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A Mathematical Representation for Mechanical Model Assessment: Numerical Model Qualification Method

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Abstract — This article illustrates a model selection management approach for virtual prototypes in interactive simulations. In those numerical simulations, the virtual prototype and its environment are modelled as a multi-agent system, where every entity (prototype, human, etc.) is modelled as an agent. In particular, virtual prototyping agents that provide mathematical models of mechanical behaviour in form of computational methods are considered. This work argues that selection of an appropriate model in a changing environment, supported by models' characteristics, can be managed by the determination *a priori* of specific exploitation and performance measures of virtual prototype models. As different models exist to represent a single phenomenon, it is not always possible to select the best one under all possible circumstances of the environment. Instead the most appropriate shall be selecting according to the use case. The proposed approach consists in identifying relevant metrics or indicators for each group of models (e.g. entity models, global model), formulate their qualification, analyse the performance, and apply the qualification criteria. Then, a model can be selected based on the performance prediction obtained from its qualification. The authors hope that this approach will not only help to inform engineers and researchers about another approach for selecting virtual prototype models, but also assist virtual prototype engineers in the systematic or automatic model selection.

Keywords — virtual prototype models; domain; qualification criterion; model qualification; model assessment; environmental modelling.

I. Introduction

MODELS are the keystone of mechanical simulations. Even if models are not perfect representation of reality [9], all the knowledge in science and engineering can be incorporated into them to resemble reality as much as possible. But, that is also impractical. Generally, some aspects of reality are preferred over others, as in computer scene graphics simulation or finite element analysis simulations. But, in the case of interactive mechanical simulations, much more aspects have to be incorporated. In these times, mechanical simulations have reached an important level of complexity in order to satisfy user's expectations. A clear example are the mechanical simulations of virtual prototypes with flexible components. More detailed prototypes are being constructed, and the expectation of real-time simulations is always imposed. Typically, the more complex a simulation becomes the less level of performance it achieves. An example of this disjunctive is found in the research of Mikchevitch et al. [15], [14] in the realistic simulation of assembly/disassembly of flexible parts. A virtual environment is created with two different component models: Real-Time Model (RTM) and Interactive Mechanical Model (IMM). The RTM is a simplified model to mimic the quasi-linear and non-linear deformation of a flexible component for a reduced range of forces, used for real-time generation of frames for an immersive VR display. The IMM accurately represents the behaviour of a flexible component for a great range of boundary conditions to provide realistic reactions and keep the models coherent with the Virtual Environment (VE). Two models are integrated due to the fact that non-linear mechanics is required to model the behaviour of flexible objects. Unfortunately, non-linear mechanics has not yet evolve to the level of computational simplicity found in linear mechanics. In the use of two models to achieve the simulation, area of opportunity found in that approach is the automation of *adaptable model selection*, given that it requires the activation of the IMM by the user with the possibility of producing immersion lost and consequently unnatural interaction.

To achieve natural interaction, a virtual prototype must satisfy simultaneously every aspect that defines or secures the interaction. Some of these aspects are objective, others are subjective. But all have to be considered. Notably, virtual prototypes can be regarded from two points of view [8]: from the computer graphics definition (VP_{CG}) and from the mechanical engineering definition (VP_{ME}). However, both definitions represent important aspects of current virtual prototypes. Mainly, the VP_{CG} is expected to be fast and realistic, while the VP_{ME} is expected to be exact and precise.

Consequently, models must be chosen carefully to preserve equilibrium between realism and speed. The ability of a VP model to balance that compromise is obtained by means of model qualification —which is presented in further sections. Thus, interactive simulation of VP requires real-time, reduced, realistic, and qualified models to achieve natural interaction.

The objective of this study is to explore the possibility of managing the selection of pertinent models by means of a proper model qualification. In this paper, model selection management is based on an *a priori* model qualification. Such

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a model qualification procedure involves the measurement of indicators that must satisfy defined criteria. The model qualification criteria are studied as a means to support decision making in the successful selection models by integrating the pertinent aspects of a model if it is to be successfully used in a realistic real-time simulation. Those criteria are classified accordingly to their nature: exploitation or results.

The next section (sect. III) provides a background of the evaluation of models and the criteria commonly used. Sections V, VI and VII present the model qualification procedures for each type of model in the virtual prototyping model taxonomy (see section IV). Section IX illustrates the application of model qualification to a cantilever beam simulation case. Section X discusses the resulting model selection management of the cantilever beam case.

II. Problem statement

A recent trend in the creation of virtual prototypes for product design is the inclusion of interactivity. Virtual prototypes are digital representations or simulations of the product concept. Simulation of interactive prototypes (or interactive simulations) can be used to explore and experiment product concepts according to the expertise and intuition of the designer[6] and the future user. Similarly, it has been suggested that the use of interactive simulation shall speed up the findings and reviewing of concept design in the early stages of the development process [4]. Interactivity in the virtual prototype is its capability to simulate the human's interaction with the design. In the past, the effectiveness of an interactive virtual prototype was limited to the following features: realistic visualisation, geometry-related constrains, and realistic simulation of physical behaviour [18]. However, human-product interaction should be included [16] as well as real-time processing and rendering [13], [4] to maintain the illusion of realism in the simulation [21]. In fact, as stated by Liu et al. [12], the key problem of virtual prototyping is how to build credible VP models. Today, virtual prototyping for product design must provide interactive simulation that ensures: realism (visual and behavioural), fast processing (computation of models), and integration of the human-object interaction. Also, extensible and reusable models are desired to simulated different design alternatives with a minimal effort. Therefore, the interactive simulation must reflect the following features:

- Accuracy and appropriate speed. Visualization and simulation of physical behaviour must be accurate to provide a realistic reliable experience to the user [18], and fast enough to maintain the sensation of immersion [21].
- Human integration. Object-object interactions as well as also human-object interaction must be integrated. [21], so that the designer is able to explore and experiment the future user reaction with the design alternatives.
- Extensibility and reusability. Quickly integration of changes in the virtual prototype [18] and easy derivation of virtual prototype variations [7] allow the creation of prototypes for the different design alternatives.

In the current research, exploration of the interactive simulation models is performed. It aims to develop a modelling methodology with the features mentioned above, except for the realistic visualization.

In this study, the assessment of appropriate models for simulation is addressed. Models are qualified to provide a model assessment (in terms of accuracy and appropriate speed) suitable for prototyping realistic real-time simulations.

III. Background

Models helps engineers and scientist to describe or predict a given phenomenon. Nonetheless, models do not capture the full complexity of the real phenomenon. Instead, they provide a less complex (but useful) abstraction: models are created with only relevant features. In the case of mathematical models (or symbolic models [9]), simplifications often alter the realism in a model, while improving the speed of analysis.

In fact, models are created to fulfil several utilization objectives or requirements: accuracy, speed, wider applicability, among others. In contrast, there are cases where the engineer or scientist would not create a new model but instead employ an existing one, as in the reuse of mechanical models (see [19]). In such a case, information about the model is required in order to be used, being aware of the advantages or disadvantages that it may present.

Consequently, model qualification has emerged to this end. Model qualification serves to assess that the model provides an adequate representation with respect to the utilization objectives [1], [2], [3]. While model qualification is defined in terms of a specification matching function [20], in this work it defined in terms of qualification criteria. Examples of these criteria can be found in product design, as in [9] where three basic criteria are used to accept a model:

1) accuracy, the model's ability to represent the real world;
2) resolution, the ability of a model to distinguish between alternative cases (as a function of sensitivity and not of accuracy); and
3) causal relationship depiction, is the ability of a model to inform how performance may change after parameter alteration.

From those criteria, accuracy is the most relevant for interactivity; but other aspects must be analysed for interactive simulation models. Real-time execution and applicability domain are very important along with accuracy and precision. While in some areas the terms accuracy and precision are used interchangeably, their difference in numeric models must be emphasized:

1) accuracy, or exactitude, is the degree of conformity of a calculated quantity to its actual (true) value; and
2) precision, or repeatability, is the degree to which further calculations will show the same or similar results.

Those criteria present their own particularities: accuracy requires an external reference to be compared against; while precision in numeric models can be analysed only in the case of iterative methods. In this work, precision would no be include as a main criterion for model qualification.

Real-time execution is a requirement for the appropriate execution of a virtual prototype in a Virtual Reality (VR) simulator. Delays in communication between devices, graphical rendering rate of the system, and required updated interval (e.g. 20 fps) impose that the numerical simulation occurs in real-time to be able to respond in a time manner to the actions of the user.

Applicability domain must be also verified; since in some cases, *specific* models are usually developed to be more efficient than their *generic* counterparts; but not without a penalty: typically the diminution of the applicability domain. The domain where a model can be applied limits its validity domain, since only there its validity can be assessed. In many situations, a model should always be accompanied of its validity domain in order to be used. Moreover, the validity domain must be verified if the model is modified: during the application of a model reduction to beam behavioural models, alteration of the resulting validity domain can be expected.

While those aspects are fundamental for simulation as well as accuracy, for the global model (the environment model) agent-related aspects must be included. Some may be related to the dynamic model selection within the simulation (as in [20]), but others to the structure and possible impact in the virtual prototype modelling and simulation.

IV. Model taxonomy for virtual prototyping

Normally, virtual prototyping aims at creating virtual representations of product concepts. As products are commonly constituted of several components, virtual prototype simulations include an environment model that is a collection of several component models. Component models allows to simulate how the component behave under different states of the environment. Typically, a component model requires at least one behavioural model; but in this work component behaviour is secured with a collection of behavioural models suited for different interactions. An organizational view of this taxonomy is shown in Fig. 1, with the indication of interactions between components. Formally, this taxonomy is expressed as shown in (1)–(3).

The set of models $\mathfrak{M}$, the set of environment models $\mathfrak{E} \subset \mathfrak{M}$, and the set of component models $\mathcal{C} \subset \mathfrak{M}$ are defined as:

$$\mathfrak{M} = \{M \mid M \text{ is a model}\} \ ; \quad (1)$$
$$\mathfrak{E} = \{\mathcal{E} = (\{C_i\}) \mid C_i \in \mathcal{C}\} \ ; \quad (2)$$
$$\mathcal{C} = \{C = (\{B_j\}) \mid B_j \in \mathcal{B} \subset \mathfrak{M}\} \ ; \quad (3)$$

where $\{C_j\}$ is a collection of component models, and $\{B_j\}$ is a collection of behavioural models.

Thereafter, a Virtual Prototype (VP) for Interactive Design (ID) is modelled in a three-level representation, see Fig. 2, as follows:

1) *Behavioural model level*, where models are basically chosen for their performance.
2) *Component model level*, where each component is constituted by a collection of behavioural models.
3) *Environment model level*, where the environment comprises several related components, organized in a particular manner.

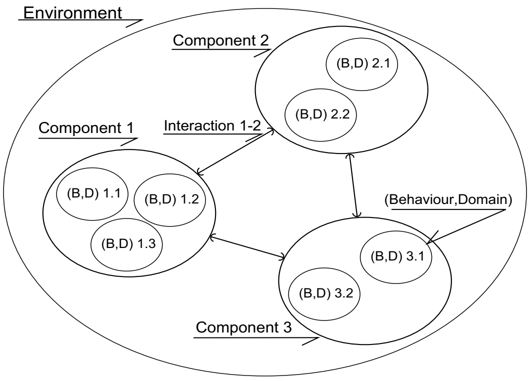

Fig. 1. Organic view for Virtual Prototype modelling. Environment model, component models, and behavioural models are included. Interactions are also indicated.

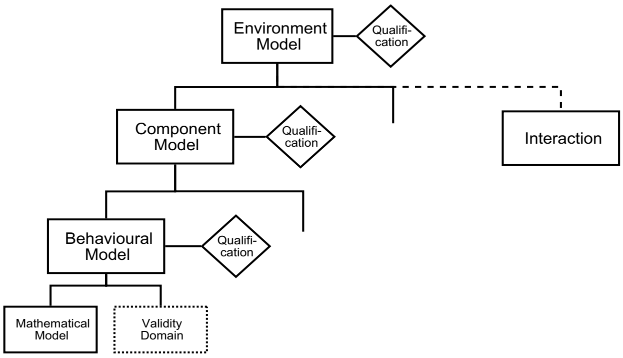

Fig. 2. Model taxonomy and qualification for Virtual Prototyping simulations. A hierarchical view of Environment model, Component model, and Behavioural model is presented. Interaction is also indicated.

A. Behavioural Models

Such a model is a mathematical abstraction of a given phenomenon that serves to estimate the response of an object to given changes of its environment. As representations of a simplified reality, these models are subject of a limited domain of applicability. This domain of applicability, also known as *validity domain*, is the extend or circumstances where the behavioural model is valid.

As an illustration of a behavioural model, the following model for the statical behaviour of an elastic body —within the domain of the linear theory— is presented. It consists of a system of field equations:

- The strain-displacement relation

$$\mathbf{E} = \frac{1}{2}\left(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}}\right) \ , \quad (4)$$

- The stress-strain relation

$$\mathbf{S} = \mathsf{C}\,[\mathbf{E}] \ , \quad (5)$$

- And the equation of equilibrium

$$\mathrm{Div}\,\mathbf{S} + \mathbf{b}_0 = \mathbf{0} \ , \quad (6)$$

where C is the elasticity tensor, $\mathbf{S}$ is the stress, $\mathbf{E}$ is the infinitesimal strain, $\mathbf{b}_0$ is the reference body force, and $\nabla\mathbf{u}$ is the displacements gradient.

The assumptions used in this model (isotropic linear elastic material, small deformations and small displacements) limit the applicability of the model. This is, it would only considered valid within the domain of Linear Elasticity and Small Perturbations (i.e. its validity domain).

Behavioural models and validity domains are represented as

$$B(\varphi, D) \quad \text{where} \quad \begin{cases} B & \text{is a behavioural model,} \\ \varphi & \text{is its mathematical model, and} \\ D & \text{is the validity domain of existence of the behavioural model.} \end{cases} \tag{7}$$

The behavioural model $B \in \mathcal{B}$ is formally defined as follows. First, its mathematical model φ:

$$\varphi : \begin{array}{ccc} X & \rightarrow & Y \\ x & \mapsto & \varphi(x) \end{array} ; \tag{8}$$

Then, the validity domain D must be included:

$$\begin{aligned} B(\varphi, D) : \quad & X \rightarrow Y \\ & x \mapsto B(x) \equiv \begin{cases} \varphi(x)\,, & \text{if } x \in D \\ \text{undefined}\,, & \text{otherwise.} \end{cases} \end{aligned} \tag{9}$$

As part of interactive simulations, these models must be evaluated in terms of accuracy, speed, complexity and validity.

B. Component Models and Interaction Models

An entity in the virtual prototype environment is an object that has a distinct, separate existence–even if it is virtual. In the environment, the virtual product (i.e. its prototype) and the human are entities. As every entity has an existence, it is characterized by performing an specific behaviour under certain circumstances of the environment.

A *component model* is an entity of a virtual environment has one or several possible behaviours that depend eventually of the nature and intensity of the external actions upon it. Commonly, these models are different in form, density and typology; but they are all characterized by the inclusion of a series of behavioural models.

An *interaction model* represents the interaction between components, by transmitting the response of a component model to another. The magnitude and type of the response serves to activate a behavioural model (in the receiving component model). It also helps to verify the equilibrium in the environment (see section VII-B).

C. Environment Models

An environment model is a representation to be simulated of the virtual prototype and its environment, comprising all the entities that may interact with it. In short, an environment model comprises a collection of component models. The environment model is implement by means of Multi-Agent Systems (MAS) modelling. One requirement of the simulation is that the environment is dynamically adaptable to ensure interaction coherence. Hence, multi-agent system modelling is an appropriate solution since it helps to eliminate complexity by the divide and conquer strategy. Intelligent agents can provide a great level of adaptability as they perform their tasks. Accordingly to [10],

> An agent is a computer system situated in some environment, and that is capable of autonomous action in this environment in order to meet its design objectives.

From the structure of an environment, a multi-agent system structure can be directly specified by assigning one agent for each component model (and its behavioural models). In short, MAS modelling represents the environment as an ensemble of *Component Agents* and *Interactions*, each one corresponding to a entity found in the previous section. Thus, a multi-agent system is

$$\mathsf{E} = \mathsf{A} \cup \mathsf{I}\,, \tag{10}$$

where A is the set of component agents, and I is the set of interaction between those agents.

V. Behavioural Model Qualification

A recurrent strategy for modelling behaviour in mechanics consists in recurring to a particular theory (e.g. Linear Elasticity theory, Non-linear Elasticity theory, Plasticity theory) to use its assumptions for abstracting the reality. This strategy has an impact in the results and exploitation of the resulting models. To assess them, five aspects of the mechanical model are formulated:

- exploitation aspects: validity domain Q_{Do}, rapidity Q_{Ra}, algorithmic complexity Q_{Cp};
- results aspect: exactitude Q_{Ex}

The determination of the values for each criteria parameter is conducted as follows:

A. Validity domain

The first assumption to assess in a behavioural model is its validity domain. While it is not a qualification criteria *per se*, it certainly provides a fundamental information of the model—the extend of the domain where the model is meant to be used. In fact, the validity domain of a behavioural model $B(\varphi, D)$, see (7), should be provided in the form of intervals:

$$\begin{array}{ll} D = [a, b] & D = (a, b) \\ D = (a, b] & D = [a, b) \end{array}$$

The generic definition of validity domain was provided in (9).

For example, in structural mechanics, intervals are established from boundaries as

- indicators for geometric boundaries: strain (ϵ) or displacement (u);
- indicators for material boundaries: stress (σ).

$$Q_{\text{Do}}(B) = \begin{cases} 1\,, & \text{if } D^* \subset D \\ 0\,, & \text{otherwise} \end{cases} ; \tag{11a}$$

or, using the Iverson bracket for simplicity:

$$Q_{\text{Do}}(B) = [D^* \subset D] ; \tag{11b}$$

TABLE I
VALIDITY DOMAINS OF MECHANICAL BEHAVIOURAL MODELS.

Domain	Indicator
Geometric Domains	
Small displacements and small deformations	$\delta \ll L$; $\epsilon_{eq} \leq 1\%$
Great displacements and small deformations	$\epsilon_{eq} \leq 1\%$
Great displacements and great deformations	$\epsilon_{eq} > 1\%$
Material Domains	
Elasticity	$\sigma_{eq} \leq \sigma_{elast}$
Elasto-plasticity	$\sigma_{0.2}, \epsilon_{plast} = 0.2\%$
Plasticity	$\sigma_{eq} \geq \sigma_{plast}$

where B is the behavioural model, D is its validity domain, D^* is the required application domain.

Equation (11) indicates that the model B can be used ($Q_{\text{Do}}(B) = 1$) in the particular domain D^* if that interval is covered by the validity domain of the model D.

B. Exactitude

The exactitude of the behavioural model is measured from the relative error of the model's response to a reference value (from experiments or from other well established and validated model). The relative error is expressed as

$$\varepsilon(y) = \left| \frac{y - y^{(\text{ref})}}{y^{(\text{ref})}} \right| . \tag{12}$$

The general form of exactitude for a behavioural model

$$\mathbf{y} = B(\varphi, D, \mathbf{x})$$

is:

$$\begin{aligned} \Delta_{\text{ref}} &= \sum \varepsilon(y_i) \\ Q_{\text{Ex}}(B) &= 1 - \Delta_{\text{ref}} \end{aligned} \tag{13}$$

For illustration, (14) presents the error measures for a cantilever beam (see Fig. 4).

$$\begin{aligned} \Delta_{\text{ref}} &= \left| \frac{\delta_x - \delta_x^{(\text{ref})}}{\delta_x^{(\text{ref})}} \right| + \left| \frac{\delta_y - \delta_y^{(\text{ref})}}{\delta_y^{(\text{ref})}} \right| + \left| \frac{\theta_z - \theta_z^{(\text{ref})}}{\theta_z^{(\text{ref})}} \right| \\ &= \varepsilon_{\delta_x} + \varepsilon_{\delta_y} + \varepsilon_{\theta_z} \end{aligned} \tag{14}$$

where δ_x, δ_y, and θ_z are the maximal displacements of the beam.

C. Rapidity

It is expected that the model responds in a timely manner. To assess this requirement, a minimum rate of model response has to be defined. This rate is related to the "frame rate" in real-time computer systems. Frame rate corresponds to the number of times per second that the system re-evaluates all necessary inputs and updates the necessary outputs under all circumstances. It can be expressed as a frequency in hertz (Hz) or as frames per second (fps) just as in the case of virtual reality applications. A series of images presented to the human eye at 25 fps produces the impression of fluidity. Furthermore, the required frame rate in VR is frequently established to 20–30 fps, as in [17]. Lower frame rates are some times accepted depending on the tasks. However, the frame rate involves the execution of behavioural models and their graphical representation (analogous to the mechanical engineering definition and the computer graphics definition of the virtual prototype). For this work, the frame rate threshold is established at 25 fps and the time required to create the graphical representation is considered negligible to ease the qualification of the behavioural model.

In short, a behavioural model will be considered rapid if it maintains a response rate of 25 Hz. In detail

$$\begin{aligned} f_{\text{resp}}(B) &= \frac{1}{t_{\text{resp}}(B)} \\ Q_{\text{Ra}}(B) &= \frac{f_{\text{resp}}(B)}{\hat{f}} \end{aligned} \tag{15}$$

where B is a behavioural model, $\hat{f}$ is the response rate threshold, and $t_{\text{resp}}(B)$ is the time of response of model B, in seconds.

As stated above, the response rate threshold would be

$$\hat{f} = 25\,\text{Hz} \tag{16}$$

D. Algorithmic complexity

The existence of the Complexity parameter is due to the fact that the time of response is affected by execution platform of the computational implementation of model. It has been shown that in computing the order of complexity of an algorithm (and hence of a model) is a more realistic measure of its performance, since time depends on the computing system where the model is executed (processor, operating system, etc.) but also of the size of the input. While a model can be appear fast to a relative small given problem, it may be slow to solve a greater problem, as in some cases the time of solution grows exponentially.

We consider that the parameters Rapidity (how fast the model can be executed) and Complexity (order of computational complexity) are strictly complementary, and that this statement is obviously observed in the case of the Finite Element Method where generally a model is constructed with a greater number of variables but solved with a relative simplicity since the computations are less complex. The determination of this parameter is shown with this example:

> Let M_1 and M_2 be two models for the same phenomenon and M_{ref} the model of reference for the qualification. With M_1 is $\mathcal{O}(f(n))$, M_2 is $\mathcal{O}(g(n))$, and M_{ref} is $\mathcal{O}(h(n))$, then the set
>
> $$\{(f(n), M_1), (g(n), M_2)(h(n), M_{\text{ref}})\}$$
>
> is ordered by the first element of the pair; i.e. if
>
> $$\begin{gathered} g(n) < h(n) < f(n) \Rightarrow \\ \langle (g(n), M_2), (h(n), M_{\text{ref}}), (f(n), M_1) \rangle \Rightarrow \\ Q_{\text{Co}_2} < Q_{\text{Co}_{\text{ref}}} < Q_{\text{Co}_1} \end{gathered}$$
>
> To obtain numerical values of complexity, it is simply a matter of evaluating the following expressions with an arbitrary value $\hat{n} >> 1$:
>
> $$f_1(n) = f(n), f_2(n) = g(n), f_{\text{ref}}(n) = h(n) ,$$

by means of the relation

$$Q_{\text{Co}}(M_i) = \left.\frac{f_i(n)}{f_{\text{ref}}(n)}\right|_{n=\hat{n}} \tag{17}$$

VI. Component Model Qualification

As stated before, the component model integrates a collection of behavioural models $C = (\{B_j\})$. Consequently, the same criteria of B are integrated for component models, with the exception of the validity domain that is evolved as Specialization Q^C_{Sp}.

A. Specialization

The validity domain of the component model D^C is formed by the aggregation of the validity domains of every behavioural model $B(\varphi, D)$ that compose it. This is

$$D^C = \bigcup D_j \qquad \forall\, C = (\{B_j\}) \in \mathcal{C}\ . \tag{18}$$

While the specialization involves the inverse verification of the application domain. This is, if the component model is valid only in the application domain it is considered as specialized for that application.

$$Q^C_{\text{Sp}} = \left[D^C = D^*\right]\ . \tag{19}$$

B. Exactitude

The exactitude of the component model is measured from the relative error of the model's response to a reference value. The relative error is expressed as in 12.

The general form of exactitude for a component model

$$\mathbf{y} = C(\mathbf{x})$$

with reference values $\mathbf{y}^*$ is:

$$\begin{aligned} \Delta_{\text{ref}} &= \sum \varepsilon(y_i) \\ Q^C_{\text{Ex}}(B) &= 1 - \Delta_{\text{ref}} \end{aligned} \tag{20}$$

C. Rapidity

Similarly to the case of behavioural models (see Sect. V-C), a component model will be considered rapid if it maintains a response rate $f_{\text{threshold}} = 25\,\text{Hz}$. In detail

$$\begin{aligned} f(C) &= \frac{1}{t_{\text{resp}}(C)} \\ Q^C_{\text{Ra}}(C) &= \frac{f(C)}{f_{\text{threshold}}} \end{aligned} \tag{21}$$

where C is a component model, $f_{\text{threshold}}$ is the response rate threshold, and $t_{\text{resp}}(C)$ is the time of response of model C which is the combined time of response of its behavioural models, in seconds.

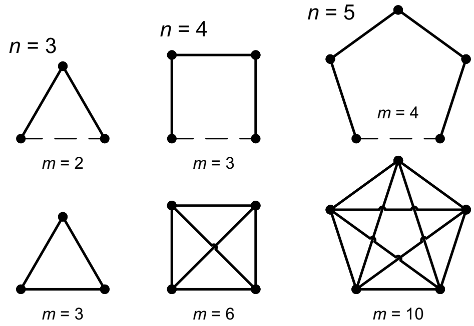

Fig. 3. Minimal and maximal connections in fully connected graphs with 3, 4, and 5 nodels

D. Algorithmic complexity

At this state, the algorithmic complexity $Qcomplexity$ is inherited from $qcomplexity$ of the behavioural model

$$Q^C_{\text{Cp}}(C) = \max(Q_{\text{Cp}}(B_j)) \tag{22}$$

as the higher complexity is the indicator of the component model.

Further development on inheritance is discussed in Sect. VIII.

VII. Environment Model Qualification

A. Density

Density relates the number of agents in the environment model with the number of connections among them. Figure 3 shows some graphs representing MAS structures, nodes as agent and edges as path of communication (connections) among agents.

The minimal and maximal densities are directly related to the minimal and maximal number of connection, respectively. Given a set of agents A, it is possible to know the minimal and maximal densities:

$$\begin{aligned} Q^{(\min)}_{\text{De}} &= \frac{m_{\min}}{n} = \frac{n-1}{n} \\ Q^{(\max)}_{\text{De}} &= \frac{m_{\max}}{n} = \frac{n(n-1)}{2n} \end{aligned} \tag{23}$$

The determination of this parameter is conducted as follows:

A higher density represents not only a greater number of connections (implying perhaps a high number of protocols) between agents, but consequently it also represents the higher number of affected agents that may need to be updated by an "isolated" incident in a dynamically adaptable Multi-Agent System.

$$\begin{aligned} \mathsf{A} = \{a_1, a_2, \cdots, a_n,\} \quad \mathsf{I} = \{i_1, i_2, \cdots, i_m,\} \\ Q_{\text{De}} = \frac{\aleph(\mathsf{I})}{\aleph(\mathsf{A})} = \frac{m}{n} \end{aligned} \tag{24}$$

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B. Equilibrium

Equilibrium corresponds to the summation of actions and reactions originated by the interacting agents in the environment. Interactions (action and reactions) are measured in terms of energy, and those must be equilibrated for the stability of the simulation. To be in equilibrium, the sum of all interactions of the environment must vanish:

$$\sum_{\mathcal{E}} \mathsf{I}_k = 0 \tag{25}$$

Consequently, the equilibrium assessment is expressed (using the Iverson bracket) as:

$$Q_{\text{Eq}}(\mathcal{E}) = \left[\sum_{\mathcal{E}} \mathsf{I}_k = 0\right] . \tag{26}$$

VIII. Inheritance of the assessment criteria

For a rapid assessment, some of the criteria selected for the behavioural models could be inherited in the component model. This particularity appears from the constitution of the component model: it is formed by the aggregation of those behavioural models. Also, a component model's response corresponds to the response obtained by executing one of its sub-models (i.e. its behavioural models). The criteria to be inherited may include exactitude, rapidity and complexity. So, in the worst case:

- exactitude of the component model:

$$Q^C_{\text{Ex}}(C) = \min(Q_{\text{Ex}}(B_j)) \tag{27}$$

- rapidity of the component model:

$$Q^C_{\text{Ra}}(C) = \min(Q_{\text{Ra}}(B_j)) \tag{28}$$

- complexity of the component model:

$$Q^C_{\text{Cp}}(C) = \max(Q_{\text{Cp}}(B_j)) \tag{29}$$

IX. Application to a simple virtual environment

A. A cantilever beam

A long thin cantilever beam, statically charged on the free end, is to be modelled for interactive simulation (see Figure 4).

The cantilever beam is considered of uniform rectangular cross section made of a homogeneous and isotropic elastic material, that follows a linear elastic constitutive law. Only small deformations are accepted, but large deflections may appear. Since large rotations move away the current configuration ($\mathcal{C}^D$) from the base configuration ($\mathcal{C}^0$), a linear model cannot be used but only for small rotations. A total lagrangian (TL) formulation model is accurate and precise enough; but the computing time exceeds the acceptable threshold for an interactive simulation since it requires an iterative solution process (usually a variant of the Newton-Raphson is used).

Table II resumes the data of the cantilever beam used as the test case. It is analogous to the problem experimented in [5]. Their results where validated experimentally. Table III presents the resulting displacements at the free end of the beam (which correspond to the maximal values).

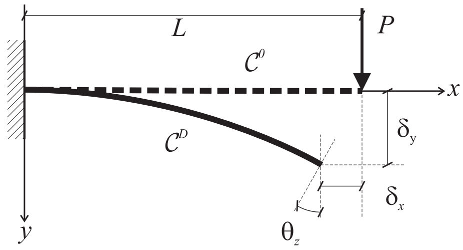

Fig. 4. Long deflection cantilever beam problem for flexible modelling

TABLE II
Data of the cantilever beam [5].

Description	Value
Length L	300 mm
Width b	30.4 mm
Height h	0.78 mm
Moment of inertia I	1.20×10^{-12} m^4
Young's modulus E	200 GPa
External force F	3.92 N

Under the assumption that deformation remains small, i.e. $\|\varepsilon\| \leq 1\%$, the domains for the simulation are restricted to:

D_S	small deflections domain: $\theta_z \leq 15°$, $\|\varepsilon\| \leq 1\%$
D_G	great deflections domain: $\theta_z > 15°$, $\|\varepsilon\| \leq 1\%$
$D_S \cup D_G$	small and great deflections domain: $\|\varepsilon\| \leq 1\%$

It is important to consider the validity of a model when used in a particular domain. For the beam described above, the non-linear model (TL formulation) is clearly more accurate than the linear model (see Fig. 5). However, the linear model is normally faster than the former.

The models presented above are the base of the creation and comparison of the reduced model. Both models are presented in the following sections.

B. Selected beam models

For the beam simulation, the proposed model organization is shown in Fig. 6 and Fig. 7: the environment model, three component models (for the beam, human hand, and the frame) with behavioural models included only for the beam. The interactions are: sensorial (Resistance Sensation) and pure physical (Deformation Energy).

The models are organized in Table IV and in Table V. Their known accuracy, speed and validity domains are included in Table IV; while their definition is presented in table Table V.

TABLE III
Displacements at the free end of the cantilever beam. Numerical results of the reference model, validated experimentally [5].

Displacements	Response of the reference model
δ_x	31.4 mm
δ_y	121.6 mm
θ_z	36.09°

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TABLE IV
BEHAVIOURAL MODELS FOR THE CANTILEVER BEAM. VALIDITY DOMAINS AND REPORTED PERFORMANCE.

Model	Validity Domain	Accuracy	Speed
Linear [22] φ_l	Small displacements	Regular	Very fast
Non-linear [11] φ_{nl}	Small and great disp.	High	Slow

TABLE V
BEHAVIOURAL MODELS FOR THE CANTILEVER BEAM. DEFINITION.

Model	Definition
Linear [22]	$\mathbf{K}\,\mathbf{u} = \mathbf{f}$ $\underbrace{\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI_{zz}}{L^3} & \frac{6EI_{zz}}{L^2} & 0 & -\frac{12EI_{zz}}{L^3} & \frac{6EI_{zz}}{L^2} \\ 0 & \frac{6EI_{zz}}{L^2} & \frac{4EI_{zz}}{L} & 0 & -\frac{6EI_{zz}}{L^2} & \frac{2EI_{zz}}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI_{zz}}{L^3} & -\frac{6EI_{zz}}{L^2} & 0 & \frac{12EI_{zz}}{L^3} & -\frac{6EI_{zz}}{L^2} \\ 0 & \frac{6EI_{zz}}{L^2} & \frac{2EI_{zz}}{L} & 0 & -\frac{6EI_{zz}}{L^2} & \frac{4EI_{zz}}{L} \end{bmatrix}}_{\mathbf{k}_l^e}$ $\mathbf{u} = \varphi_l\left(\mathbf{f}, (p_g, p_m)\right)$
Non-linear [11]	$\mathbf{K}(\mathbf{u})\,\mathbf{u} = \mathbf{f}$ $\mathbf{k}_{nl}^e = \mathbf{k}_l^e + \mathbf{k}_{nlgeo}^e\,;\; N = \frac{EA}{L}\left(L' - L\right)$ $\underbrace{N \begin{bmatrix} 0 & & & & & \\ 0 & \frac{6}{5L} & & & \text{sym.} & \\ 0 & \frac{1}{10} & \frac{2L}{15} & & & \\ 0 & 0 & 0 & 0 & & \\ 0 & -\frac{6}{5L} & -\frac{1}{10} & 0 & \frac{6}{5L} & \\ 0 & \frac{1}{10} & -\frac{L}{30} & 0 & -\frac{1}{10} & \frac{2L}{15} \end{bmatrix}}_{\mathbf{k}_{nlgeo}^e}$ $\mathbf{u} = \varphi_{nl}(\mathbf{f}, \mathbf{u}, (p_g, p_m))$

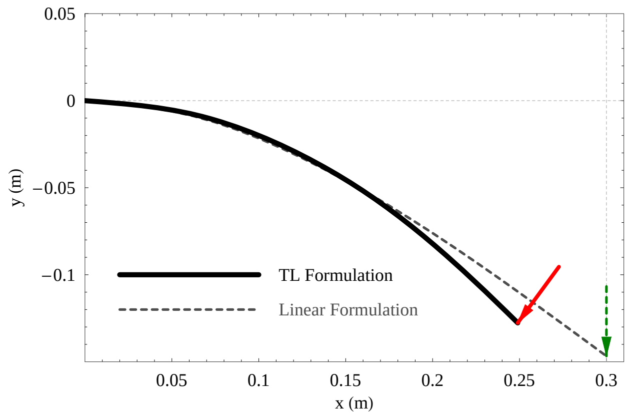

Fig. 5. Comparison of a non-linear model against a linear model in the behaviour of a cantilever beam under great displacements.

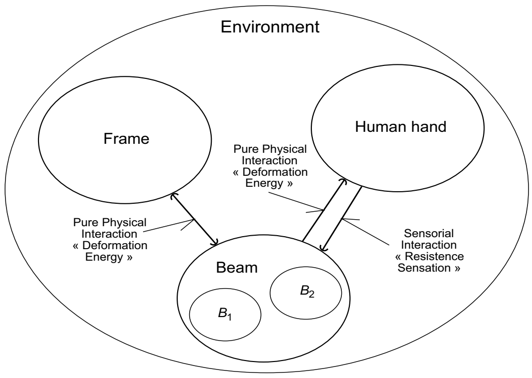

Fig. 6. Organic view for the beam simulation.

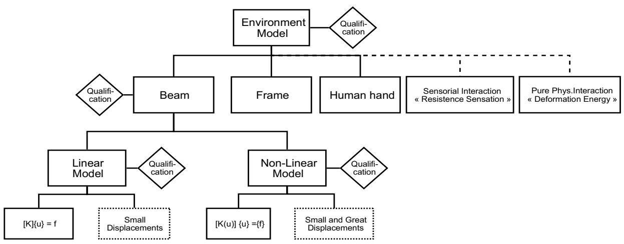

Fig. 7. Model taxonomy and qualification for the beam simulation.

The linear model [22] corresponds to the Euler-Bernoulli Beam Theory. The non-linear model [11] only takes geometric non-linearities into account.

X. RESULTS AND DISCUSSION

There are now two alternative models to simulate the problem under great displacements. Each beam model has been tested with six different configurations (30, 10 and 3 loading steps; and 5 and 2 discrete elements). Those results

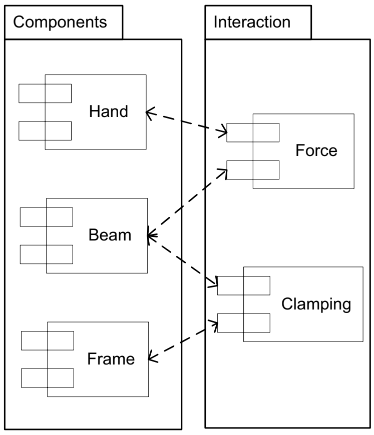

Fig. 8. Multi-Agent System (MAS) model

TABLE VII
MODEL COMPARISON WITH 30 STEPS AND 5 ELEMENTS

Measures	Reference	Lin	Non-Lin
δ_x (cm)	3.14	3.62	5.10
δ_y (cm)	12.16	13.26	12.77
θ_z (rad)	0.630	0.695	0.634
t_{resp} (s)	—	0.05	14.88
algorithmical order[a]	—	$\mathcal{O}(n_{st} \cdot n_{el}^3)$	$\mathcal{O}(n_{it} \cdot n_{st} \cdot n_{el}^3)$
ℵ(A)	—	3	3
ℵ(I)	—	2	2

[a] The dimension n_{el} is the number of discrete elements, n_{st} is the number of loading steps, and n_{it} is the number of iterations in for an iterative non-linear system solver.

are shown in Table VI. Also, the diagrams of each model compared with a pure Linear Formulation are shown with a 30 steps and 5 elements configuration (Figure fig:tl30st5el and Figure 10), and with a 3 steps and 2 elements configuration (Figure 11 and Figure 12), at the Appendix. The simulation of the models was done on a personal computer with 512 MiB of RAM and a Pentium-M @ 1.6 GHz processor.

The other qualifications are calculated only for the 30-steps and 5-elements configuration. Table VII summarizes the results and estimations for both models.

It is possible to appreciate that there is an important discrepancy in the accuracy of the models. In this application, time of solution is the most changing quantity between the models. Both can be considered as realistic when compared to a pure linear formulation, but the excessive elongation of the beam with the incremental formulation (Fig. 12) may cause a lost of credibility of the model in a VR simulation.

As presented above, the three model qualification steps provide an important. The behavioural model qualification.

Table VIII resumes the qualification of the models for the simulation.

XI. CONCLUSION

A model qualification method has been presented as a tool for mechanical model assessment in interactive simulations.

TABLE VIII
MODEL QUALIFICATION FOR ENVIRONMENT, COMPONENT, AND BEHAVIOURAL MODELS

Model	Qualification parameter	Value
Non-linear (TL)	Q_{Do}	1
	Q_{Ex}	0.32
	Q_{Ra}	0.002
	Q_{Cp}	1×10^1
Linear (Incremental)	Q_{Do}	1
	Q_{Ex}	0.65
	Q_{Ra}	0.8
	Q_{Cp}	1×10^0
Beam	Q^C_{Sp}	1
	Q^C_{Ex}	0.32
	Q^C_{Ra}	0.002
	Q^C_{Cp}	1×10^1
Environment	Q_{De}	$2/3$
	Q_{Eq}	1

The qualification is conducted at three levels: environment level, component level, behavioural level.

The selected parameters include exactitude, rapidity, validity domain, and complexity. For the special case of the environment model, as it is based on MAS modelling, two parameters were chosen for its assessment: density and equilibrium.

The application case of the beam modelling for simulation shows that the qualification is relatively easy to be employed, since most parameters can be obtained without great effort. Only the algorithmic al complexity can be tedious to obtain, but the order of complexity of many numerical methods is already reported in the literature. The need of specialized models for interactive simulation was also found. The use of model reduction techniques is fostered to obtain a better compromise between accuracy and rapidity.

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TABLE VI
COMPARISON OF NON-LINEAR (TOTAL LAGRANGIAN) AND LINEAR (INCREMENTAL) MODELS WITH VARIATION OF NUMBER OF ELEMENTS AND NUMBER OF CHARGING STEPS

Formulation	Num. Elements	Steps	δ_x (cm)	δ_y (cm)	θ_z (rad)	t_{resp} (s)
Linear (Incremental)	5	30	-3.62	-13.26	-0.6947	0.06
		10	-3.48	-13.53	-0.7036	0.02
		3	-2.81	-14.38	-0.7301	0.01
	2	30	-3.50	-13.35	-0.6974	0.02
		10	-3.36	-13.61	-0.7059	0.01
		3	-2.70	-14.41	-0.7310	< 0.01
Non-linear (Total Lagrangian)	5	30	-5.10	-12.77	-0.6344	14.95
		10	-5.10	-12.77	-0.6344	4.54
		3	-5.10	-12.77	-0.6343	1.05
	2	30	-4.07	-12.77	-0.6343	6.07
		10	-4.07	-12.77	-0.6343	1.93
		3	-4.07	-12.77	-0.6343	0.44
Reference (from [5])			-3.14	-12.16	-0.630	—
Formulation	Num. Elements	Steps	ε_{δ_x}	ε_{δ_y}	ε_{θ_z}	Δ_{ref}
Linear (Incremental)	5	30	15.43%	9.07%	10.27%	34.78%
		10	10.89%	11.31%	11.68%	33.88%
		3	10.51%	18.23%	15.88%	44.62%
	2	30	11.51%	9.79%	10.69%	32.00%
		10	7.00%	11.94%	12.04%	30.99%
		3	13.97%	18.54%	16.03%	48.54%
Non-linear (Total Lagrangian)	5	30	62.52%	4.98%	0.69%	68.19%
		10	62.52%	4.98%	0.69%	68.19%
		3	62.52%	4.98%	0.69%	68.18%
	2	30	29.78%	4.98%	0.69%	35.44%
		10	29.77%	4.98%	0.69%	35.44%
		3	29.77%	4.98%	0.69%	35.44%

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APPENDIX

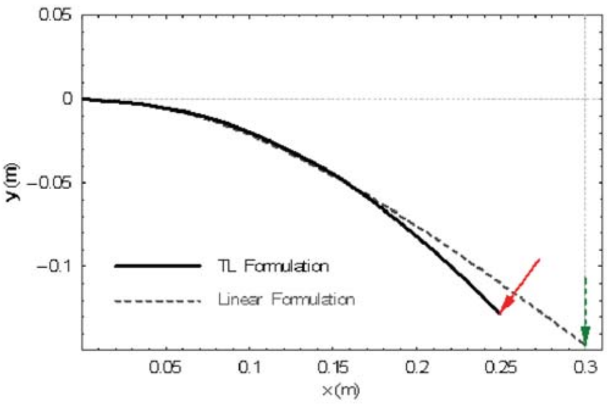

Fig. 9. Total lagrangian formulation vs. linear formulation with 30 charging steps and 5 elements

NOMENCLATURE

$\aleph(\cdot)$	cardinality of a set
$[P]$	Iverson bracket, where P is a proposition
$[a, b]$	interval
Δ_{ref}	difference of a calculated quantity from a reference value
$\{M_i\}$	collection of models
A	set of (component) agents
$\mathcal{B}$	set of behavioural models
B	behavioural model

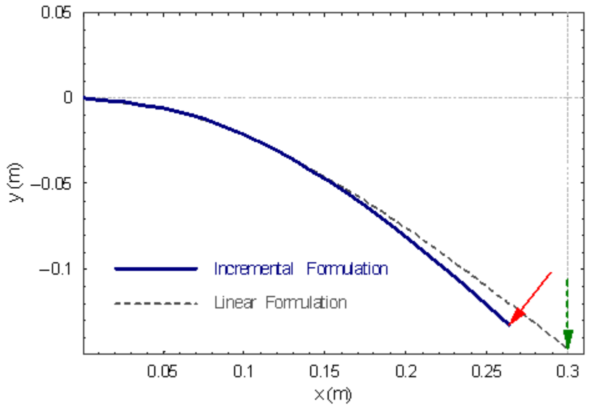

Fig. 10. Incremental formulation vs. linear formulation with 30 charging steps and 5 elements

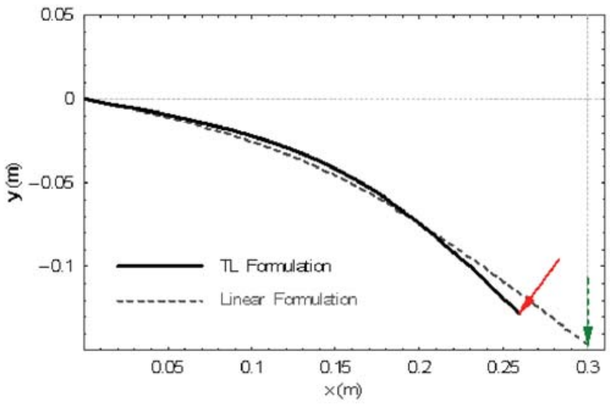

Fig. 11. Total lagrangian formulation vs. linear formulation with 3 charging steps and 2 elements

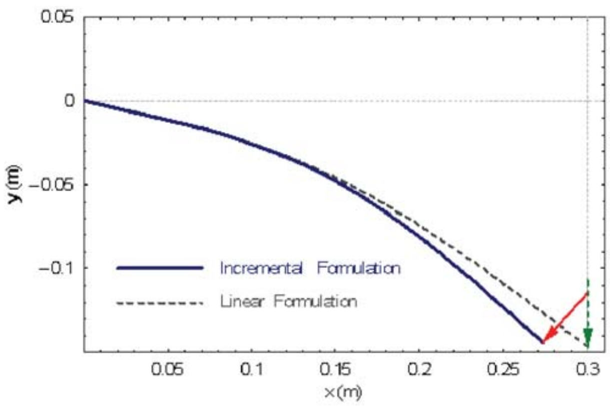

Fig. 12. Incremental formulation vs. linear formulation with 3 charging steps and 2 elements

$\mathbf{b}_0$	the reference body force
$\mathcal{C}$	set of component models
C	component model
$\mathcal{C}^0$	undeformed configuration
$\mathcal{C}^D$	deformed configuration
C	elasticity tensor
δ	displacement (m)
D	validity domain
D^C	validity domain of the component model
D_G	domain of great displacements
D_S	domain of small displacements
D^*	expected domain of application
$\mathbf{E}$	infinitesimal strain field
$\mathcal{E}$	environment model
$\mathfrak{E}$	set of environment models
E	Environment MAS
ε	relative error
E	Young's modulus (MPa)
ϵ	strain (%)
ϵ_{eq}	equivalent strain (%)
ϵ_{plast}	plastic strain (%)
$\mathbf{f}$	solicitations (N)
fps	frames per second
f_{resp}	response rate (Hz)
$\hat{f}$	response rate threshold (Hz)
I	set of interactions
I	moment of inertia of the cross section (m^4)
ID	Interactive design
$\mathbf{k}_l^e$	linear element stiffness matrix
$\mathbf{k}_{nl}^e$	non-linear element stiffness matrix
$\mathbf{k}_{nlgeo}^e$	non-linear geometric contribution to the element stiffness matrix
L	length of the beam (m)
L'	length of the deformed beam (m)
$\mathfrak{M}$	set of models
M	model
MAS	Multi-Agent System
ν	Poisson's ratio
$\mathcal{O}(\cdot)$	O-notation, the order of complexity of an algorithm
p_g	geometric properties
φ	behavioural model
φ_l	linear model
φ_{nl}	non-linear model
p_m	material properties
Q^C_{Cp}	Algorithmic complexity of a component model
Q_{Cp}	Algorithmic complexity of a behavioural model
Q_{De}	Density of an environment model
Q_{Do}	Validity domain of a behavioural model
Q_{Eq}	Equilibrium of an environment model
Q^C_{Ex}	Exactitude of a component model
Q_{Ex}	Exactitude of a behavioural model
Q^C_{Ra}	Rapidity of a component model
Q_{Ra}	Rapidity of a behavioural model
Q^C_{Sp}	Specialization of a component model
$\mathbf{S}$	stress field
σ	stress (MPa)
$\sigma_{0.2}$	elasto-plastic transition when $\epsilon_{eq} = 0.2\%$ (MPa)
σ_{elast}	elastic limit (MPa)
σ_{eq}	equivalent stress (MPa)
σ_{plast}	plastic limit (MPa)
θ_z	rotation at the free end of the beam (rad)
t_{resp}	time of response of a model (s)
$\mathbf{u}$	displacements (m)
$\nabla\mathbf{u}$	the displacements gradient
VE	Virtual environment
VP_{CG}	computer graphics definition of a VP
VP_{ME}	mechanical engineering definition of a VP
VP	Virtual prototyping or virtual prototype
VR	Virtual reality