 C_2 = \begin{bmatrix} 0 & 1 \\ -0.8187 + 0.1725K_2 & 1.819 - 0.1906K_2 \end{bmatrix}$$

According to [21], the minimal overvaluing matrix relatively to the regular vector norm p given by (15) is such as:

$$M_1 = \begin{bmatrix} \alpha & 1 \\ \gamma_1^1 & \gamma_1^2 \end{bmatrix} \text{ and } M_2 = \begin{bmatrix} \alpha & 1 \\ \gamma_2^1 & \gamma_2^2 \end{bmatrix}$$

with:

$$\begin{cases} \gamma_1^1 = -P_1(\alpha) = \\ -[\alpha^2 + (-1.905 + 0.4K_1)\alpha + (0.9048 - 0.3619K_1)] \\ \gamma_1^2 = -(-1.905 + 0.4K_1 + \alpha) \end{cases}$$

and:

$$\begin{cases} \gamma_2^1 = -P_2(\alpha) = \\ -[\alpha^2 + \alpha(-1.819 + 0.1906K_2) + 0.8187 - 0.1725K_2] \\ \gamma_2^2 = -(-1.819 + 0.1906K_2 + \alpha) \end{cases}$$

The stability conditions for example 2 given by corollary 1 are the following:

- i) $|\alpha| < 1$
- ii) $P_1(\alpha) = [\alpha^2 + (-1.905 + 0.4K_1)\alpha + 0.9048 - 0.3619K_1] < 0$
- iii) $P_2(\alpha) = [\alpha^2 + (-1.819 + 0.1906K_2)\alpha + 0.8187 - 0.1725K_2] < 0$
- iv) $P_1(1) = [1 + (-1.905 + 0.4K_1) + 0.9048 - 0.3619K_1] > 0$
- v) $P_2(1) = [1 + (-1.819 + 0.1906K_2) + 0.8187 - 0.1725K_2] > 0$
- vi) $\alpha + a_1^1 = [\alpha - 1.905 + 0.4K_1] < 0$
- vii) $\alpha + a_2^1 = [\alpha - 1.819 + 0.1906K_2] < 0$

When we take $\alpha = 0.1$, conditions (ii), (iii), (iv), (v), (vi) and (vii) allows deducing the following stability conditions:

$$\begin{cases} 2.539 < K_1 < 4.519 \\ 4.8 < K_2 < 9.018 \end{cases}$$

So, the stability domain found by the controller parameter K_2 as a function of the controller parameter K_1 is given by Fig. 2.

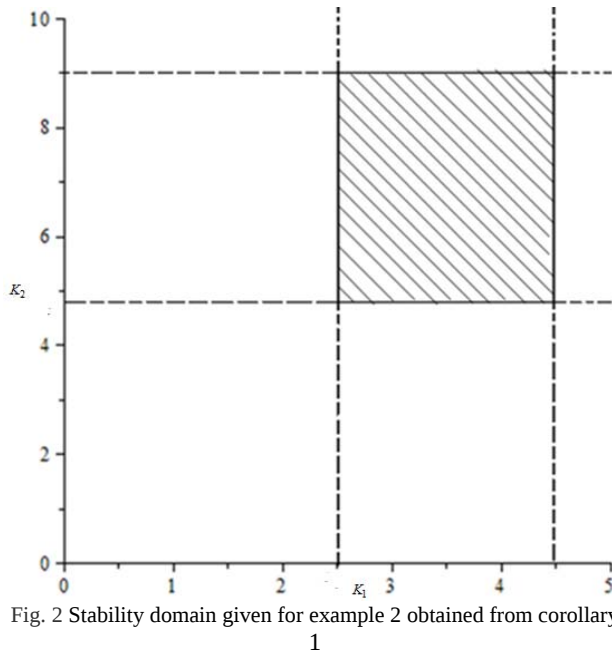


Fig. 2 Stability domain given for example 2 obtained from corollary 1

IV. CONCLUSION

This paper has investigated new stability conditions for discrete-time switched linear systems. These conditions were deduced from stability studies of overvaluing systems built on vector norms and the application of Borne-Gentina criterion.

The main advantages of this approach are that it can be applied to a very large class of switching systems and it avoids the problem of existence of Lyapunov functions.

As a validation, this approach is used in order to determine a stability domain of the conditions obtained according to controllers with state and static output feedback parameters.

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