

# A New Nonlinear Excitation Controller for Transient Stability Enhancement in Power Systems

M. Ouassaid, A. Nejmi, M. Cherkaoui, and M. Maaroufi

**Abstract**—The very nonlinear nature of the generator and system behaviour following a severe disturbance precludes the use of classical linear control technique. In this paper, a new approach of nonlinear control is proposed for transient and steady state stability analysis of a synchronous generator. The control law of the generator excitation is derived from the basis of Lyapunov stability criterion. The overall stability of the system is shown using Lyapunov technique. The application of the proposed controller to simulated generator excitation control under a large sudden fault and wide range of operating conditions demonstrates that the new control strategy is superior to conventional automatic voltage regulator (AVR), and show very promising results.

**Keywords**—Excitation control, Lyapunov technique, non linear control, synchronous generator, transient stability, voltage regulation.

## I. INTRODUCTION

THE high complexity and nonlinearity of power systems, together with their almost continuously time varying nature, have deal of challenge of power system control engineers for decades. A particular issue encountered at the generating plant level is to maintain stability under various operating conditions. In order to obtain high quality for synchronous generator controllers, many researches has been established and numerous paper are published.

Conventional excitation controllers are mainly designed by using linear control theory. The principal conventional excitation controller is the automatic voltage regulator (AVR). Many different AVR models have been developed to represent the various types used in a power system. The IEEE defined several AVR types, the main one of which (Type 1) is shown in Fig. 1. The modern AVR employing conventional, fixed parameter compensators, whilst capable of providing good steady state voltage regulation and fast dynamic response to

disturbances, do suffer from considerable variations in voltage control performance as the generator operating change. Several forms of adaptive control have been investigated to address the problem of performance variation [1].

Adversely, the generator automatic voltage regulator which reacts only to the voltage error weakens the damping introduced by damper windings. This detrimental effect of the AVR can be compensated using supplementary control loop which is the power system stabiliser. These stabilizers introduced additional system damping signals derived from the machine speed or power through the excitation system in order to improve the damping of power swings [2]. Conventional fixed parameter stabilizers work reasonably well over medium range of operating conditions. However may diminish as the generator load changes or the network configuration is altered by faults or other switching conditions which lead to deterioration in the stabilizer performance. Remarkable efforts have been devoted to the design of appropriate PSS; various methods, such as root locus, eigenvalue techniques, pole placement, adaptive control, etc have been used. But in all these methods model uncertainties cannot be considered explicitly at the design stage [3]. Hence, attention has been focused on the application of nonlinear controllers, which are independent of the equilibrium point and take into account the important non-linearities of the power system model.

The application of nonlinear control techniques to solve the transient stabilisation problem has been given much attention [4], [5], [6]. Most of these controllers are based on feedback linearization technique [7], [8]. It was shown in the literatures that the dynamics of the power system could be exactly linearized by employing nonlinear state feedback. The essence of this technique is to first transform a nonlinear system into a linear on by a nonlinear feedback, and then uses the well-known linear design techniques to complete the controller design. Consequently one can use conventional linear control to give acceptable performance [9], [10] and [11]. Nevertheless in many cases the feedback linearization method requires precise parameters plant and often cancels some useful non-linearities. On the other hand we are frequently faced with uncertainty in practical power systems. In this case, it is difficult to exactly linearize the system with nominal parameters. Adaptive versions of the feedback linearizing controls are then developed in [12], [13]. Feedback linearization is recently enhanced by using robust control

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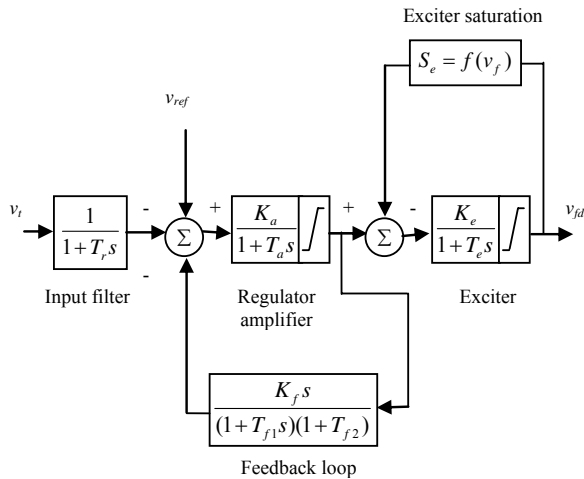


Fig. 1 Bloc diagram of the conventional IEEE type 1 AVR

designs such as  $H_\infty$  control and  $L_2$  disturbance attenuation [14], [15].

Lyapunov theory has for a long time been an important tool in linear as well as nonlinear control [16]. However, its use within nonlinear control has been hampered by the difficulties to find a Lyapunov function for a given system. If one can be found, the system is known to be stable, but the task of finding such a function has often been left to the imagination and experience of the designer.

The aim of this paper is the design of a control law for a nonlinear excitation controller to enhance the transient stability and to ensure good post-fault voltage regulation for synchronous generator connected to an infinite bus through a transmission line, as shown in Fig. 2. The model of the synchronous machine used is a 7<sup>th</sup> order model, 5 for the electrical dynamics and 2 for the mechanical dynamics, which takes into account the stator dynamics as well as the damper winding effects and practical limitation on control. The feedback system is globally asymptotically stable in the sense of the Lyapunov stability theory.

The rest of this paper is organized as follows. In section II, we describe the single-machine-infinite-bus power system model in a state space form suitable for Lyapunov-based control design. In section III, the nonlinear excitation controller is derived. The stability of this controller is proven. Some illustrative simulation results are presented and compared to the performance of a standard regulator voltage AVR in section IV to validate the proposed controller and some concluding remarks are mentioned in the final section.

## II. MATHEMATICAL MODEL OF GENERATOR SYSTEM STUDIED

The generator to be controlled, studied in this work, is shown in Fig. 2. It consists of synchronous generator connected to an infinite bus via a transmission line (SMIB). The synchronous generator is described by a 7<sup>th</sup> order nonlinear mathematical model [17], [18] which comprises three stator windings, on field winding and two damper windings. The generator winding are magnetically coupled.

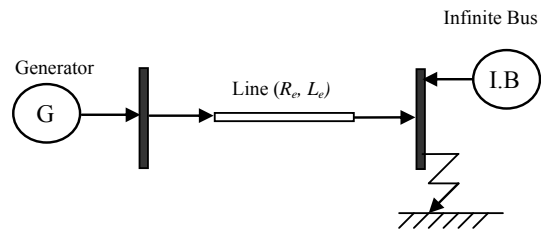


Fig. 2 Single machine infinite bus

This coupling is function of the rotor position, and therefore, the flux linking each winding is also a function of the rotor position.

The synchronous machine equations in terms of Park's d-q axis are expressed as follows [18]:

Armature windings

$$v_d = -R_s i_d - \omega \lambda_q + \frac{d\lambda_d}{dt} \quad (1)$$

$$v_q = -R_s i_q + \omega \lambda_d + \frac{d\lambda_q}{dt} \quad (2)$$

Where

$$\lambda_d = -L_d i_d + L_{md} (i_{fd} + i_{kd}) \quad (4)$$

$$\lambda_q = -L_q i_q + L_{mq} i_{kq} \quad (5)$$

Field winding

$$v_{fd} = R_s i_{fd} - L_{md} \frac{di_d}{dt} + L_{fd} \frac{di_{fd}}{dt} + L_{md} \frac{di_{kd}}{dt} \quad (6)$$

Damper windings

$$0 = R_{kd} i_{kd} - L_{md} \frac{di_d}{dt} + L_{md} \frac{di_{fd}}{dt} + L_{kd} \frac{di_{kd}}{dt} \quad (7)$$

$$0 = R_{kq} i_{kq} - L_{mq} \frac{di_q}{dt} + L_{kq} \frac{di_{kq}}{dt} \quad (8)$$

Where

$v_d, v_q$	Direct and quadrature axis stator terminal voltage components, respectively.
$v_{fd}$	Excitation control input.
$v_t$	Terminal voltage.
$i_d, i_q$	Direct and quadrature axis stator current components, respectively.
$i_{fd}$	Field winding Current
$i_{kd}, i_{kq}$	Direct and quadrature axis damper winding current components, respectively.
$\lambda_d, \lambda_q$	Direct and quadrature axis flux linkages, respectively.
$R_s$	Stator resistance.
$R_{fd}$	Field resistance.
$R_{kd}, R_{kq}$	Damper winding resistances.
$L_d, L_q$	Direct and quadrature self inductances, respectively.
$L_{fd}$	Rotor self inductance.
$L_{id}, L_{kq}$	Direct and quadrature damper winding self inductances, respectively.

$L_{md}, L_{mq}$  Direct and quadrature magnetizing inductances, respectively.

$$\frac{di_{kq}}{dt} = a_{51}i_d\omega + a_{52}i_{fd}\omega + a_{53}i_q + a_{54}i_{kd}\omega + a_{55}i_{kq} + a_{56}\sin(-\delta + a) \quad (18)$$

Mechanical equations

$$\frac{d\delta}{dt} = \omega - 1 \quad (9)$$

$$2H \frac{d\omega}{dt} = T_m - T_e - D\omega \quad (10)$$

$$\frac{d\delta}{dt} = \omega_R(\omega - 1) \quad (20)$$

Where

$\omega$  Angular speed of the generator.  
 $\delta$  Rotor angle of the generator.  
 $T_m$  Mechanical torque.  
 $T_e$  Electromagnetic torque.  
 $D$  Damping constant.  
 $H$  Inertia constant.

The electromagnetic torque is

$$T_e = (L_q - L_d)i_d i_q + L_{mfd}i_{fd}i_q + L_{md}i_{kd}i_q - L_{mq}i_d i_{kq} \quad (11)$$

The equation for transmission network with external resistance  $R_e$  and inductance  $L_e$ , in the Park transformed coordinates are

$$v_d = R_e i_d + L_e \frac{di_d}{dt} - \omega L_e i_q + V^\infty \cos(\delta - a) \quad (12)$$

$$v_q = R_e i_q + L_e \frac{di_q}{dt} + \omega L_e i_d - V^\infty \sin(\delta - a) \quad (13)$$

Where  $V^\infty$  is the infinite bus voltage and  $a$  is its phase angle.

In state space form the resulting system by combining equations (1) to (13) is highly nonlinear not only in the state but in the input and output as well [8]. The mathematical model of the generator system, in per unit, has the following form

$$\frac{di_d}{dt} = a_{11}i_d + a_{12}i_{fd} + a_{13}\omega i_q + a_{14}i_{kd} + a_{15}i_{kq}\omega + a_{16}\cos(-\delta + a) + b_1v_{fd} \quad (14)$$

$$\frac{di_{fd}}{dt} = a_{21}i_d + a_{22}i_{fd} + a_{23}\omega i_q + a_{24}i_{kd} + a_{25}i_{kq}\omega + a_{26}\cos(-\delta + a) + b_2v_{fd} \quad (15)$$

$$\frac{di_q}{dt} = a_{31}i_d\omega + a_{32}i_{fd}\omega + a_{33}i_q + a_{34}i_{kd}\omega + a_{35}i_{kq}\omega + a_{36}\sin(-\delta + a) \quad (16)$$

$$\frac{di_{kd}}{dt} = a_{41}i_d + a_{42}i_{fd} + a_{43}i_q\omega + a_{44}i_{kd} + a_{45}i_{kq}\omega + a_{46}\cos(-\delta + a) + b_3v_{fd} \quad (17)$$

Where  $\omega_R$  is the electrical frequency.

### III. THE CONTROL STRATEGIES AND STABILITY ANALYSES

Lyapunov's second or direct method is a very powerful tool of assessing stability of a nonlinear system [19]. In this paper, the concept of Lyapunov's stability criterion is used to select the control strategy in order to ensure steady and transient stability of Single Machine Infinite Bus (SMIB). To reach this objective, we define the terminal voltage error as

$$e = v_t - v_t^* \quad (21)$$

Where  $v_t^*$  is the desired trajectory and

$$v_t = \sqrt{v_d^2 + v_q^2} \quad (22)$$

The expressions of  $v_d$  and  $v_q$  as a function of the state variables can be expressed as follow

$$v_d = c_{11}i_d + c_{12}i_{fd} + c_{13}\omega i_q + c_{14}i_{kd} + c_{15}i_{kq}\omega + c_{16}\cos(-\delta + a) + c_{17}v_{fd} \quad (23)$$

$$v_q = c_{21}i_d\omega + c_{22}i_{fd}\omega + c_{23}i_q + c_{24}i_{kd}\omega + c_{25}i_{kq}\omega + c_{26}\sin(-\delta + a) \quad (24)$$

A positive definite Lyapunov function of the SMIB can be considered as

$$V = \frac{1}{2}e^2 \quad (25)$$

The basis of the Lyapunov's stability theory is that the time derivative of  $V(e)$  must be negative semi definite along the post fault trajectory.

The time derivative of the  $V(e)$  can be written as

$$\frac{dV}{dt} = e \frac{de}{dt} \quad (26)$$

From the derivative of the terminal voltage error and by using (14)-(18) and (22)-(24), we obtains the following expression

$$\begin{aligned}\frac{de}{dt} &= \frac{dv_t}{dt} = \frac{1}{v_t} \left( v_d \frac{dv_d}{dt} + v_q \frac{dv_q}{dt} \right) \\ &= c_{17} \frac{v_d}{v_t} \frac{dv_{fd}}{dt} + b_3 c_{14} \frac{v_d}{v_t} v_{fd} + \frac{v_d}{v_t} \beta_1 + \beta_2 c_{14} \frac{v_d}{v_t} \\ &\quad + \frac{v_q}{v_t} \frac{dv_q}{dt}\end{aligned}\quad (27)$$

Where

$$\begin{aligned}\beta_1 &= c_{11} \frac{di_d}{dt} + c_{12} \frac{di_{fd}}{dt} + c_{13} \left( \omega \frac{di_q}{dt} + i_q \frac{d\omega}{dt} \right) \\ &\quad + c_{15} \left( \omega \frac{di_{kq}}{dt} + i_{kq} \frac{d\omega}{dt} \right) + c_{16} \sin(-\delta + a)\end{aligned}\quad (28)$$

And

$$\begin{aligned}\beta_2 &= a_{41} i_d + a_{42} i_{fd} + a_{43} i_q \omega + a_{44} i_{kd} + a_{45} i_{kq} \omega \\ &\quad + a_{46} \cos(-\delta + a)\end{aligned}\quad (29)$$

Then the derivative of the Lyapunov function is computed as

$$\begin{aligned}\frac{dV}{dt} &= c_{17} \frac{v_d}{v_t} \frac{dv_{fd}}{dt} e + b_3 c_{14} \frac{v_d}{v_t} v_{fd} e + \frac{v_d}{v_t} \beta_1 e \\ &\quad + \beta_2 c_{14} \frac{v_d}{v_t} e + \frac{v_q}{v_t} \frac{dv_q}{dt} e\end{aligned}\quad (30)$$

Thus, the Lyapunov's stability criterion can be satisfied by making term on the right hand side of (30) negative semi definite in order to guarantee the global asymptotic stability of the system.

The candidates of  $v_{fd}$  that guarantees the semi definiteness criterion of equation (30) can be considered as

$$\frac{dv_{fd}}{dt} = -\frac{v_t}{c_{17} v_d} \left[ Ke + b_3 c_{14} \frac{v_d}{v_t} v_{fd} + \frac{v_d}{v_t} \beta_1 + c_{14} \frac{v_d}{v_t} \beta_2 + \frac{v_q}{v_t} \frac{dv_q}{dt} \right]\quad (31)$$

Where  $K$  is a positive constant feedback gain.

Substituting (31) into (30) the derivative of the Lyapunov function becomes

$$\frac{dV}{dt} = -Ke^2\quad (32)$$

Define the following equation

$$W(t) = Ke^2 \geq 0\quad (33)$$

Furthermore, by using LaSalle Yoshizawa's theorem [18], it can be shown that  $W(t)$  tend to zero as  $t \rightarrow \infty$ . Therefore,  $e$  will converge to zero as.

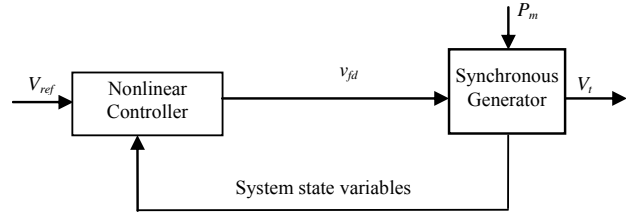


Fig. 3 Bloc diagram of the proposed nonlinear scheme

## VI. SIMULATION RESULTS AND DISCUSSION

To show the validity of the mathematical analysis and, hence, to investigate the performance of the proposed nonlinear control scheme, Simulations works are carried out for the Single Machine Infinite Bus System. The system configuration is presented as shown in Fig. 3.

The performance of the nonlinear controller was tested on the complete 7<sup>th</sup> order model of the generator system with the physical limit of the excitation voltage of the generator. Digital simulations have been carried out using MATLAB/SIMULINK. The parameter values used in the ensuing simulation are given in the appendix. In order to prove the robustness of the proposed controller, the results are compared with those of the conventional IEEE type 1 AVR. The fault considered in this paper is a symmetrical three-phase short circuit, which occurs on the infinite bus. The following temporary fault sequence is simulated:

Stage 1 : The system is in prefault steady state.

Stage 2 : A fault occurs at time  $t = 0.1$  seconds.

Stage 3 : The fault is cleared after  $t = 0.1$  seconds.

Stage 4 : The system is in post fault-state.

Consider tow cases in the simulation. Firstly, consider the operating point  $P_m = 0.3$  p. u. The simulation results are presented in figure 4 to 7. Terminal voltage, excitation voltage, rotor angle and rotor speed are shown, respectively. It is seen how the stabilisation of  $v_t$  is improved using the nonlinear controller compared to the one obtained using the linear AVR. We can see that the proposed controller can quickly and accurately track the desired terminal voltage.

It is seen how both dynamics of the rotor angle and the rotor speed exhibit large overshoots during post fault state before they settles to its steady states value with the standard linear scheme than with the proposed controller. It is obvious that with the derived control high and accuracy stability can be achieved. In the next simulation, we consider the operating point  $P_m = 0.9$  p. u. The simulation results given in Fig. 8 to Fig. 11 indicate that the proposed controller can maintain the system stability under all realistic operating conditions. Adversely, it can be seen that with the AVR, terminal voltage, excitation voltage, rotor angle and rotor speed display oscillations of constant amplitude showing that system stability is deteriorated. From the results presented earlier, it is quite evident that the proposed technique gives good and high performances transient stability.

## V. CONCLUSION

This paper has successfully demonstrated the design, and stability analysis of Lyapunov technique approach for the transient stability of a SMIB power system based on the complete 7<sup>th</sup> order model of the generator system. The nonlinear behavior of the system limits the performance of classical linear controllers used for this purpose.

The feedback system is globally asymptotically stable in the sense of Lyapunov method. The design of the controller is independent of the operating point.

Simulation results demonstrate that generator excitation with the proposed controller can effectively improve the voltage stability damp oscillation and enhance the transient stability of power system under a large sudden fault. The proposed controller demonstrates consistent superiority and most importantly reliability and robustness compared to the conventional AVR controller.

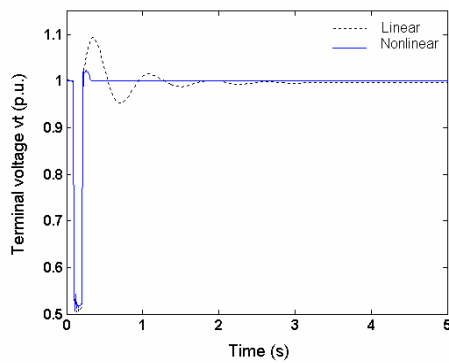


Fig. 4 Terminal voltage with operating point  $P_m = 0.3$  p. u

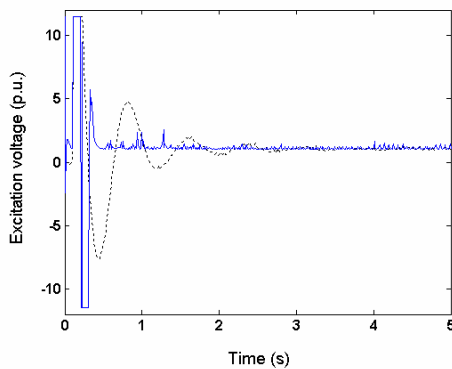


Fig. 5 Excitation voltage with operating point  $P_m = 0.3$  p. u

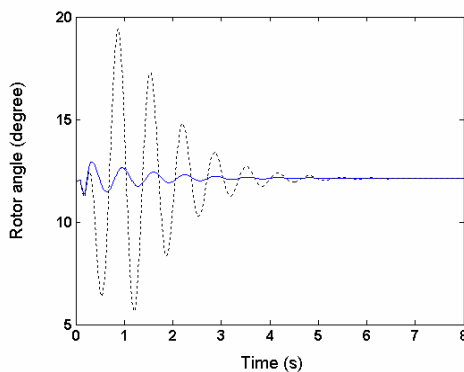


Fig. 6 Rotor angle with operating point  $P_m = 0.3$  p. u

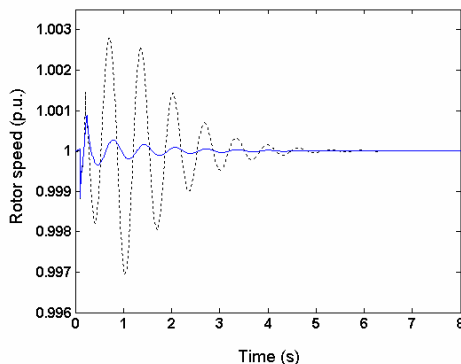


Fig. 7 Rotor speed with operating point  $P_m = 0.3$  p. u

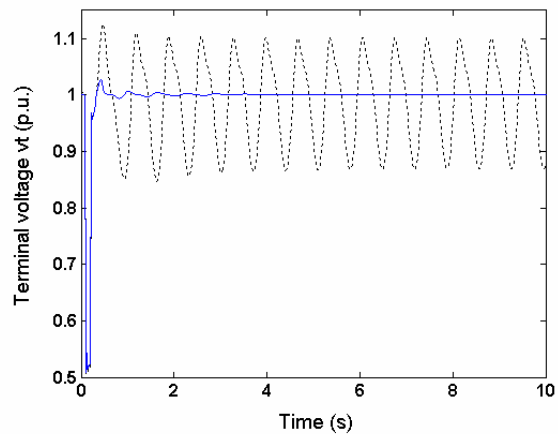


Fig. 8 Terminal voltage with operating point  $P_m = 0.9$  p. u

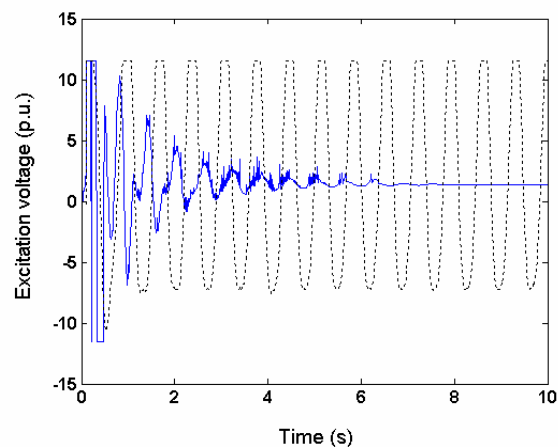
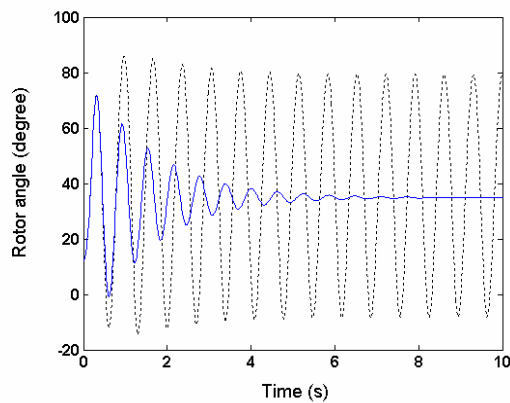
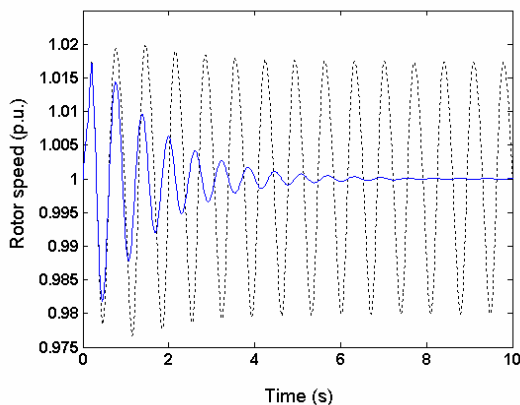


Fig. 9 Excitation voltage with operating point  $P_m = 0.9$  p. u

Fig. 10 Rotor angle with operating point  $P_m = 0.9$  p. uFig. 11 Rotor speed with operating point  $P_m = 0.9$  p. u

## APPENDIX

TABLE I

PARAMETERS OF THE POWER SYNCHRONOUS GENERATOR IN P.U.

Parameter	Value
$R_s$ , stator resistance.	$3.10^{-3}$
$R_{fd}$ , field resistance.	$6.3581.10^{-4}$
$R_{kd}$ , direct damper winding resistance.	$4.6454.10^{-3}$
$R_{kq}$ , quadrature damper winding resistance.	$6.8460.10^{-3}$
$L_{fd}$ , rotor self inductance.	1.083
$L_{kd}$ , direct damper winding self inductance.	0.9568
$L_{kq}$ , quadrature damper winding self inductance.	0.2321
$L_{md}$ , direct magnetizing inductance.	$9.1763.10^{-1}$
$L_{mq}$ , quadrature magnetizing inductance.	$2.1763.10^{-1}$
$V^\infty$ , infinite bus voltage	1
$D$ , damping constant.	0
$H$ , inertia constant.	3.195

TABLE II

PARAMETERS OF THE TRANSMISSION LINE

Parameter	Value
$L_e$ , inductance of the transmission line.	$11.16.10^{-3}$
$R_e$ , resistance of the transmission line.	$60.10^{-3}$

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