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A Model for Optimal Design of Mixed Renewable Warranty Policy for Non-Repairable Weibull Life Products under Conflict between Customer and Manufacturer Interests

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Abstract**—**A model is presented to find the optimal design of the mixed renewable warranty policy for non-repairable Weibull life products. The optimal design considers the conflict of interests between the customer and the manufacturer: the customer interests are longer full rebate coverage period and longer total warranty coverage period, the manufacturer interests are lower warranty cost and lower risk. The design factors are full rebate and total warranty coverage periods. Results showed that mixed policy is better than full rebate policy in terms of risk and total warranty coverage period in all of the three bathtub regions. In addition, results showed that linear policy is better than mixed policy in infant mortality and constant failure regions while the mixed policy is better than linear policy in ageing region of the model. Furthermore, the results showed that using burn-in period for infant mortality products reduces warranty cost and risk.

Keywords**—**Reliability, Mixed warranty policy, Optimization, Weibull Distribution.

I. INTRODUCTION

AN important problem facing the manufacturers is the design of the warranty policy. Warranty can serve as a marketing tool to show the customer that the product is reliable. In one hand, warranty can increase customer satisfaction, which leads to an increase in the market share of the product and ultimately increases the profit. On the other hand, the warranty is considered as an obligation to the company and it will cost the company money; therefore, the warranty reduces the company's profit dramatically if it is not well designed. Accordingly, warranty policy must be optimized such that a balance between the customer and the manufacturer interests is reached.

For mixed warranty policy, the warranty starts with a full rebate period followed by a linearly prorated period such that if a unit failed within the full rebate period, a full compensation will be paid to the customer and if the unit failed after the full rebate period and before the end of the warranty duration, a linear compensation will be paid to him/her [1]. Thomas [2] has studied several configurations of warranty policies. Peter [3] studied the linear and full warranty policies for exponential life distribution under risk averse situation. Mutlu [4] has studied models to maximize the expected profit of products that are subjected to a burn-in and sold with warranty. The decision factors in his study were the burn-in time, warranty period, and price. Murthy and Djamaludin [5] provided a literature review on new product warranty policies. Number of different warranty policies and warranty cost analysis, among several other topics, were covered in their literature review. Shaharudin et.al. [6] studied the warranty factor, among other factors, that affect the customer satisfaction after sales.

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II. MODEL FORMULATION

In mixed warranty policy, the customers are mainly concerned about the full rebate and total warranty coverage periods while the manufacturers are mainly concerned about the warranty cost and risk associated with this cost. Warranty designers should consider these four conflicting objectives in their design. This study presents a model to find the optimal design of mixed renewable warranty policy for non-repairable Weibull life products under conflict customer and manufacturer interests. The design factors for this model are the full rebate coverage period and the total warranty coverage period that compromise between the customer's and the manufacturer's interests.

Following Peter [3], the per unit warranty cost under mixed warranty policy can be modeled mathematically as follows:

$$UC(t_i) = \begin{cases} R_0 & , 0 \le t_i \le \tau_1 \\ R_0 \frac{(\tau_2 - t_i)}{(\tau_2 - \tau_1)} & , \tau_1 \le t_i \le \tau_2 \end{cases}, \quad (1)$$

where, $UC(t_i)$ is the cost of failure under mixed warranty policy, t_i is the time between failures, τ_2 and τ_1 are total warranty coverage period and full rebate coverage period respectively, R_0 is the replacement cost for one failed unit. It should be noticed that the total warranty coverage period, τ_2, for Weibull products can be expressed as a function of failure percentile, P, as follows:

$$\tau_2 = \theta[-ln(1-P)]^{\frac{1}{\beta}}, \tag{2}$$

where β and θ are the shape and the characteristic life of the product respectively. This means that the decision factor τ_2 can be replaced with P.

The warranty cost (WC) can be found by adding all the unit warranty costs, $UC(t_i)$, for (N) failures that failed within time less than τ_2 as follows :

$$WC = \sum_{i=1}^{N} UC(t_i). \tag{3}$$

The expected warranty cost can be derived by taking the expectation of both sides of equation (3):

$$E[WC] = E[N]E[UC(t_i)]. \tag{4}$$

Since N has a geometric distribution with probability of success (τ_2) , $E[N]$ for Weibull products can be written as:

$$E[N] = \frac{1-e^{-\left(\frac{\tau_2}{\theta}\right)^{\beta}}}{e^{-\left(\frac{\tau_2}{\theta}\right)^{\beta}}}, \tag{5}$$

Furthermore, $E[UC(t_i)]$ can be written as:

$$E[UC(t_i)] = R_0 \int_0^{\tau_1} \frac{\beta}{\theta}\left(\frac{t_i}{\theta}\right)^{\beta-1} e^{-\left(\frac{t_i}{\theta}\right)^{\beta}} dt_i + \frac{R_0}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2}(\tau_2 - t_i)\frac{\beta}{\theta}\left(\frac{t_i}{\theta}\right)^{\beta-1} e^{-\left(\frac{t_i}{\theta}\right)^{\beta}} dt_i \tag{6}$$

Substituting equations (5) and (6) into equation (4) and do some mathematical arrangements, the expected warranty cost can be given as:

$$E[WC] = \frac{1-e^{-\left(\frac{\tau_2}{\theta}\right)^{\beta}}}{e^{-\left(\frac{\tau_2}{\theta}\right)^{\beta}}} \times \frac{c_o}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2} 1 - e^{-\left(\frac{t_i}{\theta}\right)^{\beta}} dt_i, \tag{7}$$

and the variance for the warranty cost , $Var[WC]$, can be given as:

$$Var[WC] = \frac{1-e^{-\left(\frac{\tau_2}{\theta}\right)^{\beta}}}{\left(e^{-\left(\frac{\tau_2}{\theta}\right)^{\beta}}\right)^2}\left(e^{-\left(\frac{\tau_2}{\theta}\right)^{\beta}} \times E\left[\left(UC(t_i)\right)^2\right] + \left(1 - e^{-\left(\frac{\tau_2}{\theta}\right)^{\beta}}\right)\left(E[UC(t_i)]\right)^2\right), \tag{8}$$

where

$$E\left[\left(UC(t_i)\right)^2\right] = {R_0}^2 \int_0^{\tau_1} \frac{\beta}{\theta}\left(\frac{t_i}{\theta}\right)^{\beta-1} e^{-\left(\frac{t_i}{\theta}\right)^{\beta}} dt_i + \left(\frac{R_0}{\tau_2-\tau_1}\right)^2 \int_{\tau_1}^{\tau_2}(\tau_2 - t_i)^2 \frac{\beta}{\theta}\left(\frac{t_i}{\theta}\right)^{\beta-1} e^{-\left(\frac{t_i}{\theta}\right)^{\beta}} dt_i. \tag{9}$$

The equivalent time, at the same expected warranty cost, for mixed warranty policy and full rebate warranty policy can be found by equating the expected warranty cost of the mixed warranty policy with the expected warranty cost of the full rebate warranty policy, the equivalent time, τ_0, can be given as:

$$\tau_0 = \theta[-ln(x_0)]^{\frac{1}{\beta}}, \tag{10}$$

where x_0 is the positive root of the following quadratic equation:

$$1 + x^2 - x(2 + E[WC]) = 0, \tag{11}$$

where $x = e^{-\left(\frac{\tau_2}{\theta}\right)^{\beta}}$.

The variance of the full rebate warranty policy at total warranty time τ_0, $Var[FWC(\tau_0)]$, can be given as:

$$Var[FWC(\tau_0)] = \frac{{R_0}^2\left(1-e^{-\left(\frac{\tau_0}{\theta}\right)^{\beta}}\right)^2}{\left(e^{-\left(\frac{\tau_0}{\theta}\right)^{\beta}}\right)^2}\left(\left(e^{-\left(\frac{\tau_0}{\theta}\right)^{\beta}}\right)^2 + 1 - e^{-\left(\frac{\tau_0}{\theta}\right)^{\beta}}\right). \tag{12}$$

The optimal mixed warranty policy can be found by determining $,\tau_1$ and τ_2 that will compromise between the four conflicting interests. The objective function, i.e. total satisfaction level, can be written as the weighted average of the linear scale of these four conflicting interests.

Each interest will have a linear membership function that will transfer its value to a (0,1) scale and then a weight is assigned to each interest to calculate the total satisfaction level of the policy as a weighted average.

The satisfaction level for each interest can be calculated using the following linear scale transformation:

$$\mu_{interest} = \frac{interest\ value - LB}{UB - LB}, \tag{13}$$

where *LB* and *UB* are the lower bound and the upper bound of the interest respectively. Thus the total satisfaction level for policy *i* can be given as:

$$\mathrm{TSL}(\tau_1, \mathrm{P}, \beta, \theta)_{\mathrm{i}} = -w_1 \times \mu_{E[\mathrm{WC}|\tau_1,\mathrm{P},\beta,\theta,R_0]_{\mathrm{i}}} - w_2 \times \mu_{Var[\mathrm{WC}|\tau_1,\mathrm{P},\beta,\theta,R_0]_{\mathrm{i}}} + w_3 \times \mu_{\tau_{1_{\mathrm{i}}}} + w_4 \times \mu_{\tau_2(\beta,\theta,\mathrm{P})_{\mathrm{i}}}, \tag{14}$$

where $\sum_{k=1}^{4} w_k = 1$.

The desired optimal plan is the plan corresponding to max ({TSL_i}).

The weights should be chosen to reflect the relative importance of the four interests. High w_1 and w_2 means that the manufacturer is concerned more about his/her interests (cost and variance) rather than the customer's interests. High w_3 and w_4 means that the manufacturer is concerned more about customer's interests (total warranty coverage period and full compensation time) rather than his/her interests.

The proposed model can be expressed mathematically as follows:

$$\text{Max}\left(-w_1 \times \mu_{E[\mathrm{WC}|\tau_1,\mathrm{P},\beta,\theta,R_0]} - w_2 \times \mu_{Var[\mathrm{WC}|\tau_1,\mathrm{P},\beta,\theta,R_0]} + w_3 \times \mu_{\tau_1} + w_4 \times \mu_{\tau_2(\beta,\theta,\mathrm{P})}\right),$$

s.t

$$\mathrm{LCB} \le E[\mathrm{WC}|\tau_1, \mathrm{P}, \beta, \theta, R_0] \le \mathrm{UCB}$$
$$\mathrm{LVB} \le Var[\mathrm{WC}|\tau_1, \mathrm{P}, \beta, \theta, R_0] \le \mathrm{UVB}$$
$$P_l \le \mathrm{P} \le P_h$$
$$0 \le \tau_1 \le \theta[-ln(1-P)]^{\frac{1}{\beta}}$$
$$\sum_{k=1}^{4} w_k = 1$$
$$P_l < P_h$$
$$0 < P_l, P_h < 1$$
$$\tau_1, \mathrm{P}, \beta, \theta, R_0 \ge 0$$

III. Results and Discussion

The optimal mixed warranty policy will be calculated for the following parameters:

$\beta = 5, 6,$ and 7,
θ between 109 and 118,
$P_l = 1\%$ and $P_h = 25\%$,
$w_1 = 0.25, w_2 = 0.1, w_3 = 0.15,$ and $w_4 = 0.5$,
$R_0 = 250$,
$\mathrm{LCB} = \mathrm{LVB} = 0$,
$\mathrm{UCB} = 15$, and
$\mathrm{UVB} = 100$.

Table I, shows the effect of the scale and shape parameters on the optimal design of the warranty policy in the ageing region of the bathtub model.

TABLE I
THE EFFECT OF THE SCALE AND SHAPE PARAMETERS ON THE OPTIMAL DESIGN OF THE WARRANTY POLICY IN THE AGEING REGION OF THE BATHTUB MODEL

Case	τ_1	τ_2	Cost	Var.	θ
			$\beta = 7$		
1	36	91	4.43	21	109
2	36	92	4.52	21	110
3	39	92	4.18	21	111
4	39	93	4.26	21	112
5	39	94	4.35	21	113
6	39	95	4.43	21	114
7	38	96	4.43	21	115
8	38	97	4.51	21	116
9	41	97	4.19	21	117
10	41	98	4.27	21	118
			$\beta = 6$		
11	33	88	4.69	22	109
12	32	89	4.70	22	110
13	32	90	4.80	22	111
14	35	90	4.52	22	112
15	35	91	4.618	22	113
16	34	92	4.63	22	114
17	34	93	4.72	22	115
18	34	94	4.81	23	116
19	34	95	4.90	23	117
20	36	95	4.56	22	118
			$\beta = 5$		
21	28	84	4.93	24	109
22	28	85	5.04	24	110
23	28	86	5.15	24	111
24	27	87	5.18	24	112
25	27	88	5.29	24	113
26	29	88	4.99	24	114
27	29	89	5.09	24	115
28	29	90	5.203	24	116
29	28	91	5.22	24	117
30	30	91	4.94	24	118

Table I shows that as the shape parameter decrease the optimal full rebate and the optimal total warranty coverage times decrease (at the same scale parameter). Furthermore, as the scale parameter increases the optimal full rebate and the optimal total warranty coverage times increase (at the same shape parameter). On the other hand, as the shape parameter increases the optimal cost and the optimal variance of the warranty policy decreases.

Table II, compares the mixed warranty policy with the full rebate warranty policy at the same cost.

Table II shows that the duration of the full rebate warranty policy is less than the duration of the mixed warranty policy while the variance of the full rebate warranty policy is higher than the variance of the mixed warranty policy. This means that, in the ageing region of the bathtub model, the mixed warranty policy is better for the manufacturer in terms of risk and also better for the customer in terms of coverage period.

TABLE II
MIXED WARRANTY POLICY AND FULL REBATE WARRANTY POLICY AT THE SAME WARRANTY COST

Mixed warranty policy					Full rebate warranty policy	
Case	τ_1	τ_2	Var.	θ	τ_0	Var.
			$\beta = 7$			
1	36	91	21	109	82	34
2	36	92	21	110	83	34
3	39	92	21	111	83	33
4	39	93	21	112	84	33
5	39	94	21	113	85	33
6	39	95	21	114	85	34
7	38	96	21	115	86	34
8	38	97	21	116	87	34
9	41	97	21	117	87	33
10	41	98	21	118	88	33
			$\beta = 6$			
11	33	88	22	109	78	35
12	32	89	22	110	79	35
13	32	90	22	111	80	35
14	35	90	22	112	80	34
15	35	91	22	113	81	34
16	34	92	22	114	82	34
17	34	93	22	115	83	35
18	34	94	23	116	83	35

19	34	95	23	117	84	35
20	36	95	22	118	85	34
$\beta = 5$						
21	28	84	24	109	74	35
22	28	85	24	110	74	36
23	28	86	24	111	75	36
24	27	87	24	112	76	36
25	27	88	24	113	77	37
26	29	88	24	114	77	36
27	29	89	24	115	78	36
28	29	90	24	116	79	36
29	28	91	24	117	79	37
30	30	91	24	118	80	35

The same analysis will be conducted for Weibull products in infant mortality and constant failure regions of the bathtub model.

Table III, shows the effect of the scale and shape parameters on the optimal design of the mixed warranty policy in the infant mortality and constant failure regions of the bathtub model.

TABLE III
THE EFFECT OF THE SCALE AND SHAPE PARAMETERS ON THE OPTIMAL DESIGN OF THE MIXED WARRANTY POLICY IN THE INFANT MORTALITY AND CONSTANT FAILURE REGIONS OF THE BATHTUB MODEL

Case	τ_1	τ_2	Cost	Var.	θ
$\beta = 0.6$					
1	0	1057	11.12	48	9900
2	0	1068	11.12	48	10000
3	0	1079	11.12	48	10100
4	0	1089	11.11	48	10200
5	0	1100	11.12	48	10300
6	0	1111	11.12	48	10400
7	0	1121	11.11	48	10500
8	0	1132	11.12	48	10600
9	0	1143	11.12	48	10700
10	0	1153	11.11	48	10800
$\beta = 0.8$					
11	0	2085	12.08	48	9900
12	0	2106	12.08	48	10000
13	0	2127	12.08	48	10100
14	0	2149	12.09	49	10200
15	0	2170	12.09	49	10300
16	0	2191	12.09	49	10400
17	0	2212	12.09	49	10500
18	0	2233	12.09	48	10600
19	0	2254	12.09	48	10700
20	0	2275	12.09	48	10800
$\beta = 1$					
21	0	2848	10.92	45	109
22	0	2876	10.91	45	110
23	0	2905	10.91	45	111
24	0	2934	10.92	45	112
25	0	2963	10.92	45	113
26	0	2991	10.91	45	114
27	0	3020	10.91	45	115
28	0	3049	10.92	45	116
29	0	3078	10.92	45	117
30	0	3106	10.91	45	118

Table III shows that the full rebate coverage period is zero which indicates that the best warranty policy is the linear warranty policy in these regions of the bathtub model. The same general trend found in Table I is also found here regarding the total warranty coverage time, as the shape and scale parameter increases the total warranty coverage time increases.

Table IV, compares the mixed warranty policy with the full rebate warranty policy at the same cost in the infant mortality and constant failure regions of the bathtub model.

TABLE IV
THE MIXED WARRANTY POLICY AND THE FULL REBATE WARRANTY POLICY AT THE SAME COST IN THE INFANT MORTALITY AND CONSTANT FAILURE REGIONS OF THE BATHTUB MODEL

Mixed warranty policy without burn-in						Full rebate warranty policy	
Case	τ_1	τ_2	Var.	θ	Cost	τ_0	Var.
$\beta = 0.6$							
1	0	1057	48	9900	11.1	737	54
2	0	1068	48	10000	11.1	745	54
3	0	1079	48	10100	11.1	753	54
4	0	1089	48	10200	11.1	760	54
5	0	1100	48	10300	11.1	767	54
6	0	1111	48	10400	11.1	775	54
7	0	1121	48	10500	11.1	782	54
8	0	1132	48	10600	11.1	790	54
9	0	1143	48	10700	11.1	797	54
10	0	1153	48	10800	11.1	804	54
$\beta = 0.8$							
11	0	2085	48	9900	12.1	1487	56
12	0	2106	48	10000	12.1	1502	56
13	0	2127	48	10100	12.1	1516	56
14	0	2149	49	10200	12.1	1532	56
15	0	2170	49	10300	12.1	1547	56
16	0	2191	49	10400	12.1	1562	56
17	0	2212	49	10500	12.1	1577	56
18	0	2233	48	10600	12.1	1592	56
19	0	2254	48	10700	12.1	1607	56
20	0	2275	48	10800	12.1	1622	56
$\beta = 1$							
21	0	2848	45	9900	10.92	2065	53
22	0	2876	45	10000	10.91	2086	53
23	0	2905	45	10100	10.91	2107	53
24	0	2934	45	10200	10.92	2128	53
25	0	2963	45	10300	10.92	2149	53
26	0	2991	45	10400	10.91	2169	53
27	0	3020	45	10500	10.91	2190	53
28	0	3049	45	10600	10.92	2211	53
29	0	3078	45	10700	10.92	2232	53
30	0	3106	45	10800	10.91	2252	53

From Table IV, the same general trend found in Table III is also found here. The full compensation period is zero which indicates that the linear warranty policy is better than the mixed warranty policy without burn-in period in these regions of the bathtub model. The duration of the full rebate warranty policy is less than the duration of the linear warranty policy while the variance of the full rebate warranty policy is higher than the variance of the linear warranty policy. This means that the linear warranty policy is better for the manufacturer in terms of risk and also better for the customer in terms of coverage period in these regions too.

The effect of the burn-in period on the mixed warranty policy is discussed next, Tables V, compares the mixed warranty policy with the full rebate policy at the same cost under burn-in period of 1% of the characteristic life of the product .

TABLE V
THE MIXED WARRANTY POLICY AND THE FULL REBATE WARRANTY POLICY AT THE SAME COST UNDER BURN-IN PERIOD OF 1% OF THE CHARACTERISTIC LIFE OF THE PRODUCT

Mixed warranty policy with burn-in						Full rebate warranty policy	
Case	τ_1	τ_2	Var.	θ	Cost	τ_0	Var.
$\beta = 0.6$							
1	0	1057	37.0	9900	6.42	714	36.2
2	0	1068	36.7	10000	6.31	713	35.9
3	0	1079	36.4	10100	6.20	712	35.5
4	0	1089	36.1	10200	6.10	711	35.1
5	0	1100	35.8	10300	6.00	709	34.8
6	0	1111	35.6	10400	5.90	708	34.4
7	0	1121	35.3	10500	5.81	707	34.1
8	0	1132	35.0	10600	5.71	706	33.8
9	0	1143	34.8	10700	5.62	705	33.4
10	0	1153	34.5	10800	5.53	704	33.1
$\beta = 0.8$							
11	0	2085	47.9	9900	11.36	1545	51.4
12	0	2106	47.5	10000	11.15	1544	50.9
13	0	2127	47.1	10100	10.95	1543	50.4
14	0	2149	46.6	10200	10.76	1542	49.9
15	0	2170	46.2	10300	10.57	1541	49.4
16	0	2191	45.8	10400	10.38	1540	48.9
17	0	2212	45.4	10500	10.20	1538	48.4
18	0	2233	45.0	10600	10.03	1537	48.0
19	0	2254	44.6	10700	9.86	1536	47.5
20	0	2275	44.2	10800	9.69	1535	47.1
$\beta = 1$							
21	0	2848	47.9	9900	12.26	2188	54.0
22	0	2876	47.5	10000	12.00	2187	53.4
23	0	2905	47.1	10100	11.75	2185	52.8
24	0	2934	46.6	10200	11.51	2184	52.2
25	0	2963	46.2	10300	11.27	2183	51.6
26	0	2991	45.8	10400	11.04	2181	51.0
27	0	3020	45.4	10500	10.82	2180	50.4
28	0	3049	45.0	10600	10.60	2179	49.9
29	0	3078	44.6	10700	10.39	2177	49.4
30	0	3106	44.2	10800	10.18	2176	48.8

From Table V, once again the linear warranty policy was selected over the mixed warranty policy. It is clear that the burn-in period can help in reducing the warranty cost and the variance in infant mortality region of the bathtub model and will increase the cost and the variance in the constant failure region of the bathtub region (as a result of lack of memory property of the exponential distribution, $\beta = 1$).

IV. CONCLUSION

Based on the results, the mixed warranty policy is better than the full rebate warranty policy in terms of risk and warranty coverage period in all regions of the bathtub model for Weibull products. In infant mortality and constant failure regions of the bathtub model, linear warranty policy is better than mixed warranty policy while in ageing region, mixed warranty policy is better than linear warranty policy. For infant mortality products, burn-in period helps in reducing the cost and the risk of the mixed warranty policy.

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