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A Method for Solving a Bi-Objective Transportation Problem under Fuzzy Environment

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Abstract**—**A bi-objective fuzzy transportation problem with the objectives to minimize the total fuzzy cost and fuzzy time of transportation without according priorities to them is considered. To the best of our knowledge, there is no method in the literature to find efficient solutions of the bi-objective transportation problem under uncertainty. In this paper, a bi-objective transportation problem in an uncertain environment has been formulated. An algorithm has been proposed to find efficient solutions of the bi-objective transportation problem under uncertainty. The proposed algorithm avoids the degeneracy and gives the optimal solution faster than other existing algorithms for the given uncertain transportation problem.

Keywords**—**Transportation problem, efficient solution, ranking function, fuzzy transportation problem.

I. Introduction

A certain class of mathematical programming problem arises very frequently in practical applications. For example, a product may be transported from factories to retail stores. The factories are the sources and the stores are the destinations. The amount of the product which is available and the demand are also known. The problem is that the different legs of the network joining the sources to the destinations have different costs associated with them. The aim is to find the minimum cost routing of products from supply point to destination, this problem is widely known as the cost minimizing transportation problem. The transportation problem with a single objective to minimize the duration of transportation has been studied in detail by many researchers [19]-[22]. And also with multiple objectives has been discussed in [15]-[17]. As in practical life, decision makers do not have the exact transportation cost and time; then, there exists the uncertainty about the cost and time. Therefore, it is very interesting to deal with the transportation problem in fuzzy environment. The idea of fuzzy set was first proposed by Zadeh [25], as a mean of handling uncertainty that is due to imprecision rather than to randomness. Bellmann and Zadeh [2] presented the technique of decision making process in fuzzy environment. After that, many authors have studied fuzzy linear programming problem techniques such as Fang [7], Rommelfanger [18] and Tanaka et al. [24] etc.

In literature, we find that there many transportation models where fuzzy linear programming have been applied or approaches to solve multi-objective fuzzy transportation problem. From this view point, Chanas [5] proposed a fuzzy programming in multi-objective linear programming solved by parametric approach. Tanakka and Asai [23] introduced fuzzy linear programming problem in fuzzy environment. Zimmermann [26] proposed a fuzzy multi-criteria decision making set, by using intersection of all fuzzy goals and constraints. Lai-Hawng [10] considered multi-objective linear programming problem with all parameters, having a triangular possibility distribution. Bit [3] considered fuzzy programming approach to a multi-criteria decision making transportation problem in which the constraints are of equality types. Later, Bit et al. [4] also considered a fuzzy programming approach to multi-objective solid transportation problem. And other several authors [1], [6], [8], [14] have proposed different models for solving fuzzy multi-objective transportation problems.

In this paper, we define an algorithm that has been proposed to find the fuzzy optimal value of a bi-objective fuzzy transportation problem. The technique gives the optimal solution faster than other existing techniques. It also reduces the computational work.

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II. Preliminaries

In this section, basic definitions, arithmetic operations and ranking functions are reviewed [9], [11].

A. Basic Definitions

In this section some basic definitions are reviewed [9].

Definition 1: The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in *X*. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the universal set *X* falls within a specified range [0, 1] i.e., $\mu_{\tilde{A}}(x): X \longrightarrow \{0,1\}$. The assigned value indicates the membership grade of the element in the set *A*.

The function $\mu_{\tilde{A}}$ is called the membership function and the *A* set $\tilde{A}=\{(x,\mu_{\tilde{A}}(x));x\in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2: A fuzzy set $\tilde{A}$, defined on the universal set of real number *R*, is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}(x): X \longrightarrow \{0,1\}$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, c] \cup [d, \infty)$.
3. Is strictly increasing on [*c*, *a*] and strictly decreasing on [*b*, *d*].
4. $x \in [a, b]$ for all $x \in [a, b]$.

Definition 3: A fuzzy number $\tilde{A} = (a, b, c, d)$, is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \le x < b \\ 1, & b \le x \le c \\ \frac{(c-x)}{(d-c)}, & c < x \le d \end{cases}$$

where $a, b, c, d \in R$.

B. Arithmetic Operations

In this subsection, arithmetic operations between two trapezoidal fuzzy numbers, defined on a universal set of real numbers *R*, are reviewed [9].

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers, then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$.

(ii) $\tilde{A}_1 \Theta \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$.

(iii) $\lambda \otimes \tilde{A}_2 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1) & \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1) & \lambda < 0 \end{cases}$.

C. Ranking Function

A convenient method for comparing fuzzy numbers is by using ranking function [12], [13]. A ranking function $\Re : F(R) \longrightarrow R$, where *F*(*R*) set of all fuzzy numbers defined on set of real numbers defined on set of real numbers, maps each fuzzy number into a real number. Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers, then:

(i) $\tilde{A} \underset{\Re}{\ge} \tilde{B}$ if $\Re(\tilde{A}) \ge \Re(\tilde{B})$.

(ii) $\tilde{A} \underset{\Re}{>} \tilde{B}$ if $\Re(\tilde{A}) > \Re(\tilde{B})$

(iii) $\tilde{A} \underset{\Re}{=} \tilde{B}$ if $\Re(\tilde{A}) = \Re(\tilde{B})$

III. Formulation of Bi-Objective Fuzzy Transportation Problem

Suppose there are *m* sources and *n* destinations. Let $a_i\,(i = 1, 2, ..., m)$ be the unit availability at source *i*, $b_j\ (j=1,2,...,n)$ be the unit demand at the destination *j*, $\tilde{c}_{ij}\ (i = 1, 2, ..., m; j = 1, 2, ..., n)$ be the fuzzy cost of transportation of unit homogeneous product from source *i* to destination *j*, $\tilde{t}_{ij}\ (i = 1, 2, ..., m;\ j = 1, 2, ..., n)$ be the fuzzy time of transportation of unit homogeneous product from i^{th} source to j^{th} destination and $x_{ij}(i = 1, 2, ..., m;\ j = 1, 2, ..., n)$ is the variable assuming the value 0 or 1 according as the entire requirement of destination *j* is not met or met from source *i*.

Let $\tilde{C}$ and $\tilde{T}$ denote the total fuzzy cost and fuzzy time of transportation respectively. The mathematical formulation of the problem is as follows. Determine x_{ij}'s which minimize the two-objective functions:

$$\tilde{C} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes x_{ij} \quad (1)$$

$$\tilde{T} = \max\{\tilde{t}_{ij} \otimes x_{ij}; i = 1, 2, ..., m;\ j = 1, 2, ..., n\} \quad (2)$$

without according priorities to them, subject to the constraints,

$$\sum_{j=1}^{n} b_j x_{ij} \le a_i;\ (i = 1, 2, ..., m) \quad (3)$$

$$\sum_{i=1}^{m} x_{ij} = 1\ ;\ (j = 1, 2, ..., n) \quad (4)$$

$$x_{ij} = 0 \text{ or } 1\ (i = 1, 2, ...m; j = 1, 2, ..., n) \quad (5)$$

IV. Solution Procedure

The proposed algorithm has three subparts, as given below, to find the fuzzy efficient optimal solution of the fuzzy bi-objective transportation problem.

A. Conversion of Two Objectives into a Sequence of Single Objective

Here we use the following process to convert bi-objective fuzzy transportation problem in single objective fuzzy transportation problem as follows,

Step 1. The set $\tilde{t}_{ij}\,(i = 1, 2, ..., m;\ j = 1, 2, ..., n)$ is partitioned into subsets $L_k(k = 1, 2, ..., q)$ in the following way. Each of the subsets L_k's consists of the $\tilde{t}_{ij}$'s having the same fuzzy value. L_1 consists of the $\tilde{t}_{ij}$ having the largest fuzzy time, L_2 consists of the $\tilde{t}_{ij}$ having the next largest fuzzy time, and so on, L_q consists of the $\tilde{t}_{ij}$ having the smallest fuzzy time.

Step 2. Preemptive priority factors $M_0, M_1, ..., M_q$ are assigned to $\tilde{C}, \sum_{L_0} x_{ij}, \sum_{L_1} x_{ij}, ..., \sum_{L_q} x_{ij}$ respectively. Here $\sum_{L_k} x_{ij}$ is the sum of the x_{ij} corresponding to the $\tilde{t}_{ij}$ belonging to L_k. All the priority factors M_k's are fixed positive real numbers and are such that the expression $\sum_{k=0}^{q} \alpha_k M_k$, where α_k's are real numbers which can be negative or zero or positive, has the same sign as the non-zero α_k with the smallest subscript in it irrespective of the values of other α_k's. This implies that $M_0, M_1, ..., M_q$ are such that $M_0 >> M_1 >> ... >> M_q$. (The symbol >> indicates that the quantity on its left side arbitrarily large compared to right hand side).

Step 3. After this, the cost–time trade-off fuzzy transportation problem with $\tilde{C}$ and $\tilde{T}$ as the first and second priority objectives, respectively, is reduced to an equivalent single-objective fuzzy transportation problem seeking to determine x_{ij}'s which minimize

$$Z = M_0 \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes x_{ij} + \sum_{k=1}^{q} M_k \sum_{L_k} x_{ij} \qquad (6)$$

subject to the constraints (3)–(5).

B. Proposed Algorithm to Obtain Fuzzy Efficient Solution

Step I: The single objective fuzzy transportation problem, obtained in Section IV.A, is transformed into the tabular form.

Step II: Consider a set S having the cells (i, j) which has the fuzzy cost with minimum rank among each entries of its corresponding row and column in the obtained table.

Step III: Calculate $\tilde{P}_{ij}$ for each cell $(i, j) \in S$ .where

$$\tilde{P}_{ij} = \frac{\text{Sum of fuzzy costs of nearest adjacent sides of cell } (i, j)}{\text{Number of fuzzy costs added}}.$$

Step IV: Allocate the cell (i, j) for which rank of $\tilde{P}_{ij}$ i.e. $\Re(\tilde{P}_{ij})$ is maximum. If two or more $\Re(\tilde{P}_{ij})$'s have the same values then allocate that cell which has least cost among all cells for which $\Re(\tilde{P}_{ij})$'s are equal. Again, if the costs of these cells are equal then randomly allocate that cell for which $a_i \neq b_j$.

Step V: Check whether the requirement of each destination is fulfilled or not. If not then repeat

Step II-V, else, the obtained fuzzy solution is our fuzzy optimal solution of fuzzy transportation problem.

C. Procedure to Obtain 2nd Subsequent and Efficient Fuzzy Solution

After finding the first efficient solution, $x_{ij}^{(1)}$ has been obtained of given fuzzy transportation problem. The second efficient solution $x_{ij}^{(2)}$ is obtained by deleting all the cells (i, j) corresponding to $\Re(\tilde{t}_{ij}) \geq \Re(\tilde{T}(x_{ij}^{(1)}))$. The resultant problem is designated the second efficient solution $x_{ij}^{(2)}$. Further, the third efficient solution is obtained by deleting those cells (i, j) in the second cost-time trade-off fuzzy transportation problem in fuzzy environment, that correspond to the $\Re(\tilde{t}_{ij}) \geq \Re(\tilde{T}(x_{ij}^{(2)}))$. Subsequent efficient solutions are obtained by proceeding exactly in the same way.

V. Numerical Example

In this section, a numerical problem is considered of four origins and five destinations and applies the algorithm as explained in Section IV. The tableau representation of the numerical problem is given in Table I. The upper entries denote the fuzzy cost of unit product which have to transport from i^{th} origin to j^{th} destination and the lower entries denote the fuzzy time of transportation from i^{th} origin to j^{th} destination.

TABLE I
BI-OBJECTIVE FUZZY TRANSPORTATION PROBLEM

	D_1	D_2	D_3	D_4	D_5	a_i
O_1	$(0,1,2,5)$ $(1,3,4,8)$	$(1,2,3,6)$ $(1,3,4,8)$	$(1,2,3,6)$ $(3,7,10,20)$	$(2,5,7,14)$ $(3,5,8,16)$	$(0,0.5,1.5,2)$ $(2,5,7,14)$	5
O_2	$(1,3,4,8)$ $(1,3,4,8)$	$(0,0.5,1.5,2)$ $(2,5,7,14)$	$(0,0.5,1.5,2)$ $(5,7,12,24)$	$(0,1,2,5)$ $(5,9,14,28)$	$(3,5,8,16)$ $(3,5,8,16)$	4
O_3	$(0,0.5,1.5,2)$ $(3,5,8,16)$	$(2,5,7,14)$ $(0,1,2,5)$	$(5,6,11,22)$ $(1,3,4,8)$	$(0,0.5,1.5,2)$ $(1,3,4,8)$	$(1,4,5,10)$ $(1,3,4,8)$	3
O_4	$(9,11,20,40)$ $(1,3,4,8)$	$(13,17,30,60)$ $(2,4,6,12)$	$(3,7,10,20)$ $(2,5,7,14)$	$(0,1,2,5)$ $(0,0.5,1.5,2)$	$(1,4,5,10)$ $(0,0.5,1.5,2)$	2
b_j	3	3	2	2	1	

In Table I, the upper entries of cell (i, j) depicts the unit fuzzy cost and the lower entries of a cell (i, j) depict fuzzy time of fuzzy transportation from origin O_i to destination D_j. In the last row and column, b_j and a_i depicts the units of the commodity required at the destinations and available at the origins, respectively. The numerical problem seeks to determine x_{ij}'s which minimize the two objective functions,

$$\tilde{C} = (0,1,2,5)x_{11} \oplus (1,2,3,6)x_{12} \oplus (1,2,3,6)x_{13} \oplus (2,5,7,14)x_{14} \oplus (0,0.5,1.5,2)x_{15} \oplus (1,3,4,8)x_{21}$$
$$\oplus (0,0.5,1.5,2)x_{22} \oplus (0,0.5,1.5,2)x_{23} \oplus (0,1,2,5)x_{24} \oplus (3,5,8,16)x_{25} \oplus (0,0.5,1.5,2)x_{31}$$
$$\oplus (2,5,7,14)x_{32} \oplus (5,6,11,22)x_{33} \oplus (0,0.5,1.5,2)x_{34} \oplus (1,4,5,10)x_{35} \oplus (9,11,20,40)x_{41}$$
$$\oplus (13,17,30,60)x_{42} \oplus (3,7,10,20)x_{43} \oplus (0,1,2,5)x_{44} \oplus (1,4,5,10)x_{45}$$

$$\tilde{T} = \max\{\tilde{t}_{ij} \otimes x_{ij} : i = 1,2,3,4;\ j = 1,2,3,4,5\}.$$

A. Solution Procedure

Using the procedure given in Section IV.A, the single-objective fuzzy transportation problem of numerical problem seeks to determine x_{ij}'s which minimize

$$\tilde{Z} = M_0((0,1,2,5)x_{11} \oplus (1,2,3,6)x_{12} \oplus (1,2,3,6)x_{13} \oplus (2,5,7,14)x_{14} \oplus (0,0.5,1.5,2)x_{15}$$
$$\oplus (1,3,4,8)x_{21} \oplus (0,0.5,1.5,2)x_{22} \oplus (0,0.5,1.5,2)x_{23} \oplus (0,1,2,5)x_{24} \oplus (3,5,8,16)x_{25}$$
$$\oplus (0,0.5,1.5,2)x_{31} \oplus (2,5,7,14)x_{32} \oplus (5,6,11,22)x_{33} \oplus (0,0.5,1.5,2)x_{34} \oplus (1,4,5,10)x_{35}$$
$$\oplus (9,11,20,40)x_{41} \oplus (13,17,30,60)x_{42} \oplus (3,7,10,20)x_{43} \oplus (0,1,2,5)x_{44} \oplus (1,4,5,10)x_{45})$$
$$\oplus (0,0.5,1.5,2)\{M_1(x_{24}) + M_2(x_{23}) + M_3(x_{13}) + M_4(x_{14} + x_{25} + x_{31}) + M_5(x_{15} + x_{22} + x_{43})$$
$$+ M_6(x_{42}) + M_7(x_{11} + x_{12} + x_{21} + x_{33} + x_{34} + x_{35} + x_{41}) + M_8(x_{32} + x_{44} + x_{45})$$

subject to the given constraints (3)-(5) after assigning numerical values to all the parameters therein.

The tableau representation of the single-objective fuzzy transportation problem is shown in Table II, where the cells (i, j)'s $(i = 1,2,3,4;\ j = 1,2,3,4,5)$ depict the fuzzy cost and it has been considered that $M_0 >> M_1 >> ... >> M_8$ while minimizing $\tilde{Z}$.

TABLE II
SINGLE OBJECTIVE FUZZY TRANSPORTATION PROBLEM

	D_1	D_2	D_3	D_4	D_5	a_i
O_1	$(0,1,2,5)M_0 \oplus$ $(0,0.5,1.5,2)M_7$	$(1,2,3,6)M_0 \oplus$ $(0,0.5,1.5,2)M_7$	$(1,2,3,6)M_0 \oplus$ $(0,0.5,1.5,2)M_3$	$(2,5,7,14)M_0 \oplus$ $(0,0.5,1.5,2)M_4$	$(0,0.5,1.5,2)M_0 \oplus$ $(0,0.5,1.5,2)M_5$	5
O_2	$(1,3,4,8)M_0 \oplus$ $(0,0.5,1.5,2)M_7$	$(0,0.5,1.5,2)M_0 \oplus$ $(0,0.5,1.5,2)M_5$	$(0,0.5,1.5,2)M_0 \oplus$ $(0,0.5,1.5,2)M_2$	$(0,1,2,5)M_0 \oplus$ $(0,0.5,1.5,2)M_1$	$(3,5,8,16)M_0 \oplus$ $(0,0.5,1.5,2)M_4$	4
O_3	$(0,0.5,1.5,2)M_0 \oplus$ $(0,0.5,1.5,2)M_4$	$(2,5,7,14)M_0 \oplus$ $(0,0.5,1.5,2)M_8$	$(5,6,11,22)M_0 \oplus$ $(0,0.5,1.5,2)M_7$	$(0,0.5,1.5,2)M_0 \oplus$ $(0,0.5,1.5,2)M_7$	$(1,4,5,10)M_0 \oplus$ $(0,0.5,1.5,2)M_7$	3
O_4	$(9,11,20,40)M_0 \oplus$ $(0,0.5,1.5,2)M_7$	$(13,17,30,60)M_0 \oplus$ $(0,0.5,1.5,2)M_6$	$(3,7,10,20)M_0 \oplus$ $(0,0.5,1.5,2)M_5$	$(0,1,2,5)M_0 \oplus$ $(0,0.5,1.5,2)M_8$	$(1,4,5,10)M_0 \oplus$ $(0,0.5,1.5,2)M_8$	2
b_j	3	3	2	2	1	

In the fuzzy matrix, given in Table II, the rank of the fuzzy cost of the cell (1,5), (2,2), (2,3), (3,1), (3,4) and (4,4) is minimum corresponding to their row and column. The values of $\tilde{P}_{ij}$ for these cells are,

$$\tilde{P}_{15} = \frac{(5,10,15,30)M_0 \oplus (0,1,3,4)M_4}{2} = (2.5,5,7.5,15)M_0 \oplus (0,0.5,1.5,2)M_4$$

$$\tilde{P}_{22} = \frac{(4,10.5,15.5,30)M_0 \oplus (0,0.5,1.5,2)M_2 \oplus (0,1,3,4)M_7 \oplus (0,0.5,1.5,2)M_8}{4}$$
$$= (1,2.625,3.875,7.5)M_0 \oplus (0,0.125,0.375,0.5)M_2 \oplus (0,0.25,0.75,1)M_7$$
$$\oplus (0,0.175,0.375,0.5)M_8$$

$$\tilde{P}_{23} = \frac{(6,9.5,17.5,35)M_0 \oplus (0,0.5,1.5,2)M_1 \oplus (0,0.5,1.5,2)M_3 \oplus (0,0.5,1.5,2)M_5 \oplus (0,0.5,1.5,2)M_7}{4}$$
$$= (1.5,8.375,4.375,8.75)M_0 \oplus (0,0.125,0.375,0.5)M_1 \oplus (0,0.125,0.375,0.5)M_3$$
$$\oplus (0,0.125,0.375,0.5)M_5$$

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$$\tilde{P}_{31} = \frac{(12,19,31,62)M_0 \oplus (0,1,3,4)M_7 \oplus (0,0.5,1.5,2)M_8}{3}$$
$$= (4,6.3,10.3,20.6)M_0 \oplus (0,0.3,1,1.3)M_7 \oplus (0,0.16,0.5,0.6)M_8$$

$$\tilde{P}_{34} = \frac{(6,12,20,42)M_0 \oplus (0,0.5,1.5,2)M_1 \oplus (0,1,3,4)M_7 \oplus (0,0.5,1.5,2)M_8}{4}$$
$$= (1.5,3,5,10.5)M_0 \oplus (0,0.125,0.375,0.5)M_1 \oplus (0,0.25,0.75,1)M_7 \oplus (0,0.125,0.375,0.5)M_8$$

$$\tilde{P}_{44} = \frac{(4,11.5,16.5,32)M_0 \oplus (0,0.5,1.5,2)M_5 \oplus (0,0.5,1.5,2)M_7 \oplus (0,0.5,1.5,2)M_8}{3}$$
$$= (1.3,3.83,5.5,10.6)M_0 \oplus (0,0.16,0.5,0.6)M_5 \oplus (0,0.16,0.5,0.6)M_7 \oplus (0,0.16,0.5,0.6)M_8$$

By using the ranking function, we can see that $\max\{\tilde{P}_{15}, \tilde{P}_{22}, \tilde{P}_{23}, \tilde{P}_{31}, \tilde{P}_{34}, \tilde{P}_{44}\} = \tilde{P}_{31}$, therefore allocate the cell (3, 1) and block S_3 and D_1, as shown in Table III.

TABLE III
AFTER 1ST ALLOCATION

	D_1	D_2	D_3	D_4	D_5	a_i
O_1	$(0,1,2,5)M_0 \oplus (0,0.5,1.5,2)M_7$	$(1,2,3,6)M_0 \oplus (0,0.5,1.5,2)M_7$	$(1,2,3,6)M_0 \oplus (0,0.5,1.5,2)M_3$	$(2,5,7,14)M_0 \oplus (0,0.5,1.5,2)M_4$	$(0,0.5,1.5,2)M_0 \oplus (0,0.5,1.5,2)M_5$	5
O_2	$(1,3,4,8)M_0 \oplus (0,0.5,1.5,2)M_7$	$(0,0.5,1.5,2)M_0 \oplus (0,0.5,1.5,2)M_5$	$(0,0.5,1.5,2)M_0 \oplus (0,0.5,1.5,2)M_2$	$(0,1,2,5)M_0 \oplus (0,0.5,1.5,2)M_1$	$(3,5,8,16)M_0 \oplus (0,0.5,1.5,2)M_4$	4
O_3	$(0,0.5,1.5,2)M_0 \oplus (0,0.5,1.5,2)M_4$ **(3)**	$(2,5,7,14)M_0 \oplus (0,0.5,1.5,2)M_8$	$(5,6,11,22)M_0 \oplus (0,0.5,1.5,2)M_7$	$(0,0.5,1.5,2)M_0 \oplus (0,0.5,1.5,2)M_7$	$(1,4,5,10)M_0 \oplus (0,0.5,1.5,2)M_7$	~~3~~
O_4	$(9,11,20,40)M_0 \oplus (0,0.5,1.5,2)M_7$	$(13,17,30,60)M_0 \oplus (0,0.5,1.5,2)M_6$	$(3,7,10,20)M_0 \oplus (0,0.5,1.5,2)M_5$	$(0,1,2,5)M_0 \oplus (0,0.5,1.5,2)M_8$	$(1,4,5,10)M_0 \oplus (0,0.5,1.5,2)M_8$	2
b_j	~~3~~	3	2	2	1	

On repeating the steps of algorithm (Section IV.B) until all the requirements of each destination is completed. The first fuzzy efficient optimal solution of the problem given in Table I is $x_{13}^{(1)} = 2, x_{15}^{(1)} = 1, x_{21}^{(1)} = 1, x_{22}^{(1)} = 3, x_{31}^{(1)} = 2, x_{44}^{(1)} = 2$ and the first efficient optimal value of fuzzy cost and fuzzy time is $\tilde{C}(x_{ij}^{(1)}) = (3,12,23,42), \tilde{T}(x_{ij}^{(1)}) = (3,7,10,20)$, respectively.

To obtain the next efficient fuzzy optimal solution, block all the cells (*i, j*) in the previous cost time trade-off fuzzy transportation problem, for which $\Re(\tilde{t}_{ij}) \geq \Re(\tilde{T}(x_{ij}^{(1)})) = 10$ units. Using this procedure, four fuzzy efficient optimal solutions have been obtained and shown in Table IV.

TABLE IV
SET OF FUZZY EFFICIENT SOLUTIONS

Fuzzy Efficient Solutions	Optimal Solution	Total Fuzzy Cost	Total Fuzzy Time
$x_{ij}^{(1)}$	$x_{13}^{(1)} = 2, x_{15}^{(1)} = 1, x_{21}^{(1)} = 1, x_{22}^{(1)} = 3, x_{31}^{(1)} = 2, x_{44}^{(1)} = 2$	$\tilde{C}(x_{ij}^{(1)}) = (3,12,23,42)$	$\tilde{T}(x_{ij}^{(1)}) = (3,7,10,20)$
$x_{ij}^{(2)}$	$x_{11}^{(2)} = 1, x_{12}^{(2)} = 1, x_{15}^{(2)} = 1, x_{22}^{(2)} = 3, x_{31}^{(2)} = 1, x_{34}^{(2)} = 2, x_{43}^{(2)} = 2$	$\tilde{C}(x_{ij}^{(2)}) = (7,20.5,35.5,65)$	$\tilde{T}(x_{ij}^{(2)}) = (3,5,8,16)$
$x_{ij}^{(3)}$	$x_{11}^{(3)} = 3, x_{15}^{(3)} = 1, x_{22}^{(3)} = 3, x_{33}^{(3)} = 1, x_{34}^{(3)} = 2, x_{43}^{(3)} = 1$	$\tilde{C}(x_{ij}^{(3)}) = (8,19,36,69)$	$\tilde{T}(x_{ij}^{(3)}) = (2,5,7,14)$
$x_{ij}^{(4)}$	$x_{12}^{(4)} = 3, x_{21}^{(4)} = 3, x_{33}^{(4)} = 2, x_{34}^{(4)} = 1, x_{44}^{(4)} = 1, x_{45}^{(4)} = 1$	$\tilde{C}(x_{ij}^{(4)}) = (17,32.5,51.5,103)$	$\tilde{T}(x_{ij}^{(4)}) = (1,3,4,8)$

VI. CONCLUSION

In this paper, a bi-objective fuzzy transportation problem has been formulated in fuzzy environment and an algorithm is proposed to find fuzzy efficient solutions. The solution obtained by the proposed algorithm shows that the decision maker has the more flexibility because the decision maker does not have the exact transportation cost and time, there then exists uncertainty about the cost and time. The proposed method provides the optimal solution faster than other existing methods for fuzzy transportation problems. It also reduces the computational work.

REFERENCES

[1] E. E. Ammar and E. A. Youness, "Study on multiobjective transportation problem with fuzzy numbers." *Appl. Math. Comput.,* Vol. 166, pp. 241–253, 2005.
[2] R. E. Bellmannand and L. A. Zadeh, "Decision making in fuzzy environment," Management sciences, Vol. 17, pp. 141-164, 1970.
[3] A. K. Bit, "Fuzzy programming with Hyperbolic membership functions for Multi-objective capacitated transportation problem, *OPSEARCH,* Vol. 41, pp. 106-120, 2004.
[4] A. K. Bit, M.P. Biswal and S.S. Alam, "Fuzzy programming approach to multicriteria decision making transportation problem," *Fuzzy sets and systems,* Vol. 50, pp. 35-41, 1992.
[5] S. Chanas, W. Kolodziejckzy and Machaj, "A fuzzy approach to the transportation problem," *Fuzzy Sets and Systems,* Vol. 13, pp. 211-221, 1984.
[6] S. K. Das, A. Goswami and S. S Alam, "Multiobjective transportation problem with interval cost, source and destination parameters." *Eur. J. Oper. Res.,* Vol. 117, pp. 100–112, 1999.
[7] S. C. Fang, C. F. Hu., H. F. Wang and S. Y. Wu., "Linear programming with fuzzy coefficients in constraints", Computers and mathematics with applications, Vol. 37, pp. 63-76, 1999.
[8] P. Gupta and M. K. Mehlawat, "An algorithm for a fuzzy transportation problem to select a new type of coal for a steel manufacturing unit," *Top.,* Vol. 15, pp. 114–137, 2007.
[9] A. Kaufmann and M. M. Gupta, "Introduction to fuzzy arithmetics, theory and applications," *Van Nostrand Reinhold*, New York, 1991.
[10] Y. J. Lai and C. L. Hawng, Fuzzy Mathematical Programming, *Lecture notes in Economics and Mathematical systems*, Springer-Verlag, 1992.
[11] T. S. Liou and M. J. Wang, "Ranking fuzzy numbers with integral values," Fuzzy sets and systems, Vol. 50, pp. 247-255, 1992.
[12] H. M. Nehi, H. R. Maleki and M. Mashinchi, "Solving fuzzy number linear programming problem by lexicographic ranking function," *Italian journal of pure and applied mathematics*, Vol. 16, pp. 9-20, 2004.
[13] A. A. Noora and P. Karami, "Ranking functions and its applications to fuzzy DEA," *International mathematical forum*, Vol. 3, pp. 1469-1480, 2008.
[14] S. Pramanik, T.K. Roy, "Multiobjective transportation model with fuzzy parameters: priority based fuzzy goal programming approach." *J. Transp. Syst. Eng. Inform. Technol*., Vol. 8, pp. 40–48, 2008.
[15] S. Prakash, "Transportation problem with objectives to minimizes the total cost and duration of transportation", *OPSEARCH,* Vol. 18, pp. 235-238, 1981.
[16] S. Prakash, A. K. Agarwal and S. Shah, "Non-dominated solutions of cost–time trade-off transportation and assignment problems", *OPSEARCH,* Vol. 25, pp. 126–131, 1988.
[17] S. Purushotam, S. Prakash and P. Dhyani, "A transportation problem with minimization of duration and total cost of transportation as high and low priority objectives respectively", *Bulletin of the technical university of Istanbul,* Vol. 37, pp. 1-11, 1984.
[18] H. Rommelfanger, J. Wolf and R. Hanuscheck, "Linear programming with fuzzy coefficients, Fuzzy sets and systems", Vol. 29, pp. 195-206, 1989.
[19] C. R. Seshan and V. G. Tikekar, "On Sharma-Sawrup algorithm for time minimizing transportation problems", *Proceeding of the Indian Academy of Sciences, Mathematical Sciences*, Vol. 89, pp. 101-102, 1980.
[20] J. K. Sharma and K. Sawrup, "Bi-level time minimizing transportation problem", *Discrete optimization*, Vol. 5, pp. 714–723, 1977.
[21] Sonia and M. C. Puri, "Two level hierarchical time minimizing transportation problem", *TOP.*, Vol. 12, pp. 301-330, 2004.
[22] Sonia, A. Khandelwal and M. C. Puri, "Bi-level time minimizing transportation problem", *Discrete optimization,* Vol. 5, pp. 714-723, 2008.
[23] H. Tanaka, K. Asai, "Fuzzy linear programming problems with fuzzy numbers", *Fuzzy Sets and Systems*, Vol. 13**,** pp. 1-10, 1984.
[24] H. Tanaka, Ichihashi and K. Asai, "A formulation of fuzzy linear programming based on comparison of fuzzy numbers", *Control and cybernetics,* Vol. 13, pp. 185-194, 1984.
[25] L. A. Zadeh, "Fuzzy sets", *Information and Control*, Vol. 8, pp. 338-353, 1965.
[26] H. J. Zimmermann, "Fuzzy programming and linear programming with several objective functions", *Fuzzy sets and System,* Vol. 1, pp. 45- 55, 1978.