A Multivariate Moving Average Control Chart for Photovoltaic Processes

Chunchom Pongchavalit

Abstract—For the electrical metrics that describe photovoltaic cell performance are inherently multivariate in nature, use of a univariate, or one variable, statistical process control chart can have important limitations. Development of a comprehensive process control strategy is known to be significantly beneficial to reducing process variability that ultimately drives up the manufacturing cost photovoltaic cells. The multivariate moving average or MMA chart, is applied to the electrical metrics of photovoltaic cells to illustrate the improved sensitivity on process variability this method of control charting offers. The result show the ability of the MMA chart to expand to as any variables as needed, suggests an application with multiple photovoltaic electrical metrics being used in concert to determine the processes state of control.

Keywords—The multivariate moving average control chart, Photovoltaic processes control, Multivariate system.

I. INTRODUCTION

IN photovoltaic processing sequences can be responsible for significant monetary losses in the form of increased scrap rates and decrease cell performance. The translation of this cell level variability into module level departures from the ideal represents a significant area for photovoltaic process and product improvement.

One variables statistical process control methodology can be a useful approach to photovoltaic process control. The benefit of optimizing and controlling one part of the photovoltaic fabrication sequence via univariate SPC methodology has been demonstrated previously [1]. However, in the case of cell electrical metric, the multiple correlated performance variables each relay a portion of information about the process. Viewing each of them in isolation for process control purposes can result in an insensitive indicator of process variability.

Multivariate methods, which consider the interrelationship between varables, have been fruitfully employed previously in semiconductor process related applications [4], [8]. The use of the MEWMA or multivariate exponentially weighted moving average control chart, within Process control schemes for chemical processes has also been [2], [9]. These applications suggest that the single crystal silicon, photovoltaic processing sequence, with its chemical and semiconductor characteristics, as well as multiple performance metrics, may be a nature area of application for multivariate methodology. This document

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presents one alternative to univariate statistical process control for photovoltaic process control sequences. The MMV or multivariate moving average is shown to provide improved sensitivity to photovoltaic process shifts.

II. A MULTIVARIATE SYSTEM

For Fig. 1, consider the current voltage relationship, or IV curve, of an illuminated photovoltaic cell. The maximum power spot on the IV curve, Pmax, is indicated by the dot. By definition, Pmax is the product of Isc, Voc and FF. Isc, the short circuit current, and Voc, the open circuit. The voltage, are labeled on the current and voltage axes respectively. Fill Factor (FF) is defined as the area of the Pmax rectangle (ISC'voc). The Slope at Voc and the slop at Isc are also shown in figure one. The IV curve can be specified via many different parameterizations. Commonly the space is defined via the three parameters Isc, Voc, and FF. While the ultimate measure of solar cell performance is Pmax, the shape of the IV curve as described by the multiple cell electrical metrics, can provide useful information about design and processing proclivities of the solar cell when considered in concert.
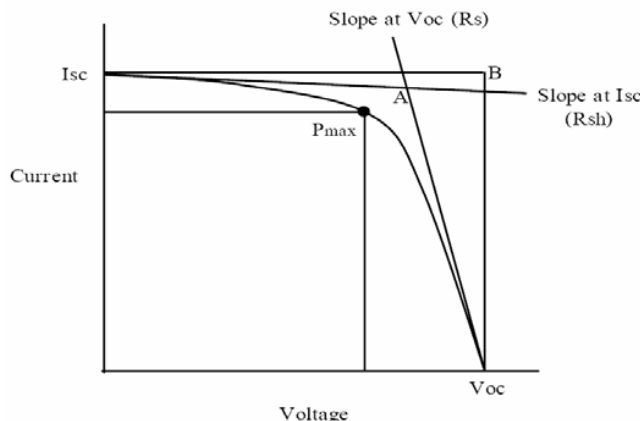

Fig. 1 IV curve of an illuminated solar cell.

Each parameter is a random variable in a system of random variables. The nature of how the random variables vary together and how this covariance might change due to process events can be studied via multivariate analysis methods including multivariate SPC. A sample correlation matrix of five of the cell performance metrics is shown in Table I.

TABLE I
ESTIMATED CORRELATION MATRIX

	Isc	Voc	Rsh	Rs
Voc	0.165			
Rsh	0.017	-0.023		
Rs	0.066	-0.275	0.165	
FF	-0.105	0.284	-0.241	-0.969

Considering the correlation between the variables and the nature of the IV curve itself, suggests that univariate statistical process control methodology, which does not consider the interrelationship between variable, may have important limitations in photovoltaic applications. In short, a group of univariate charts may all individually tell a story that the process is in control but they do not answer the question as to whether the process is in control in a multivariate sense. Multivariate control charts take into account the correlation between the variables and can improve detection of events that affect multiple variables [6]. We now present the multivariate moving average or MMA, as an alternative to univariate control of photovoltaic cell electrical metrics.

A. A Multivariate Moving Average (MMA) Control Chart

In multivariate statistical, the data forms a matrix with a column for each variable and a row for each observation. Once control limits for a multivariate chart are established, a new row of data may be evaluated as to whether it is statistically consistent with the control limits. The new row, which is a vector of however many variables are being observed, must be combined to a single measurement of statistical consistency. For the multivariate moving average or MMA control chart, this is the T^2 statistic. Step for the MMV statistic may be defined as follows:

Step 1 To calculate T_j^2, the vector of the sample means by Moving Average is given by:

$$\overline{M}_j = \begin{bmatrix} \overline{M}_{1j} \\ \overline{M}_{2j} \\ \vdots \\ \overline{M}_{pj} \end{bmatrix}, \qquad j = 1,2,...,m \qquad (1)$$

where $\overline{M}_{ij}$ represents the samples mean by Moving average of the ith characteristic for the jth sample and is found from

$$\overline{M}_{ij} = \frac{\sum_{k=1}^{n} M_{ijk}}{n} \qquad \begin{cases} i = 1,2,...,p \\ j = 1,2,...,m \end{cases} \qquad (2)$$

where M_{ijk} represents the value of the kth observation of the ith characteristic in the jth sample

M_i is then defined as a matrix of weighted observations and $M_0=0$

Also: $\qquad T_i^2 = M_i' \sum_{M_i}^{-1} M_i \qquad (3)$

Step 2 The sample variances for the ith characteristic in the jth sample Moving Average are given by

$$S_{ij}^2 = \frac{1}{n-1} \sum_{k=1}^{n} (M_{ijk} - \overline{M}_{ij})^2 \quad \begin{cases} i = 1,2,...,p \\ j = 1,2,...,m \end{cases} \qquad (4)$$

The covariance between characteristic i and characteristic h in the jth sample is calculated from

$$S_{ihj} = \frac{1}{n-1} \sum_{k=1}^{n} (M_{ijk} - \overline{M}_{ij})(M_{hjk} - \overline{M}_{hj}),$$

where $\begin{cases} j = 1,2,...,m \\ i \neq h \end{cases} \qquad (5)$

The vector $\overline{\overline{M}}$ of target means of each characteristic for m samples is estimated as $\qquad \overline{\overline{M}}_i = \frac{\sum_{j=1}^{m} \overline{M}_{ij}}{m} \quad i = 1,2,...,p$

Step 3 The element of the variance-covariance matrix S in eq.(8) are estimated from the following averages for m samples:

$$S_i^2 = \frac{\sum_{j=1}^{m} S_{ij}^2}{m}, i = 1,2,...,p \text{ and } S_{ih} = \frac{\sum_{j=1}^{m} S_{ihj}}{m} \quad i \neq h \qquad (6)$$

where eq(6) is the weighted covariance matrix and T_i^2 is the vector of actual values plotted on the control chart. One calculation of the upper control limit for the MMV is given by eq(9)

Final Step The vector $\overline{\overline{M}}$ is estimated using the elements $\{\overline{\overline{M}}_i\}$, and the matrix S is estimated as follows (only the upper diagonal part is shows because the matrix is symmetric):

$$S = \begin{bmatrix} S_1^2 & S_{12} & S_{13} & \cdots & S_{1p} \\ & S_2^2 & S_{2p} & \cdots & S_{2p} \\ & & & & \vdots \\ & & & & S_p^2 \end{bmatrix} \qquad (7)$$

The use of eq. (3) requires inverting this matrix

Because $V(M_t) = \frac{1}{w^2} \sum_{t=t-w+1}^{t} V(\overline{x}_i) = \frac{1}{w^2} \sum_{t=t-w+1}^{t} \frac{\sigma^2}{n} = \frac{\sigma^2}{nw}$

Then eq(7) will be change to

$$S = \begin{bmatrix} \dfrac{S_1^2}{nw_1} & S_{12} & S_{13} & \cdots & S_{1p} \\ & \dfrac{S_2^2}{nw_2} & S_{2p} & \cdots & S_{2p} \\ & & & & \vdots \\ & & & & \dfrac{S_p^2}{nw_p} \end{bmatrix} \qquad (8)$$

From $UCL = (\dfrac{mnp - mp - np + p}{mn - m - p + 1})F_{\alpha, p, (mn-m-p+1)}$ will be

change to $UCL = (\dfrac{mwp - mp - wp + p}{mw - m - p + 1})F_{\alpha, p, (mw-m-p+1)}$ (9)

where p equals the number of variables and the average in control run length.

B. A Multivariate Moving Average Example

Consider the following example using actual data (N=20). During the period in which the sampling was made, the process maintained a relatively stable pattern of variation, indicative of normal production. Calibration of the cell electrical measurement device is accomplished via a reference cell once per shift of production. In the interest of brevity, the example presented only contains the electrical metrics Isc and FF. However, the multivariate techniques applied could be extended to as many variables as needed, provided a multivariate relationship exists between the variables.

A dataset collected on the same production day was used to estimate the means, variances and covariance used in all control limit initial calculations, a necessary step in proper control charting application. Based on these data the estimated population means for Isc and Fill Factor were 82.32 and 20.19 respectively. Table II presents the estimated covariance matrix used in the control chart construction to follow.

TABLE II
ESTIMATED COVARIANCE MATRIX VALUES

	Isc	FF
Isc	0.00024	-0.0080
FF	-0.0080	2.7540

III. NUMERICAL

All control chart limits for both univariate and multivariate charts were estimated using this same data set to provide a consistent basis for comparison. The actual sample of observations plotted (N=20) had means standard deviations of 82.32(0.97) and 20.19(.47) for Isc and FF respectively. The sample bivaraite correlation coefficient between the variable was 0.374, indicating these variables are good candidates for a multivariate control scheme.

The intent of this contrivance was to incorporate a clear process shift to illustrate the benefit of the multivariate approach to photovoltaic process control. While normal production data offered numerous examples of the benefit of the MMA over univariate charts, examining the response of the chart to known effects within this controlled scenario has benefits from a tutorial perspective. The illustration proceeds with an examination of the univariate control chart effectiveness in detection of the one sigma shift. Fig. 2 displays the Isc data on an individuals chart with two sigma warning limits, use data from Table III and three sigma control limits shown on the right side axis.

TABLE III
DATA OF ISC AND FF

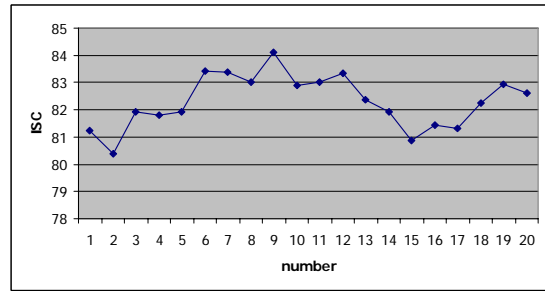| NUMBER	ISC-MV	FF-MV	NUMBER	ISC-MV	FF-MV
1	81.25	20.25	11	83.00	19.38
2	80.37	20.63	12	83.34	20.06
3	81.92	20.92	13	82.38	20.25
4	81.81	20.13	14	81.94	21.06
5	81.94	20.31	15	80.88	20.50
6	83.43	20.38	16	81.44	20.19
7	83.37	20.19	17	81.31	20.38
8	83.00	20.5	18	82.25	20.06
9	84.12	19.63	19	82.94	20.19
10	82.88	19.00	20	82.63	20.00

Fig. 2 Moving average control chart for Isc.

A similar situation exists with the moving average for Fill Factor or MA control chart of FF shown in Fig. 3.
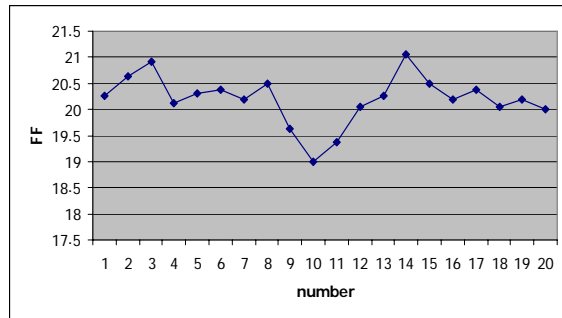

Fig. 3 Moving average control chart for Fill Factor

The univariate EWMA chart was chosen for the FF metric because of its robustness to non-normality [6]. Fill Factor data are significantly skewed and an individuals chart of the king used for Isc would be highly sensitive to this departure. As with the Isc chart, control limit values appear on the right side axis.

The MA chart shows Fig. 2 and Fig. 3. The MMA control chart show in Fig. 4 utilizes the totality of variation in both metrics o offer a more sensitive representation of the processes state of control.
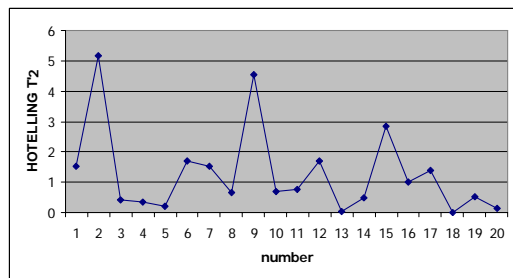
Fig. 4 Multivariate moving average control chart for Chart of Isc, Fill Factor

In this case, the upper control limit, or UCL, of 5.89 shown on the right side axis, does not have a direct interpretation relative to Isc of FF units. T^2 values are plotted on the MMA and compared to the UCL. In this example, the MEWMA chart detects the one sigma univariate shift in the Isc and FF metrics CONCLUSION.

Utilization of the MMA control chart considers the multivariate relationship of the photovoltaic cell electrical metrics. Further, the MMA incorporate the information in the current data point as well as previous ones, a design that provides improved sensitivity to small process shifts [6]. In its totality, the use of the MEWMA control chart may offer a more sensitive approach to process control situations commonly encountered in photovoltaic processing.

Additionally, the ability of the MMA chart to expand to as any variables as needed, suggests an application with multiple photovoltaic electrical metrics being used in concert to determine the processes state of control. Improved insight into the causes of process events is possible with a multivariate perspective because the overall nature of the correlation structure of the photovoltaic variables is considered.

In closing, the use of multivariate methodology for analysis of photovoltaic process data may offer a more appropriate alternative than one-variable at a time approaches. Continuing applied research will seek to further investigate the potential usage of multivariate methodology in photovoltaic settings.

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