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A New Proportional - Pursuit Coupled Guidance Law with Actuator Delay Compensation

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***Abstract*—**The aim of this paper is to present a new three-dimensional proportional-pursuit coupled (PP) guidance law to track highly maneuverable aircraft. Utilizing a 3-D polar coordinate frame, the PP guidance law is formed by collecting proportional navigation guidance in Z-R plane and pursuit guidance in X-Y plane. Feedback linearization control method to solve the guidance accelerations is used to implement PP guidance. In order to compensate the actuator time delay, the time delay compensated version of PP guidance law (CPP) was derived and proved the effectiveness of modifying the problem of high acceleration in the final phase of pursuit guidance and improving the weak robustness of proportional navigation. The simulation results for intercepting Max G turn situation show that the proposed proportional-pursuit coupled guidance law guidance law with actuator delay compensation (CPP) possesses satisfactory robustness and performance.

***Keywords*—**Feedback linearization control, time delay, guidance law, robustness, proportional navigation guidance, pursuit guidance.

I. Introduction

In modern warfare, we have to hold the superiority of air combat to ensure victory and to reduce casualties of ground troops. Therefore, air-to-air missile is the master key. When air combat enters beyond visual range stage (BVR), the Advanced Medium Range Air-to-Air Missile (AMRAAM), such as U.S. AIM-120, is the most prominent weapon with a range of fire which is between twenty and one hundred kilometers [1], [2]. AMRAAM needs no further intervention once fired. This feature, known as "fire-and-forget", enables the aircrew to aim and take evasive maneuvers at the same time. Thus AMRAAM greatly enhance an aircraft's effectiveness and improve the survival of pilots. Because of this, missiles play an important role in modern air combat and the missile theory has been incorporated into air combat simulation systems [3]-[5].

In air intercept, a missile's navigation system calculates the relative position of the target to determine the flight path, and guides missile to track its target effectively. Nowadays many new guidance laws have been proposed to intercept the highly maneuverable targets. Imado compared proportional navigation and augmented proportional navigation in the pursuit-evasion problem [6], [7].

Yang [8] described the concept of a generalized guidance law and the closed-form solution of a homing missile pursuing a maneuvering target. A new zero-sliding guidance law for the terminal guidance system was proposed by Yeh [9], whose purpose is to eliminate the sliding velocity between the missile and the target in the normal direction of LOS. Siouris and Berglund [10], [11] presented many useful guidance laws, such as three-point guidance, pursuit guidance and proportional navigation guidance. The problem of reducing the miss distance with different guidance laws and with different navigation constants, included PNG, APNG and PID guidance law has also been investigated [12]. A geometric approach to capture analysis of proportional navigation was studied by Li [13]. Lin [14] presented a concept of robust guidance and control parameters design for a guidance and control system using genetic approach.

However, most commonly used guidance laws are the pursuit guidance and the proportional navigation (PN) guidance which are simple to implement and useful in air combats. In essence, pursuit guidance is considered a bit to be impractical as it is used for intercepting with the requirement of ending the attack in a tail chase. Another disadvantage of the guidance law is that the missile’s speed must be greater than that of the target. That is, the interceptor with pursuit guidance needs high acceleration. Therefore, there exists large curvature during the whole pursuit course. A large miss distance happened when the missile cannot perform the acceleration requirement [10], [11]. On the other hand, proportional navigation has many advantages, but it is not satisfied with robustness. Wind or uncertainty may reduce its performance.

Motivated by the need to deal with the high acceleration and performance robustness problems, a three-dimensional proportional-pursuit coupled (PP) guidance law is considered in this paper. PP guidance is constructed by adopting proportional navigation guidance along the pitch-axis and pursuit guidance along the yaw-axis. Moreover, the pitch- and yaw- axes acceleration commands of PP guidance are derived respectively in this paper.

A comparative study of several evasive tactics of a fighter against proportional navigation missile was conducted by Tsao [15], of which some parameters, e.g. navigation gain, time constant, maximum acceleration capability, were claimed to have significant effects on evasive region of a fighter. Similarly, in the implementation of PP guidance, we found the impact of

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time delay on pursuit-evasion game was great.

Therefore, this paper attempts to derive the actuator time delay compensated proportional-pursuit coupled (CPP) guidance law to make simulations more realistic. Moreover, the intercept performance, the maximum time rate of change of the acceleration and the robustness of PN, PP, CPP and pursuit guidance will be compared by intercepting Max G turn situation in the simulation demonstrations.

II. Missile Dynamics

Consider now the free-flight dynamic model of a missile. Assume for simplicity that the missile can be modeled as a point mass. Here we will assume that the thrust is constant during motor burn and that the drag coefficient, air density, and missile mass are also constant. With reference to Fig. 1, from the missile's balanced forces, we may write the following approximate expression for the missile motion [7].

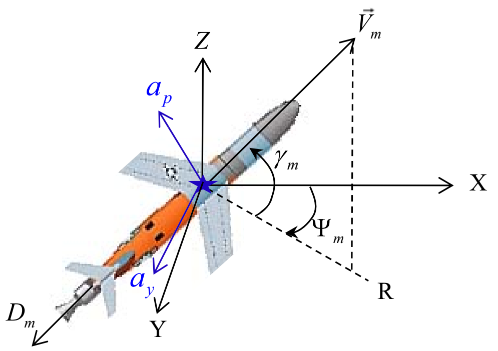

Fig. 1 Representation of the missile in the local reference frame

$$\dot{V}_m = (T_m - D_m)/m_m - g\sin\gamma_m \tag{1}$$

$$\dot{\gamma}_m = (a_p - g\cos\gamma_m)/V_m \tag{2}$$

$$\dot{\Psi}_m = a_y / V_m\cos\gamma_m \tag{3}$$

$$\dot{x}_m = V_m\cos\gamma_m\cos\Psi_m \tag{4}$$

$$\dot{y}_m = V_m\cos\gamma_m\sin\Psi_m \tag{5}$$

$$\dot{z}_m = V_m\sin\gamma_m \tag{6}$$

The acceleration commands due to time delay are given by

$$\dot{a}_p = (a_{pc} - a_p)/\tau \tag{7}$$

$$\dot{a}_y = (a_{yc} - a_y)/\tau \tag{8}$$

The missile drag D_m is given by the expression

$$D_m = k_d v_m^2 + k_d\left[a_p^2 + a_y^2 / v_m^2\right] \tag{9}$$

The above equations can be used to provide velocity, flight path angle and azimuth angle information to the tracking system on the location of the missile, Furthermore, we can obtain the relative range of the missile and target by transforming the 3-D Cartesian coordinates into the spherical coordinates. The missile movement is controlled by two actuators (see Eq. (7) and Eq. (8)), which receive acceleration commands a_{pc} and a_{yc} from the guidance system and cause control surfaces to move so as to attain these commanded accelerations. The guidance design of acceleration commands and the compensation technique of time delay will be discussed in Section 4 and Section 5, respectively.

III. Two-Dimensional Guidance Law

In this section we will review both proportional navigation guidance and pursuit guidance [10], [11], which will be employed in this paper. The two guidance laws usually explore pursuit-evasive games in two-dimensional coordinates. The discussion presented in this section is useful in the development of a three-dimensional proportional-pursuit coupled (PP) guidance law, which will be discussed in Section 4.

A. Proportional navigation guidance law

Proportional navigation guidance law is used extensively in present. The main reasons are easy to implement and effective to intercept the maneuvering aircraft. In addition, the guidance law requires less normal acceleration in the final intercept stage. Proportional navigation guidance law states that the rate of change of a missile's flight path angle (σ) is directly proportional to the rate of change of the line-of-sight (LOS) angle (q) from the missile to the target. The interception rule of the guidance law is as Eq. (10), and its geometry is shown in Fig. 2.

$$d\sigma/dt = K \cdot dq/dt \tag{10}$$

Where K is a navigation constant. The two-dimensional kinematic equations in polar coordinates are displayed as Eqs. (11)-(14).

$$dr/dt = V_T\cos\eta_T - V_M\cos\eta \tag{11}$$

$$dq/dt = \left(-V_T\sin\eta_T + V_M\sin\eta\right)/r \tag{12}$$

$$\eta = q - \sigma \tag{13}$$

$$\eta_T = q - \sigma_T \tag{14}$$

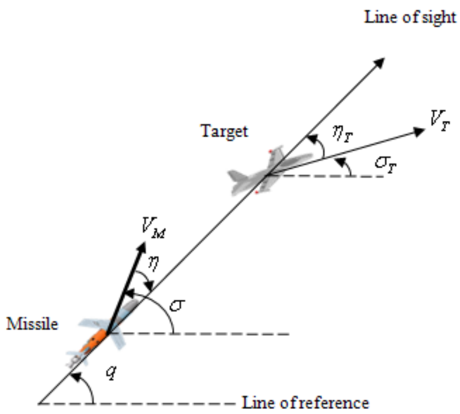

Fig. 2 Two-dimensional geometry for PN guidance

B. Pursuit Guidance law

Pursuit guidance is the first navigation law and is easy to implement. In the pursuit trajectory, a missile flies directly toward the target at all times and constantly turns to pursue the LOS between the guided missile and the target. Missiles fly a pursuit course usually end up in a tail-chase situation. An observation from the following equation is that a tail-chase intercept occurs at $\eta = 0$, these equations describe the kinematics of pursuit guidance are

$$dr/dt = V_T \cos\eta_T - V_M \tag{15}$$

$$dq/dt = \left(-V_T \sin\eta_T + V_M\right)/r \tag{16}$$

IV. Three-dimensional Proportional-pursuit Coupled (CPP) Guidance Law

In previous sections, we discussed proportional navigation and pursuit guidance. The two navigational rules were based on two-dimensional models. Since in general, there are two lateral missile coordinate axes, the general three-dimensional attack geometry can be resolved into these two directions. Therefore, in this section we will use the simple approach for the extensions from two-dimensional guidance laws to the three-dimensional case. On the other hand, we hope to use the advantages of PN guidance and pursuit guidance to reduce the normal acceleration and to improve anti-disturbance capability, hence, the three-dimensional proportional-pursuit coupled (PP) guidance law is proposed at present. PP guidance combines proportional navigation along the pitch-axis and pursuit guidance along the yaw-axis. The analytic solution of pitch and yaw acceleration commands to implement PP guidance will be derived in the next two subsections.

A. The acceleration command of PP guidance along the pitch -axis

Referring to Fig. 3, assume the variables θ and ϕ are independent, the three-dimensional geometric relationship between the missile and the target can be projected on the Z-R plane, in which the O-R axis is baseline. The three-dimensional proportional navigation guidance law can be constructed by two two-dimensional guidance laws that are employed in the X-Y and Z-R planes, respectively. Compared with three-dimensional missile-body coordinate frame in Fig. 1 and Earth-centered inertial reference frame in Fig. 3, the two-dimensional proportional navigation guidance law Eq. (10) shown in Fig. 2 can rewrite as following two equations:

$$\dot{\gamma}_m = K_p \phi \tag{17}$$

$$\dot{\Psi}_m = K_y \dot{\theta} \tag{18}$$

When the pitch-axis acceleration enables γ_m to track ϕ , Eq. (17) can combine with Eq. (2) as follows.

$$\dot{\gamma}_m = \left(a_p - g\cos\gamma_m\right)/V_m = K_p \dot{\phi} \tag{19}$$

Hence, the pitch-axis acceleration command can be obtained by

$$a_p = k_p \phi V_m + g\cos\gamma_m \tag{20}$$

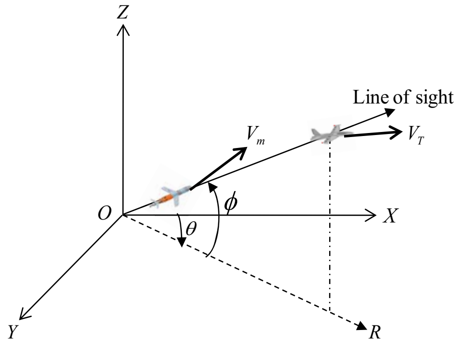

Fig. 3 Inertial reference frame for missile and target

B. The acceleration command of PP guidance along the yaw -axis

The three-dimensional pursuit guidance can be constructed by two two-dimensional ones that are employed in the X-Y and the Z-R planes. Being similar to previous subsection, we assume the variables θ and ϕ are independent. The three-dimensional geometric relationship between the missile and the target can be projected on the Z-R plane and the O-R axis is baseline, as compared with three-dimensional pursuit guidance. It's obvious that $q \neq \phi$. Next, the three-dimensional geometric relationship can also be projected on the X-Y plane, and in which X-axis is baseline. We obtain $q = \theta$.

As mentioned in subsection 0, the interception rule of pursuit guidance is that the tail-chase intercept occurs at $\eta = 0$. It can be considered by according to Fig. 1 that the missile flight path angle γ_m and azimuth angle Ψ_m has to track angle ϕ and angle θ exactly. The problem can be solved by feedback linearization control method and the tracking process is shown in Fig. 4, where K_y is the yaw-axis navigation constant.

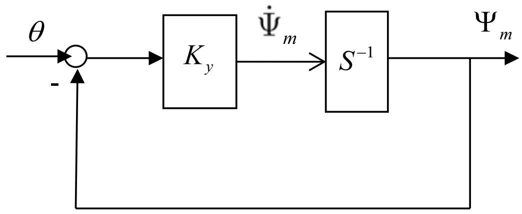

Fig. 4 Feedback linearization control diagram

If we wish azimuth angle Ψ_m to track angle θ , the equation of control system is as follows:

$$\dot{\Psi}_m + K_y \Psi_m = K_y \theta \tag{21}$$

Assume yaw-axis acceleration as Eq. (22) and take it into Eq. (3), we can get Eq. (21). Therefore, the acceleration command of PP guidance along the yaw-axis in Eq. (21) can enable the flight path angle Ψ_m to track angle θ .

$$a_y = K_y V_m \cos\gamma_m \left(\theta - \Psi_m\right) \tag{22}$$

However, observing Eqs. (7) and (8), we understand that the

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pitch and yaw acceleration commands a_{pc} and a_{yc} of PP guidance should pass through actuator to change to a_p and a_y, respectively. Its processes are shown in Fig. 5. Ignoring the time delay of the actuator, we can assume $a_{pc} \approx a_p$, $a_{yc} \approx a_y$ and then acceleration commands are given by

$$a_{pc} \approx a_p = k_p \phi V_m + g\cos\gamma_m \tag{23}$$

$$a_{yc} \approx a_y = K_y V_m \cos\gamma_m \left(\theta - \Psi_m\right) \tag{24}$$

The expressions (23) and (24) enable missile flight path angle γ_m and azimuth angle Ψ_m to track angles ϕ and θ respectively. When the angle γ_m approaches ϕ and the angle Ψ_m approaches θ, we can rewrite the equations of velocity Eqs. (4)-(6) for the target tracking of a missile in the form

$$\dot{x}_m = V_m \cos\phi\cos\theta \tag{25}$$

$$\dot{y}_m = V_m \cos\phi\sin\theta \tag{26}$$

$$\dot{z}_m = V_m \sin\phi \tag{27}$$

V. Actuator Time Delay Compensated Proportional-pursuit Coupled (CPP) Guidance Law

As indicated above, the acceleration commands have to face the problem of time delay with Eqs. (7) and (8), We now design new acceleration commands with actuator time delay compensation to overcome this problem.

A. Pitch-axis acceleration command with actuator time delay compensation

Assuming the new acceleration command can substitute for the item (a_{pc}) in Eq. (7) to be valid with a_p expressed in Eq. (20). Therefore,

$$\begin{aligned} a_{pc} &= \tau \dot{a}_p + a_p \\ &= \tau\left(K_p \ddot{\phi} V_m + K_p \dot{\phi} \dot{V}_m - g\sin\gamma_m \dot{\gamma}_m\right) \\ &\quad + K_p \dot{\phi} V_m + g\cos\gamma_m \end{aligned} \tag{28}$$

In above equation, $\dot{V}_m$ and $\dot{\gamma}_m$ can be substituted with Eq. (1) and Eq. (2)

$$\begin{aligned} a_{pc} &= \tau K_p \ddot{\phi} V_m + \tau K_p \dot{\phi}\left[\left(T_m - D_m\right)/m_m - g\sin\gamma_m\right] \\ &\quad - \tau g \sin\gamma_m \left[\left(a_p - g\cos\gamma_m\right)/V_m\right] + K_p \dot{\phi} V_m + g\cos\gamma_m \end{aligned} \tag{29}$$

When the pitch-axis acceleration enables γ_m to track ϕ, using the solution of a_p from Eq. (20) to combine it with Eq. (29), we then have

$$\begin{aligned} a_{pc} &= \tau K_p \ddot{\phi} V_m + \tau K_p \dot{\phi}\left[\left(T_m - D_m\right)/m_m\right] - \tau K_p \dot{\phi} g \sin\gamma_m \\ &\quad - \tau g \sin\gamma_m \left(K_p \dot{\phi}\right) + K_p \dot{\phi} V_m + g\cos\gamma_m \end{aligned} \tag{30}$$

After arranging, we get the new acceleration command which is expressed in the form

$$\begin{aligned} a_{pc} &= \tau K_p \dot{\phi}\left[\left(T_m - D_m\right)/m_m - 2g\sin\gamma_m + V_m/\tau\right] \\ &\quad + \tau K_p \ddot{\phi} V_m + g\cos\gamma_m \end{aligned} \tag{31}$$

The new acceleration command in Eq. (31) with actuator time delay compensation will guarantee the acceleration to approach to the value expressed in Eq. (20) through the actuator.

■Proof

Assume the acceleration command is expressed in Eq. (31), which was incorporated into Eq. (7) to observe the output acceleration value through an actuator. Rewrite Eq. (7) as follows:

$$\begin{aligned} a_p &= a_{pc} - \tau \dot{a}_p \\ &= \tau K_p \dot{\phi}\left[\left(T_m - D_m\right)/m_m - 2g\sin\gamma_m + V_m/\tau\right] \\ &\quad + \tau K_p \ddot{\phi} V_m + g\cos\gamma_m \\ &\quad - \tau\left(K_p \ddot{\phi} V_m + K_p \dot{\phi} \dot{V}_m - g\sin\gamma_m \dot{\gamma}_m\right) \end{aligned} \tag{32}$$

The value of $\left(T_m - D_m\right)/m_m$ is defined in Eq. (1). Substitute it into Eq. (32) to simplify as follows:

$$\begin{aligned} a_p &= \tau K_p \dot{\phi}\left(\dot{V}_m - g\sin\gamma_m + V_m/\tau\right) + \tau K_p \ddot{\phi} V_m \\ &\quad + g\cos\gamma_m - \tau\left(K_p \ddot{\phi} V_m + K_p \dot{\phi} \dot{V}_m - g\sin\gamma_m \dot{\gamma}_m\right) \end{aligned} \tag{33}$$

After arranging, we get the form

$$a_p = \tau g \sin\gamma_m \left(\dot{\gamma}_m - K_p \dot{\phi}\right) + K_p \dot{\phi} V_m + g\cos\gamma_m \tag{34}$$

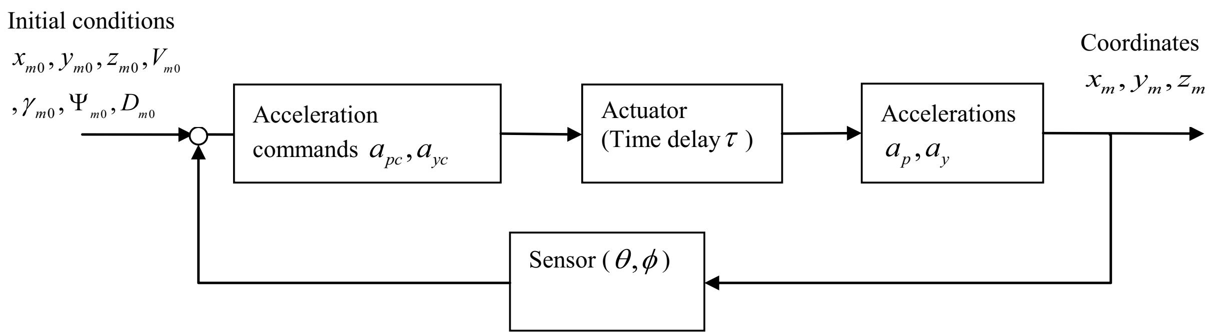

Fig. 5 Missile acceleration command working diagram

When the pitch-axis acceleration enables γ_m to track ϕ, we can incorporate the definition of two-dimensional proportional navigation in Eq. (17) into Eq. (34), and we get the pitch-axis acceleration equation:

$$a_p = K_p \dot{\phi} V_m + g\cos\gamma_m$$

The result is equal to the value expressed in Eq. (20). It proves that actuator translates the pitch-axis acceleration command a_{pc} in Eq. (31) with time delay into acceleration a_p which is exactly the same as Eq. (20).

Let us draw a comparison between Eq. (31) and Eq. (20). We found that many parameters appeared in Eq. (31), such as angular acceleration $\ddot{\phi}$, time constant τ, missile thrust T_m, drag D_m and missile mass m_m, which are the compensatory items for time delay.

B. Yaw-axis acceleration command with actuator time delay compensation

In this section, the derivation of the analytic solution of yaw-axis acceleration command with actuator time delay compensation is shown. Assuming the new acceleration command can substitute for the item (a_{yc}) in Eq. (8) to be valid with expressed in Eq. (22), so

$$\begin{aligned} a_{yc} &= \tau \dot{a}_y + a_y \\ &= \tau \begin{bmatrix} K_y \dot{V}_m \cos\gamma_m (\theta - \Psi_m) \\ +K_y V_m (-\sin\gamma_m) \dot{\gamma}_m (\theta - \Psi_m) \\ +K_y V_m \cos\gamma_m (\dot{\theta} - \dot{\Psi}_m) \end{bmatrix} \\ &\quad + K_y V_m \cos\gamma_m (\theta - \Psi_m) \end{aligned} \tag{35}$$

In the above equation, $\dot{V}_m$, $\dot{\gamma}_m$ and $\dot{\Psi}_m$ can be substituted with Eq. (1), Eq. (2) and Eq. (3).

$$\begin{aligned} a_{yc} &= \tau \left\{ \begin{aligned} &K_y \left((T_m - D_m)/m_m - g\sin\gamma_m \right) \cos\gamma_m (\theta - \Psi_m) \\ &-K_y V_m \sin\gamma_m \left(a_p - g\cos\gamma_m \right)/V_m (\theta - \Psi_m) \\ &+K_y V_m \cos\gamma_m \left(\dot{\theta} - a_y / (V_m \cos\gamma_m) \right) \end{aligned} \right\} \\ &\quad + K_y V_m \cos\gamma_m (\theta - \Psi_m) \end{aligned} \tag{36}$$

When the pitch-axis acceleration enables γ_m to track ϕ and the yaw-axis acceleration enables Ψ_m to track θ, using the solutions of a_p from Eq. (20) and from Eq. (22) to combine Eq. (36), we then have

$$\begin{aligned} a_{yc} &= \left[\left(\tau K_y \cos\gamma_m (T_m - D_m) \right)/m_m \right] (\theta - \Psi_m) \\ &\quad - \tau K_y g \sin\gamma_m \cos\gamma_m (\theta - \Psi_m) \\ &\quad - \tau K_y V_m \sin\gamma_m (\theta - \Psi_m) \left(\left(K_p V_m (\dot{\phi}) \right)/V_m \right) \\ &\quad + K_y V_m \cos\gamma_m (\theta - \Psi_m) \\ &\quad + \tau K_y V_m \cos\gamma_m \\ &\quad \cdot \left(\dot{\theta} - \left(K_y V_m \cos\gamma_m (\theta - \Psi_m) \right) / (V_m \cos\gamma_m) \right) \end{aligned} \tag{37}$$

After arranging, we get the new acceleration command with actuator time delay compensator which is expressed in the form

$$\begin{aligned} a_{yc} &= \begin{pmatrix} \left(\cos\gamma_m (T_m - D_m) \right)/m_m - (1/2) g \sin 2\gamma_m \\ -K_p V_m \sin\gamma_m \dot{\phi} + (1/\tau - K_y) V_m \cos\gamma_m \end{pmatrix} \\ &\quad \cdot \tau K_y (\theta - \Psi_m) + \tau K_y V_m \cos\gamma_m \dot{\theta} \end{aligned} \tag{38}$$

The new acceleration command in Eq. (38) with actuator time delay compensator will guarantee the acceleration a_y to approach to the value expressed in Eq. (22) through the actuator.

■Proof

Assume the yaw-axis acceleration command is expressed in Eq. (38), we incorporate Eq. (38) into Eq. (8) to observe the output acceleration value through an actuator. Rewrite Eq. (8) as follows:

$$\begin{aligned} a_y &= a_{yc} - \tau \dot{a}_y \\ &= \begin{pmatrix} \left(\cos\gamma_m (T_m - D_m) \right)/m_m - (1/2) g \sin 2\gamma_m \\ -K_p V_m \sin\gamma_m \dot{\phi} + (1/\tau - K_y) V_m \cos\gamma_m \end{pmatrix} \\ &\quad \cdot \tau K_y (\theta - \Psi_m) \\ &\quad - \tau \begin{pmatrix} K_y \dot{V}_m \cos\gamma_m (\theta - \Psi_m) \\ +K_y V_m (-\sin\gamma_m) \dot{\gamma}_m (\theta - \Psi_m) \\ +K_y V_m \cos\gamma_m (\dot{\theta} - \dot{\Psi}_m) \end{pmatrix} + \tau K_y V_m \cos\gamma_m \dot{\theta} \end{aligned} \tag{39}$$

The value of $(T_m - D_m)/m_m$ is defined in Eq. (1). Substitute it into Eq. (39) to simplify as follows:

$$\begin{aligned} a_y &= \begin{pmatrix} \dot{V}_m \cos\gamma_m - K_p V_m \sin\gamma_m \dot{\phi} \\ +(1/\tau - K_y) V_m \cos\gamma_m \end{pmatrix} \tau K_y (\theta - \Psi_m) \\ &\quad - \tau \begin{pmatrix} K_y \dot{V}_m \cos\gamma_m (\theta - \Psi_m) \\ +K_y V_m (-\sin\gamma_m) \dot{\gamma}_m (\theta - \Psi) \\ +K_y V_m \cos\gamma_m (\dot{\theta} - \dot{\Psi}) \end{pmatrix} + \tau K_y V_m \cos\gamma_m \dot{\theta} \end{aligned} \tag{40}$$

After arranging, we get the form

$$\begin{aligned} a_y &= \tau K_y V_m \sin\gamma_m (\theta - \Psi_m) (\dot{\gamma}_m - K_p \dot{\phi}) \\ &\quad + \tau K_y V_m \cos\gamma_m \left[\dot{\Psi}_m - K_y (\theta - \Psi_m) \right] \\ &\quad + K_y V_m \cos\gamma_m (\theta - \Psi_m) \end{aligned} \tag{41}$$

When the pitch-axis acceleration enables γ_m to track ϕ and the yaw-axis acceleration enables Ψ_m to track θ, we can incorporate the definitions of two-dimensional proportional navigation in Eq. (17) and two-dimensional pursuit guidance in Eq. (21) into Eq. (41). Then, we get the yaw-axis acceleration equation:

$$a_y = K_y V_m \cos\gamma_m (\theta - \Psi_m)$$

The result is equal to the value expressed in Eq. (22). It proves that actuator translates the yaw-axis acceleration command a_{yc} in Eq. (38) with time delay into acceleration a_y which is exactly the same as Eq. (22).

■Q.E.D.

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Let us draw a comparison between Eq. (38) and Eq. (22). We found that many parameters appeared in Eq. (38), such as angular velocities $\dot{\phi}$, $\dot{\theta}$, time constant τ, missile thrust T_m, drag D_m and missile mass m_m, which are the compensatory items for time delay.

For convenience of derivation of these parameters $\dot{\theta}$, $\dot{\phi}$ and $\ddot{\phi}$ in Eq. (18), Eq. (31) and Eq. (38), the target and missile coordinates are defined as (x_T, y_T, z_T) and (x_M, y_M, z_M), respectively. The relative coordinates in three-dimensional space are given by

$$x = x_T - x_M \tag{42}$$

$$y = y_T - y_M \tag{43}$$

$$z = z_T - z_M \tag{44}$$

The relation between the 3-D Cartesian coordinate system (x,y,z) and the spherical coordinate frame (γ, θ, ϕ) are given by

$$x = r\cos\phi\cos\theta \tag{45}$$

$$y = r\cos\phi\sin\theta \tag{46}$$

$$z = r\sin\phi \tag{47}$$

where

$$r = \sqrt{x^2 + y^2 + z^2} \tag{48}$$

Then Eq. (49) is easily obtained by trigonometry.

$$\cos\theta = x\Big/\sqrt{x^2 + y^2} \tag{49}$$

Using Eq. (49), we have the first derivative of given by

$$\dot{\theta} = \left(\left(x\dot{y} - \dot{x}y\right)/x^2\right)\cos^2\theta = \left(x\dot{y} - \dot{x}y\right)\Big/\left(x^2 + y^2\right) \tag{50}$$

Similarly, the first derivative of ϕ is given by

$$\cos\phi = \sqrt{x^2 + y^2}\Big/r \tag{51}$$

$$\begin{aligned}\dot{\phi} &= \left(\dot{z}r - z\dot{r}\right)/r^2\sec\phi \\ &= \left(\dot{z}\left(x^2 + y^2\right) - z\left(x\dot{x} + y\dot{y}\right)\right)\Big/\left(\sqrt{x^2 + y^2}\left(x^2 + y^2 + z^2\right)\right)\end{aligned} \tag{52}$$

To differentiate parameter $\dot{\phi}$, the second derivative of ϕ is given by

$$\ddot{\phi} = \left\{\begin{matrix}\begin{bmatrix}\ddot{z}\left(x^2 + y^2\right) + \dot{z}\left(x\dot{x} + y\dot{y}\right) \\ -z\left(\dot{x}^2 + x\ddot{x} + \dot{y}^2\right)\end{bmatrix} \\ \cdot\left(x^2 + y^2\right)\left(x^2 + y^2 + z^2\right) \\ -\left[\dot{z}\left(x^2 + y^2\right) - z\left(x\dot{x} + y\dot{y}\right)\right] \\ \cdot\begin{bmatrix}\left(2x\dot{x} + 2y\dot{y} + 2z\dot{z}\right)\left(x^2 + y^2\right) \\ +\left(x^2 + y^2 + z^2\right)\left(x\dot{x} + y\dot{y}\right)\end{bmatrix}\end{matrix}\right\} \cdot \begin{bmatrix}\left(x^2 + y^2\right)^{3/2} \\ \cdot\begin{pmatrix}x^2 + y^2 \\ +z^2\end{pmatrix}\end{bmatrix}^{-1} \tag{53}$$

Where the relative velocity and acceleration are defined as the follows:

$$\dot{x} = \dot{x}_T - \dot{x}_m, \dot{y} = \dot{y}_T - \dot{y}_m, \dot{z} = \dot{z}_T - \dot{z}_m \tag{54}$$

$$\ddot{x} = \ddot{x}_T - \ddot{x}_m, \ddot{y} = \ddot{y}_T - \ddot{y}_m, \ddot{z} = \ddot{z}_T - \ddot{z}_m \tag{55}$$

The missile velocity components are shown as Eqs. (4)-(6) and the ones of target's are given by

$$\dot{x}_T = V_T\cos\gamma_T\cos\Psi_T \tag{56}$$

$$\dot{y}_T = V_T\cos\gamma_T\sin\Psi_T \tag{57}$$

$$\dot{z}_T = V_T\sin\gamma_T \tag{58}$$

Then, we use chain rule to obtain $\ddot{x}_m$ as follows:

$$\begin{aligned}\ddot{x}_m &= \dot{V}_m\cos\gamma_m\cos\Psi_m + V_m\left(-\sin\gamma_m\right)\dot{\gamma}_m\cos\Psi_m \\ &\quad + V_m\cos\gamma_m\left(-\sin\Psi_m\right)\dot{\Psi}_m\end{aligned} \tag{59}$$

Combining Eqs. (1)-(3) into Eq. (59), we then have

$$\begin{aligned}\ddot{x}_m &= \left[\left(T_m - D_m\right)/m_m - g\sin\gamma_m\right]\cos\gamma_m\cos\Psi_m \\ &\quad - V_m\sin\gamma_m\cos\Psi_m\left[\left(a_p - g\cos\gamma_m\right)/v_m\right] \\ &\quad - V_m\cos\gamma_m\sin\Psi_m\left[a_y/\left(V_m\cos\gamma_m\right)\right]\end{aligned} \tag{60}$$

Then combining Eqs. (20) and (22) into Eq. (60), we get the equation of

$$\begin{aligned}\ddot{x}_m &= \left[\left(T_m - D_m\right)/m_m - g\sin\gamma_m\right]\cos\gamma_m\cos\Psi_m \\ &\quad - \sin\gamma_m\cos\Psi_m\left(K_p\dot{\phi}V_m\right) \\ &\quad - K_yV_m\cos\gamma_m\left(\theta - \Psi_m\right)\sin\Psi_m\end{aligned} \tag{61}$$

The derivations of remaining components of acceleration ($\ddot{y}_m, \ddot{z}_m, \ddot{x}_T, \ddot{y}_T, \ddot{z}_T$) are the same as $\ddot{x}_m$. The equations are given by the following:

$$\begin{aligned}\ddot{y}_m &= \left[\left(T_m - D_m\right)/m_m - g\sin\gamma_m\right]\cos\gamma_m\sin\Psi_m \\ &\quad - \sin\gamma_m\sin\Psi_m\left(K_p\dot{\phi}V_m\right) \\ &\quad + K_yV_m\cos\gamma_m\left(\theta - \Psi_m\right)\cos\Psi_m\end{aligned} \tag{62}$$

$$\ddot{z}_m = \left[\left(T_m - D_m\right)/m_m - g\sin\gamma_m\right]\sin\gamma_m - \cos\gamma_m\left(K_p\dot{\phi}V_m\right) \tag{63}$$

$$\begin{aligned}\ddot{x}_T &= \left[\left(T_T - D_T\right)/m_T - g\sin\gamma_T\right]\cos\gamma_T\cos\Psi_T \\ &\quad - \sin\gamma_T\cos\Psi_T\left(K_p\dot{\phi}V_T\right) \\ &\quad - K_yV_T\cos\gamma_T\left(\theta - \Psi_T\right)\sin\Psi_T\end{aligned} \tag{64}$$

$$\begin{aligned}\ddot{y}_T &= \left[\left(T_T - D_T\right)/m_T - g\sin\gamma_T\right]\cos\gamma_T\sin\Psi_T \\ &\quad - \sin\gamma_T\sin\Psi_T\left(K_p\dot{\phi}V_T\right) \\ &\quad + K_yV_T\cos\gamma_T\left(\theta - \Psi_T\right)\cos\Psi_T\end{aligned} \tag{65}$$

$$\ddot{z}_T = \left[\left(T_T - D_T\right)/m_T - g\sin\gamma_T\right]\sin\gamma_T - \cos\gamma_T\left(K_p\dot{\phi}V_T\right) \tag{66}$$

The procedures for determining the acceleration commands are described.

Step 1: Using the Runge-kutta numerical analysis method to get the solutions of the missile's dynamic Eqs. (1)-(6) and the solutions of target's dynamic Eqs. (56)-(58). These solutions include missile's coordinates (x_M, y_M, z_M), velocity (V_m), path angle (γ_m), azimuth angle (Ψ_m), drag (D_m) and the target's coordinates (x_T, y_T, z_T).

Step 2: Pursuit-evasion game between the missile and the target is a relative motion. So this paper defines the frame of reference (x,y,z) by the missile's and the target's coordinates, which are expressed by Eqs. (42)-(44).

Step 3: Then, the frame of reference (x,y,z) is transformed into a spherical coordinate system to obtain the relative distance r, elevation angle θ and azimuth angle ϕ. A missile can intercept target accurately according to these parameters. Next, the three-dimensional geometry is used to get angular velocities ($\dot{\theta},\dot{\phi}$), which are expressed by Eq. (50) and Eq. (52).

Step 4: In order to analyze the missile-target relative motion, the relative velocity ($\dot{x},\dot{y},\dot{z}$) and the relative acceleration ($\ddot{x},\ddot{y},\ddot{z}$) must be defined as Eq. (54)-(55). Parameters ($\ddot{x}_m,\ddot{y}_m,\ddot{z}_m,\ddot{x}_T,\ddot{y}_T,\ddot{z}_T$) that have obtained from Eqs. (61)-(66) can be used for Eqs. (54)-(55).

Step 5:The relative velocity and acceleration components ($\dot{x},\dot{y},\dot{z},\ddot{x},\ddot{y},\ddot{z}$) that are obtained in step 4 are combined into Eq. (53) to find the angular acceleration $\ddot{\phi}$. The parameter is used for deriving the analytic solution of the missile's pitch-axis acceleration command a_{pc}.

Step 6: By using the parameters, such as $\theta,\phi,V_m,\gamma_m,\Psi_m,D_m$, and combining them into Eq. (20) and Eq. (22), the analytic solution of the missile's pitch and yaw acceleration (a_p,a_y) respectively, can then be obtained.

Step 7: Then, by using the missile dynamics of Eqs. (7)-(8), the analytic solution of the missile's pitch and yaw acceleration commands (a_{pc},a_{yc}) can be derived. They are shown as Eq. (31) and Eq. (38).

Step 8: At time ($t=t+1$), the program repeats step 1 to step 7 to derive a_{pc} and a_{yc} until the missile hits its target or fails.

VI. Simulation Results

We compared the four guidance laws that included the proportional-pursuit (PP) coupled guidance in Eqs. (23) and (24); the actuator time delay compensation proportional-pursuit (CPP) coupled guidance shown as Eqs. (31) and (38); proportional navigation (PN) and pursuit guidance. The initial conditions of missile and target are shown in TABLE I

In Fig. 6, the target employs a Max G turn maneuver to avoid hitting. We draw a comparison of the flight paths and find out that the final trajectory of PP guidance is between that of PN guidance and pursuit guidance. Thus, we can use PP guidance to avoid the disadvantage of sharpest curvature in final stage of pursuit guidance and shorten the tracking time. Compared with PP guidance, CPP has a smaller tracking time to keep high maneuverability.

As shown in Fig. 7, pursuit guidance has the largest acceleration demand in final stage and the maximum rate of change of the acceleration in initial stage. As compared with PP guidance, CPP guidance has smaller values.

An external environmental disturbance, such as wind, can interfere with a missile's performance in the intercept, which might affect the missile's velocity, accuracy, effective range and probability of kill. In order to test the robustness of the four guidance laws, this paper adds external environmental disturbance to Eqs. (4)-(6) indicated into be Eqs. (67)-(69) as follows:

$$\dot{x}_{md}=V_m\cos(\gamma_m)\cos(\Psi_m)+C\cdot N(t) \tag{67}$$

$$\dot{y}_{md}=V_m\cos(\gamma_m)\sin(\Psi_m)+C\cdot N(t) \tag{68}$$

$$\dot{z}_{md}=V_m\sin(\gamma_m)+C\cdot N(t) \tag{69}$$

Where $N(t)$ is white noise, maximum value is 1 (N_{max} =1(m/s)) and C is a chosen constant. Simulation results are showed in Fig. 8. It is clear that the flight trajectories are interfered by noise. The missile can still track the target within the maximum noise, but performance drops a lot. For example, in normal conditions the tracking time of PP and CPP is 54.0 seconds and 51.5 seconds, respectively. In maximum noise, the tracking time of PP and CPP guidance is 297.1 seconds and 260.5 seconds, respectively. If the noise exceeds the limit, the missile will fail. It should be pointed out that PP and CPP and pursuit guidance can all withstand the maximum noise (38 N_{max}), but PN can only bear 37 N_{max}.

TABLE I
Missile and Target Initial Values

Item	X-axis(m)	Y-axis (m)	Z-axis (m)
Missile	0	0	1000
Target	500	500	2000
Item	**Velocity (m/sec)**	**Integral time interval Δt**	**Time constant τ**
Missile	100	0.1s	0.2s
Target	40	0.1s	0.1s

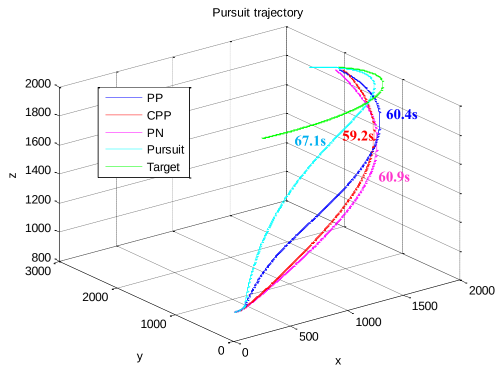

Fig. 6 Target employs Max G turn maneuver

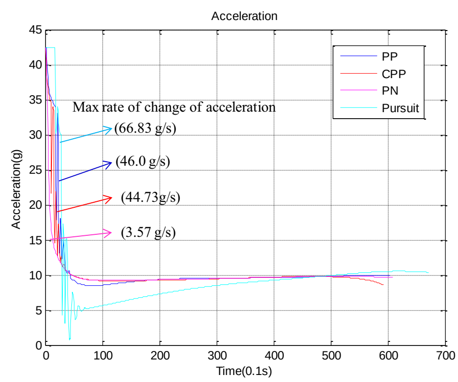

Fig. 7 Acceleration for Max G turn

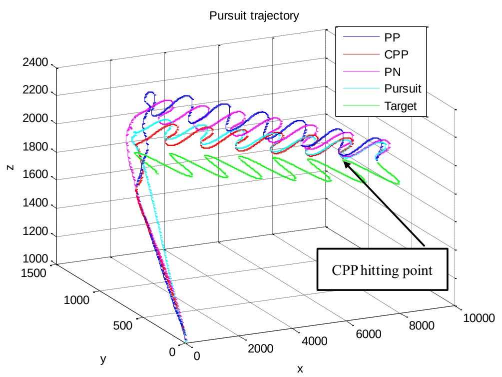

Fig. 8 Pursuit trajectory in maximum noise

VII. Conclusion

This paper has offered a new three-dimensional proportional-pursuit coupled (PP) guidance law and the actuator time delay compensation version (CPP), which connected proportional navigation guidance with pursuit guidance. The proof and the procedures to develop CPP are demonstrated in this paper. From the simulation results, proportional-pursuit coupled guidance law with actuator time delay compensation can modify the problem of high acceleration in final phase and the rate of change of the acceleration in the initial phase of pursuit guidance, and also can improve the weak robustness of proportional navigation.

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