International Journal of Electrical, Electronic and Communication Sciences
ISSN: 2517-9438
Vol:3, No:7, 2009

A Modified Genetic Based Technique for Solving the Power System State Estimation Problem

A. A. Hossam-Eldin, E. N. Abdallah, and M. S. El-Nozahy

***Abstract*—**Power system state estimation is the process of calculating a reliable estimate of the power system state vector composed of bus voltages' angles and magnitudes from telemetered measurements on the system. This estimate of the state vector provides the description of the system necessary for the operation and security monitoring. Many methods are described in the literature for solving the state estimation problem, the most important of which are the classical weighted least squares method and the non-deterministic genetic based method; however both showed drawbacks. In this paper a modified version of the genetic algorithm power system state estimation is introduced, Sensitivity of the proposed algorithm to genetic operators is discussed, the algorithm is applied to case studies and finally it is compared with the classical weighted least squares method formulation.

***Keywords*—**Genetic algorithms, ill-conditioning, state estimation, weighted least squares.

I. Introduction

THE heart of the data processing activities of the modern electric utility energy control center is the power system state estimator using both real time measurements and a historical database. The power system state estimator detects errors in the measurements and calculates an optimal estimate of the system state vector of bus voltages' magnitudes and angles. This optimal state estimate is then used by the security monitoring, operation and control functions of the center [1].

The state estimation process is based on a statistical criterion that estimates the true value of the state variables of the system to minimize or maximize the selected criterion. The most common and familiar criterion used with state estimation is the weighted least squares method where the objective function is to minimize the sum of the squares of the differences between each measured value and the true estimated value with each squared difference divided or "weighted" by the variance of the meter error [2],[3] as follows:

Minimize

$$J(x) = \sum_{i=1}^{m} [\frac{z_i - f_i(E_1,........,E_n,\theta_1,.....,\theta_{n-1})}{\sigma_i^2}]^2 \quad (1)$$

Where

f_i = function that is used to calculate the value being measured by the i^{th} measurement

σ_i^2 = variance for the i^{th} measurement

J(x) = measurement residual

m = number of independent measurements

n = the number of buses which mean that we have 2n-1 unknown parameters.

z_i = i^{th} measurement

This problem maybe optimized by a deterministic iterative procedure, the Newton method [4]. Another method to solve the state estimation problem is by using genetic algorithms [5]; however both showed drawbacks. A modified version of the genetic algorithm power system state estimation is introduced in this paper, Sensitivity of the proposed algorithm to genetic operators is discussed, the algorithm is applied to case studies and finally it is compared with the classical weighted least squares method formulation.

II. Weaknesses of the Methods Described in the Literature

A. Weaknesses of the Classical Weighted Least Squares Estimator

The WLS State Estimator leads to the iterative solution of the so-called normal equation (NE) [4]:

$$x_{est} = \left[H^T R^{-1} H\right]^{-1} H^T R^{-1} \begin{bmatrix} z_1 - f_1(x) \\ z_2 - f_2(x) \\ \end{bmatrix} \quad (2)$$

Where

R^{-1} = the weighting matrix ($diag^{-1}(\sigma^2)$)

H = the Jacobian of f(x)

$H^T R^{-1} H$ = the gain matrix G

It is clear that the Jacobian matrix is a sparse matrix i.e. a matrix populated primarily with zeros. Sparsity arises due to

A. A. Hossam-Eldin, E. N. Abdallah, and M. S. El-Nozahy are with the department of Electrical Engineering, Faculty of Engineering Alexandria University, P. O. BOX 21544, Alexandria, Egypt.

systems which are loosely coupled as in power systems (for 2 bus bars to be coupled they should have a line connection between them which is not usually the case in practical power systems). The sparsity pattern of G can be directly deduced from that of H which, in turn, is determined by the network topology and measurement configuration. This implies that, G will in general be much less sparse (containing many less zeros) than the Jacobian matrix and than the bus admittance matrix. Consequently, solving the NE will involve significantly more computations than those required by the power flow solution for the same network to obtain the inverse of G. i.e. obtaining the inverse of G needs more computations than those required in obtaining the inverse of the bus admittance matrix because G is less sparse than the admittance matrix.

Another and perhaps a more important drawback for the WLS estimation is the numerical conditioning of the solution equations. A linear equation system is said to be ill-conditioned if small errors in the entries of the coefficient matrix and/or the right hand side vector translate into significant errors in the solution vector. The more singular a matrix is, the more ill-conditioned its associated system will be. The degree, to which a system is ill conditioned, can be quantified by a measure called the condition number, which is defined as:

$$\kappa(A) = \|A^{-1}\| \cdot \|A\| \tag{3}$$

Where $\|A\|$ represents the norm of matrix A

This value is equal to unity for identity matrices and tends to infinity for matrices approaching singularity. Condition numbers are typically approximately computed, due to the high computing cost of κ as evident from its definition above. One such approximation which yields a good estimate of the condition number is the ratio $\lambda_{max} / \lambda_{min}$ where λ_{max} and λ_{min} are the largest and smallest absolute eigenvalues respectively of a normalized matrix. It can be shown that:

$$\kappa(A A^T) = (\kappa(A))^2 \tag{4}$$

This means that the NE are intrinsically ill-conditioned (because the gain matrix contains $H^T.H$ element) A combination of poor word-length and severe ill-conditioning may cause convergence problems or even divergence.

Furthermore, for the WLS state estimation, the following specific sources of ill-conditioning have been described in the literature [6]:

- Very large weighting factors used to enforce virtual measurements.
- Short and long lines simultaneously present at the same bus.
- A large proportion of injection measurements.

For the above mentioned reasons several alternative techniques which try to circumvent the shortcomings of the normal equations by avoiding the use of G and/or handling measurements in a different manner were adopted such as the orthogonal transformations which are more numerically stable than other methods. By applying them, the issue of ill-conditioning is solved. But even this algorithm can suffer from divergence.

B. Weaknesses of the Power System State Estimation Using Genetic Algorithms

Genetic Algorithms Power System State Estimation (GAPSSE) is introduced in [5] however the proposed algorithms showed some drawbacks, the most important of which is that in many cases, the optimal solution is obtained before the predetermined maximum number of generations is reached (early convergence) however the algorithm has no mean to detect the occurrence of convergence and it continues till the maximum number of generations is computed, which is time consuming as shown in Fig. 1.

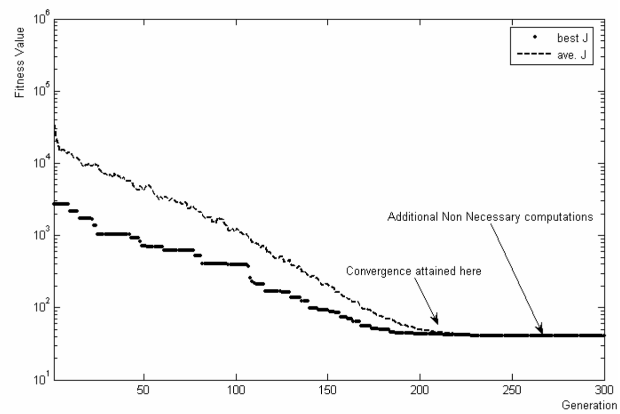

Fig. 1 Solution of the GAPSSE

This figure is adopted from [5], it shows that convergence is attained at the 234th generation, however the program continues till the maximum predefined number of generations is reached (which is 300).

III. Modified GAPSSE

A. Representation of Variables

Based on the results obtained in [5] real number encoding was adopted. Binary encoding results in a lengthy chromosome and thus increasing the computational burden introduced to the software.

B. Fitness Function

Genetic algorithms are mainly used to optimize maximization problems and since the problem of state estimation is minimization problem, the fitness function is given by [5]:

$$\text{Fit}(x) = \begin{cases} F_{max} - F(x) & , \text{if } F(x) < F_{max} \\ 0 & , \text{otherwise} \end{cases} \tag{5}$$

Where F(x) is the raw fitness function = -J(x) (the sum of weighted squares required to be minimized), F_{max} is the largest value of F(x) in the current population.

This converts the minimization problem into a maximization one suitable to be handled by the genetic algorithms. Another representation for the fitness function was tested:

$$\text{Fit}(x) = \frac{1}{J(x)} \tag{6}$$

However it didn't give satisfactory results, it also proved to be time consuming and suffered convergence problems (in some cases it showed divergence) so we settled on the representation given in (5).

C. Generating the Initial Population

Initial population is generated in a complete random way between the pre-specified maximum and minimum values for the system state variables as in [5]. It is recommended that the difference between maximum and minimum range for system state variables is as small as possible to decrease the search space and obtain fast convergence.

D. Genetic Operators for the Modified GAPSSE

Population size: Determining the size of the population is a crucial factor. Choosing a population size too small increases the risk of converging prematurely to local minima, since the population does not have enough genetic material to sufficiently cover the state space. A larger population has a greater chance of finding the global optimum at the expense of more CPU time. According to [5] a population size of 70 was found to give satisfactory results for the 6 bus bars test system.

Crossover probability: The effect of varying the crossover probability on the fitness value was studied on IEEE standard 6 bus bar system ; It was found that crossover probability doesn't affect the performance of the program significantly thus we used a crossover probability of 0 (no crossover) to reduce the computational burden. However for any other value for the crossover probability, the algorithm utilizes the arithmetical crossover method as follows:

$$X^{t+1} = a.X^{t} + (1-a).Y^{t} \tag{7}$$

$$Y^{t+1} = a.Y^{t} + (1-a).X^{t} \tag{8}$$

Where

X^t, Y^t are the parents at generation t.

X^{t+1}, Y^{t+1} are the offsprings at generation t+1.

a is a random number between the interval [0, 1].

Fig. 2 shows that the variation in the value of the objective function with the variation in the crossover probability is almost negligible thus the crossover isn't a key factor in tuning the modified GAPSSE.

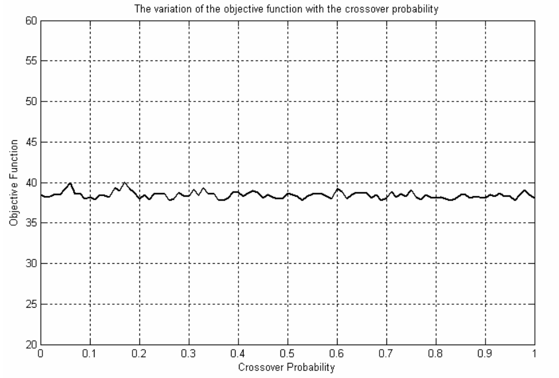

Fig. 2 Effect of varying crossover probability on the fitness value

Selection: Roulette wheel selection was adopted as in [5].

Mutation: Non uniform mutation was used to apply random changes on the genetic properties of chromosomes as follows:

$$X_t' = \begin{cases} X_t + \Delta(t, y) \quad , \quad y = X_t^{max} - X_t \\ \quad \text{if a random digit is smaller than 0.5} \\ \\ X_t - \Delta(t, y) \quad , \quad y = X_t - X_t^{min} \\ \quad \text{if a random digit is smaller than 0.5} \end{cases} \tag{9}$$

And

$$\Delta(t, y) = y.r.(1 - \frac{t}{T})^{b} \tag{10}$$

Where

X' = The value of offsprings in the t^{th} generation after mutation.

X_t = The value of offsprings in the t^{th} generation before mutation.

X_t^{max} = The maximum of all offsprings in the t^{th} generation.

X_t^{min} = The minimum of all offsprings in the t^{th} generation.

T = The maximal generation number.

B = System parameter determining the degree of non-uniformity.

r = random number from [0, 1].

The function Δ (t, y) returns a value in the range [0, y] such that the probability of Δ (t, y) approaches 0 as t increases. This property causes this operator to search the space uniformly initially (when t is small), and very locally at later generations.

It was found that the mutation probability as well as the constant of non uniformity "b" are key factors that affect the performance of the algorithm. Tests were carried on the IEEE standard 6 bus bars system. Each test was repeated for 10 times and the average and the standard deviation of the 10 runs were computed.

Fig. 3 shows the effect of varying the mutation probability on the fitness value, it is clear that the relation isn't deterministic thus we developed Fig. 4 that shows the same data after excluding those results with high noise (tests with standard deviation greater than 2%). It is shown that the minimum value for the objective function can be obtained at mutation probability between 0.5 and 0.8 which are the same results obtained by [5], a mutation value of 0.6 was used in the program. Fig. 5 studies the effect of varying the constant on non uniformity on the fitness value and again after excluding results with high noise it is evident from Fig. 6 that best performance for the modified GAPSSE can be obtained when the constant of non uniformity ranges between 3 and 6 which are again the same results obtained by [5], a value of 3.6 was used throughout the rest of the calculations.

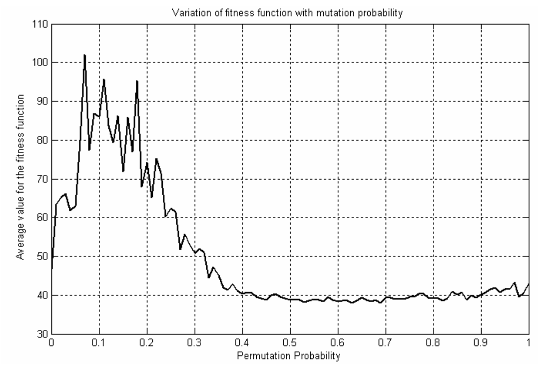

Fig. 3 Effect of varying mutation probability on the fitness value

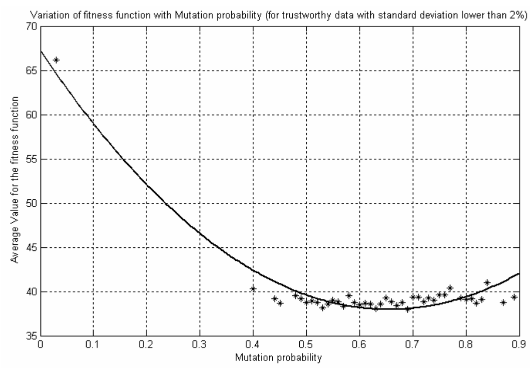

Fig. 4 Effect of varying mutation probability on the fitness value considering only tests with low level of noise (standard deviation less than 2%)

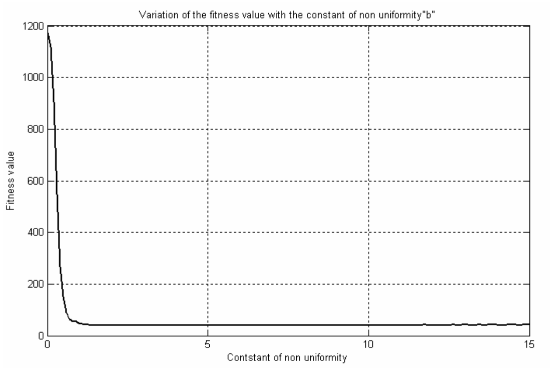

Fig. 5 Effect of varying constant of non uniformity on the fitness value

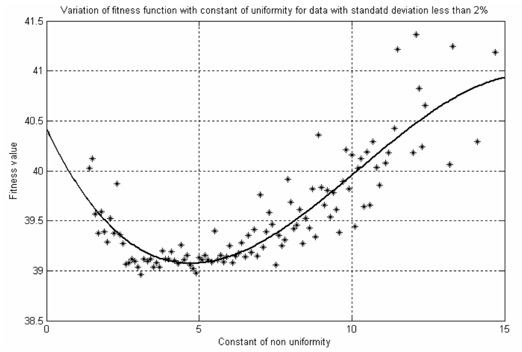

Fig. 6 Effect of varying constant of non uniformity on the fitness value considering only tests with low noise (standard deviation less than 2%)

Tuning those 2 parameters (mutation probability and constant of non uniformity) affects greatly the results obtained by the modified GAPSSE.

E. Stalling Detection Technique

One of the weaknesses of the GAPSSE is that even if the optimal solution is obtained before the predetermined maximum number of generations is reached the program continues till the maximum number of generations is calculated, which is time consuming and very unprofessional.

A stalling technique is proposed in is paper that checks on the following criterion:

*If gen. count >=21

*If (absolute value (best value (current gen.)-average value (current gen.)) <0.1

& (absolute value (best value (current gen.)-best value (10 previous gens)) <0.1

Stalling occurred

End

End

That is when the difference between the average values for the raw fitness function is larger than the best value by a small tolerance (taken as 0.1) and at the same time the best value for the raw fitness functions didn't improve for 10 successive iterations the program has reached convergence. We start to check this criterion after the 21st generation to avoid premature convergence.

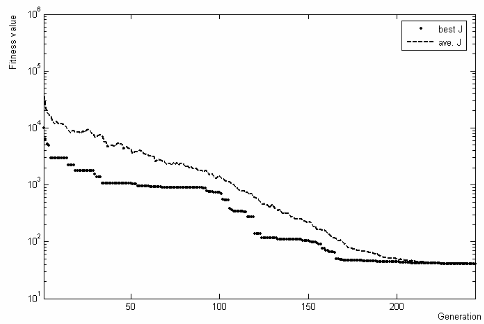

Fig. 7 Solution of the GAPSSE after applying the stalling detection technique

Solving the same problem showed in Fig. 1 using the modified GAPSSE we were able to attain convergence after 244 generations instead of 300 as shown in Fig. 7 corresponding to a time of 11.11 seconds instead of 18.22 seconds for the GAPSSE proposed in [5].

IV. MODIFIED GAPSSE VS. WLS ESTIMATOR

The WLS state estimator will be compared to the modified GAPSSE with tuned genetic operators. Studying the reasons behind the weaknesses of the WLS state estimator we notice that such reasons aren't probable to occur in a practical large scale power system (short lines aren't usually connected to the same bus bars with long lines and flow measurements are usually more than the injection measurements) thus the ill-conditioning problem isn't likely to occur unless in very small systems (up to 4 bus bars) so no additional improvements have to be done to the classical WLS approach. In the next sections we will apply both techniques on test cases, compare the results obtained by each to justify the selection of a certain technique to be used for solving the state estimation problem.

A. State Estimation of the 6 Bus Bars Test System

Applying both estimators to estimate the states of the standard IEEE 6 bus bars test system we get the following results:

i. State estimation of the 6 bus bars test system using modified GAPSSE

The modified GAPSSE was able to solve the state estimation problem in 244 generations corresponding to 11.11 seconds resulting in a sum of residuals (J) that is equal to 41.0894; the estimated values are shown in Appendix A.

ii. State estimation of the 6 bus bars test system using WLS estimator

The WLS estimator solved the state estimation problem in 0.156 seconds resulting in a sum of residuals (J) that is equal to 40.9730. The estimated values are shown in Appendix B.

Table I shows that the state variables estimated by the WLS estimator are more accurate than those estimated by the modified GAPSSE; however for both techniques the maximum error is within the accepted range.

TABLE I
ESTIMATED STATE VARIABLES FOR THE IEEE 6 BUS BARS TEST SYSTEM

State Variable	Base case values	Modified GAPSSE		WLS estimator	
		States	% error	States	% error
Θ_2	-0.0642	-0.0670	4.3614	-0.0669	4.2056
Θ_3	-0.0747	-0.0781	4.5515	-0.0780	4.4177
Θ_4	-0.0734	-0.0760	3.5422	-0.0758	3.2698
Θ_5	-0.0922	-0.0963	4.4469	-0.0961	4.2299
Θ_6	-0.1040	-0.1077	3.5577	-0.1075	3.3654
V_1	1.0500	1.0440	0.5714	1.0459	0.3905
V_2	1.0500	1.0405	0.9048	1.0424	0.7238
V_3	1.0700	1.0617	0.7757	1.0637	0.5888
V_4	0.9881	0.9805	0.7692	0.9825	0.5667
V_5	0.9861	0.9769	0.9330	0.9790	0.7200
V_6	1.0000	0.9977	0.2300	0.9999	0.0100

B. State Estimation of the 14 Bus Bars Test System

Applying both estimators to estimate the states of the standard IEEE 14 bus bars test system; we obtained the following results:

i. State estimation of the 14 bus bars test system using modified GAPSSE

The modified GAPSSE failed to solve the state estimation problem of the 14 bus bars test system moreover it showed premature convergence at J=327.8 after 312 seconds as shown in Fig. 8.

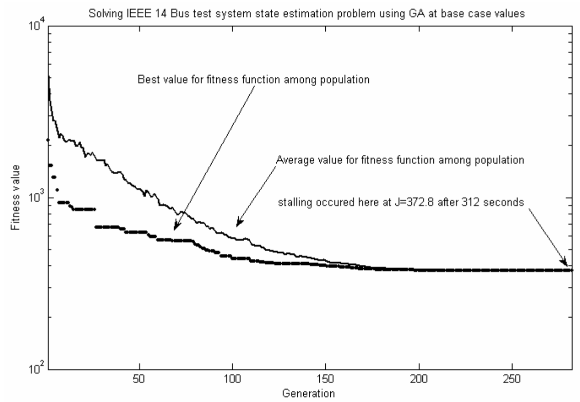

Fig. 8 Solving the IEEE 14 bus test system state estimation using modified GAPSSE

ii. State estimation of the 14 bus bars test system using WLS estimator

The WLS estimator solved the state estimation problem in 0.312 seconds resulting in a sum of residuals (J) that is equal to 1.6956; the estimated values are shown in Appendix C.

Table II shows the estimated state variables using the WLS estimator for the 14 bus bar test system and their deviation from the base case values:

TABLE II
ESTIMATED STATE VARIABLES FOR THE IEEE 14 BUS BARS TEST SYSTEM

State Variable	Base case values	Modified GAPSSE		WLS estimator	
		States	% error	States	% error
Θ_2	-0.0869	N/A*	N/A	-0.0843	3.0257
Θ_3	-0.2220	N/A	N/A	-0.2193	1.2075
Θ_4	-0.1803	N/A	N/A	-0.1765	2.0979
Θ_5	-0.1532	N/A	N/A	-0.1500	2.1448
Θ_6	-0.2482	N/A	N/A	-0.2447	1.3892
Θ_7	-0.2334	N/A	N/A	-0.2290	1.8736
Θ_8	-0.2332	N/A	N/A	-0.2285	2.0030
Θ_9	-0.2608	N/A	N/A	-0.2568	1.5037
Θ_{10}	-0.2635	N/A	N/A	-0.2596	1.4957
Θ_{11}	-0.2581	N/A	N/A	-0.2544	1.4386
Θ_{12}	-0.2630	N/A	N/A	-0.2592	1.4435
Θ_{13}	-0.2646	N/A	N/A	-0.2610	1.3471
Θ_{14}	-0.2800	N/A	N/A	-0.2759	1.4522
V_1	1.0600	N/A	N/A	1.0588	0.1107
V_2	1.0450	N/A	N/A	1.0446	0.0353
V_3	1.0100	N/A	N/A	1.0090	0.0961
V_4	1.0190	N/A	N/A	1.0186	0.0365
V_5	1.0200	N/A	N/A	1.0195	0.0490
V_6	1.0700	N/A	N/A	1.0694	0.0568
V_7	1.0620	N/A	N/A	1.0612	0.0755
V_8	1.0900	N/A	N/A	1.0887	0.1209
V_9	1.0560	N/A	N/A	1.0552	0.0743
V_{10}	1.0510	N/A	N/A	1.0501	0.0819
V_{11}	1.0570	N/A	N/A	1.0562	0.0802
V_{12}	1.0550	N/A	N/A	1.0544	0.0535
V_{13}	1.0500	N/A	N/A	1.0494	0.0551
V_{14}	1.0360	N/A	N/A	1.0355	0.0471

*Not Available

V. RESULTS AND DISCUSSION

It's clear from the above 2 sections that using the WLS estimator is more favored in solving the state estimation problem as:

- It doesn't exhibit divergence problems.
- About 70 times faster than the modified GAPSSE (WLS estimator needed 0.156 second compared to 11.11 seconds for the modified GAPSSE).
- Obtains more accurate results.

However it might show ill-conditioning problems in case of small power systems (less than 4 bus bars), thus we can conclude that:

- For very small systems (less than 4 bus bars) we have to use the modified GAPSSE as WLS estimator shows ill-conditioning.
- For power systems containing 4-6 bus bars we might use either estimator; however we favor using the WLS estimator.
- For large power systems (more than 6 bus bars) the modified GAPSSE fails to find a solution and hence we have to use the WLS estimator.

REFERENCES

[1] Gu J., Clements K., Krumpholz G., Davis P., "The Solution of ill conditioned Power System State Estimation Problems via the Method of Peters and Wilkinson". *Power Industry Computer Applications Conference Proceedings*, Houston, May 1983, pp.239-246.

[2] Schweppe F.C. and Wildes J., "Power System Static-State Estimation, Part I: Exact Model", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-89, January 1970, pp. 120-125.

[3] Schweppe F.C. and Rom D.B., "Power System Static-State Estimation, Part II: Approximate Model", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-89, January 1970, pp.125-130.

[4] Schweppe F.C., "Power System Static-State Estimation, Part III: Implementation", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-89, January 1970, pp. 130-135.

[5] Emtethal N.Abdallah, Amr A.Ghazala and Norhan Hanafy,"Power System State Estimation Using Genetic Algorithms", *Proceedings of the tenth middle east power systems conference*, December 2005,pp.669-676.

[6] A. Abur and A. G. Exposito, "Power System State Estimation—Theory and Implementations", New York: Marcel Dekker, 2004, p.37.

APPENDIX A

SOLVING THE STATE ESTIMATION PROBLEM FOR THE IEEE 6 BUS BARS TEST SYSTEM USING MODIFIED GAPSSE

	The Base case value			The Measured value			The Estimated value		
Measurement	**KV**	**MW**	**MVAR**	**KV**	**MW**	**MVAR**	**KV**	**MW**	**MVAR**
Mv1	241.5			238.4			240.1		
MG1		107.9	16		113.1	20.2		111.73	18.84
M1-2		28.7	-15.4		31.5	-13.2		30.30	-14.28
M1-4		43.6	20.1		38.9	21.2		44.72	21.26
M1-5		35.6	11.3		35.7	9.4		36.71	11.86
Mv2	241.5			237.8			239.3		
MG2		50	73.4		48.4	71.9		47.52	70.23
M2-1		-27.8	12.8		-33.9	9.7		-29.32	11.90
M2-3		2.9	-12.3		8.6	-11.9		3.02	-12.64
M2-4		33.1	46.1		32.8	38.3		32.35	45.23
M2-5		15.5	15.4		17.4	22		15.58	14.84
M2-6		26.2	12.4		22.3	15		25.89	10.90
Mv3	246.1			250.7			244.2		
MG3		60	89.6		55.1	90.6		59.49	87.77
M3-2		-2.9	5.7		-2.1	10.2		-2.98	6.23
M3-5		19.1	23.2		17.7	23.9		19.17	23.01
M3-6		43.8	60.7		43.3	58.3		43.30	58.53
Mv4	227.7			225.7			225.5		
MG4		-70	-70		-71.8	-71.9		-70.18	-70.07
M4-1		-42.5	-19.9		-40.1	-13.3		-43.55	-20.69
M4-2		-31.6	-45.1		-29.8	-43.3		-30.88	-44.33
M4-5		3.1	-3.9		0.7	-17.4		4.25	-5.05
Mv5	226.8			225.2			224.7		
MG5		-70	-70		-72	-67.7		-71.60	-69.51
M5-1		-33.5	-13.5		-36.6	-17.5		-35.55	-13.65
M5-2		-15	-18		-11.7	-22.2		-15.09	-17.44
M5-3		-18	-26.1		-25.1	-29.9		-18.07	-25.82
M5-4		-4	-2.8		-2.1	-1.5		-4.21	-2.53
M5-6		1.6	-9.7		-2.1	-0.8		1.32	-10.07
Mv6	230			228.9			229.5		
MG6		-70	-70		-72.3	-60.9		-68.93	-65.96
M6-2		-25.7	-16		-19.6	-22.3		-25.33	-14.52
M6-3		-42.8	-57.9		-46.8	-51.1		-42.34	-55.83
M6-5		-1.6	3.9		1	2.9		-1.26	4.39

International Journal of Electrical, Electronic and Communication Sciences
ISSN: 2517-9438
Vol:3, No:7, 2009

APPENDIX B
SOLVING THE STATE ESTIMATION PROBLEM FOR THE IEEE 6 BUS BARS TEST SYSTEM USING WLS ESTIMATOR

	The Base case value			The Measured value			The Estimated value		
Measurement	**KV**	**MW**	**MVAR**	**KV**	**MW**	**MVAR**	**KV**	**MW**	**MVAR**
Mv1	241.5			238.4			240.5		
MG1		107.9	16		113.1	20.2		111.9	18.7
M1-2		28.7	-15.4		31.5	-13.2		30.3	-13.3
M1-4		43.6	20.1		38.9	21.2		43.8	21.1
M1-5		35.6	11.3		35.7	9.4		36.8	11.8
Mv2	241.5			237.8			239.8		
MG2		50	73.4		48.4	71.9		47.5	70.1
M2-1		-27.8	12.8		-33.9	9.7		-29.4	11.9
M2-3		2.9	-12.3		8.6	-11.9		3	-12.7
M2-4		33.1	46.1		32.8	38.3		32.3	45.2
M2-5		15.5	15.4		17.4	22		15.6	13.8
M2-6		26.2	12.4		22.3	15		25.9	10.8
Mv3	246.1			250.7			243.6		
MG3		60	89.6		55.1	90.6		59.5	87.8
M3-2		-2.9	5.7		-2.1	10.2		-3	6.3
M3-5		19.1	23.2		17.7	23.9		19.2	23
M3-6		43.8	60.7		43.3	58.3		43.3	58.5
Mv4	227.7			225.7			226		
MG4		-70	-70		-71.8	-71.9		-70.2	-70.1
M4-1		-42.5	-19.9		-40.1	-13.3		-43.6	-20.7
M4-2		-31.6	-45.1		-29.8	-43.3		-30.9	-43.3
M4-5		3.1	-3.9		0.7	-17.4		3.3	-5.1
Mv5	226.8			225.2			225.2		
MG5		-70	-70		-72	-67.7		-71.8	-69.5
M5-1		-33.5	-13.5		-36.6	-17.5		-35.6	-13.6
M5-2		-15	-18		-11.7	-22.2		-15.1	-17.4
M5-3		-18	-26.1		-25.1	-29.9		-18.1	-25.9
M5-4		-4	-2.8		-2.1	-1.5		-3.2	-2.5
M5-6		1.6	-9.7		-2.1	-0.8		1.3	-10.1
Mv6	230			228.9			230		
MG6		-70	-70		-72.3	-60.9		-68.9	-65.9
M6-2		-25.7	-16		-19.6	-22.3		-25.4	13.5
M6-3		-42.8	-57.9		-46.8	-51.1		-42.4	-55.8
M6-5		-1.6	3.9		1	2.9		-1.2	3.4

APPENDIX C
SOLVING THE STATE ESTIMATION PROBLEM FOR THE IEEE 14 BUS BARS TEST SYSTEM USING WLS ESTIMATOR

	The Base case value			The Measured value			The Estimated value		
Measurement	**KV**	**MW**	**MVAR**	**KV**	**MW**	**MVAR**	**KV**	**MW**	**MVAR**
Mv1	243.8			245.2			243.5		
MG1		232.3	-22.5		224.4	-21.8		225.3	-22.8
M1-2		156.7	-23.4		149.1	-23.8		151.5	-23.6
M1-5		75.6	0.9		77.3	0.8		73.8	0.7
Mv2	240.3			239.9			240.3		
MG2		18.5	20.7		19.0	21.4		22.3	20.8
M2-1		-152	24.8		-146	24.4		-147	24.2
M2-3		73.2	1.2		73.6	1.2		73.2	1.5
M2-4		56.2	-4.2		55.2	-4.1		55.5	-4.0
M2-5		41.5	-1.0		43.2	-1.0		41.1	-0.8
Mv3	232.3			230.0			232.1		
MG3		-94.3	2.4		-97.3	2.3		-94.9	2.1
M3-2		-70.9	-0.6		-68.9	-0.7		-70.8	-0.9
M3-4		-23.4	3.1		-24.2	2.9		-24.0	3.0
Mv4	234.4			231.3			234.3		
MG4		-48.8	-11.2		-46.7	-11.4		-46.6	-11.2
M4-2		-54.5	2.0		-55.1	2.0		-53.8	1.8
M4-3		23.8	-4.8		23.9	-4.7		24.4	-4.6
M4-5		-61.1	17.9		-58.3	18.3		-59.8	17.8
M4-7		27.4	-20.2		26.7	-19.3		27.1	-20.0
M4-9		15.5	-6.2		15.4	-6.1		15.5	-6.1
Mv5	234.6			235.3			234.5		
MG5		-10.8	-38.3		-11.1	-38.3		-10.2	-38.7
M5-1		-72.8	-0.1		-72.1	-0.1		-71.2	-0.4
M5-2		-40.6	-3.6		-41.2	-3.7		-40.3	-3.8
M5-4		61.6	-16.3		63.6	-16.4		60.3	-16.2
M5-6		41.1	-18.3		40.5	-18.6		40.9	-18.2
Mv6	246.1			248.8			246.0		
MG6		-8.0	36.7		-8.4	37.1		-8.2	36.9
M6-5		-41.1	23.2		-42.9	22.6		-40.9	23.1
M6-11		7.3	3.5		7.5	3.6		7.3	3.7
M6-12		7.8	2.6		7.4	2.5		7.6	2.6
M6-13		17.9	7.4		17.3	7.1		17.8	7.5
Mv7	244.3			244.4			244.1		
MG7		0.4	11.9		0.4	12.2		0.9	11.9

Measurement	The Base case value			The Measured value			The Estimated value		
	KV	MW	MVAR	KV	MW	MVAR	KV	MW	MVAR
M7-4		-27.4	22.6		-26.5	22.3		-27.1	22.3
M7-8		-0.1	-16.9		-0.1	-16.9		-0.3	-16.6
M7-9		27.9	6.2		29.0	6.2		28.4	6.2
Mv8	250.7			253.2			250.4		
MG8		0.1	17.3		0.1	16.6		0.3	17.0
M8-7		0.1	17.3		0.1	17.1		0.3	17.0
Mv9	242.9			240.8			242.7		
MG9		-28.8	10.0		-29.5	10.1		-29.3	10.0
M9-4		-15.5	7.7		-15.2	7.3		-15.5	7.6
M9-7		-27.9	-5.4		-27.8	-5.2		-28.4	-5.3
M9-10		5.3	4.3		5.4	4.2		5.3	4.4
M9-14		9.4	3.5		9.0	3.3		9.3	3.4
Mv10	241.7			243.9			241.5		
MG10		-9.1	-5.9		-9.1	-6.1		-9.0	-6.0
M10-9		-5.3	-4.2		-5.2	-4.4		-5.3	-4.3
M10-11		-3.8	-1.6		-4.0	-1.7		-3.7	-1.7
Mv11	243.1			241.4			242.9		
MG11		-3.4	-1.7		-3.5	-1.8		-3.5	-1.8
M11-6		-7.3	-3.4		-7.3	-3.3		-7.2	-3.6
M11-10		3.8	1.7		3.7	1.7		3.7	1.7
Mv12	242.6			239.8			242.5		
MG12		-6.0	-1.7		-6.0	-1.7		-5.8	-1.8
M12-6		-7.7	-2.4		-7.6	-2.3		-7.6	-2.5
M12-13		1.7	0.8		1.7	0.7		1.8	0.7
Mv13	241.5			241.9			241.4		
MG13		-13.8	-6.2		-14.3	-6.3		-13.9	-6.1
M13-6		-17.7	-7.0		-18.2	-7.0		-17.6	-7.0
M13-12		-1.7	-0.7		-1.6	-0.7		-1.8	-0.7
M13-14		5.6	1.5		5.6	1.5		5.4	1.6
Mv14	238.3			240.1			238.2		
MG14		-14.8	-4.7		-15.0	-4.8		-14.5	-4.6
M14-9		-9.3	-3.2		-9.0	-3.1		-9.2	-3.2
M14-13		-5.5	-1.4		-5.6	-1.5		-5.4	-1.5