# A Low Complexity Frequency Offset Estimation for MB-OFDM based UWB Systems

Wang Xue, Liu Dan, Liu Ying, Wang Molin, Qian Zhihong

**Abstract**—A low-complexity, high-accurate frequency offset estimation for multi-band orthogonal frequency division multiplexing (MB-OFDM) based ultra-wide band systems is presented regarding different carrier frequency offsets, different channel frequency responses, different preamble patterns in different bands. Utilizing a half-cycle Constant Amplitude Zero Auto Correlation (CAZAC) sequence as the preamble sequence, the estimator with a semi-cross contrast scheme between two successive OFDM symbols is proposed. The CRLB and complexity of the proposed algorithm are derived. Compared to the reference estimators, the proposed method achieves significantly less complexity (about 50%) for all preamble patterns of the MB-OFDM systems. The CRLBs turn out to be of well performance.

*Keywords*—CAZAC, Frequency Offset, Semi-cross Contrast, MB-OFDM, UWB

#### I. INTRODUCTION

ULTI-band orthogonal frequency division multiplexing (MB-OFDM) is an attractive wideband technology, dividing the allocated 7.5 GHz ultra-wideband (UWB) spectrum into 14 bands, each with a bandwidth of 528 MHz whereby information is transmitted using OFDM modulation on each band. The fourteen bands are organized into five band groups: four groups of three bands each and one group of two bands. The very high data rate (480 Mbps and beyond) capability of the UWB technology would provide a compelling cable-replacement wireless technology. OFDM carries are efficiently generated using a 128-point Inverse Fast Fourier Transform/Fast Fourier Transform (IFFT/FFT) operation. Information is coded across all bands in use to exploit frequency diversity and provide robustness against multi-path and interference. MB-OFDM-based UWB system has been proposed for the IEEE802.15.3a ultra-wideband standard[1],

the new Wireless-USB PHY layer standard, the standard ECMA-368[2] and ECMA-369.

Although OFDM has several advantages such as low complexity equalization in dispersive channels and the spectral density scalability, it has some disadvantages such as larger susceptibility to nonlinear distortion at the transmit power amplifier[3] and larger sensitivity to frequency offsets[4]. Frequency offset causes a loss of orthogonality among the subcarriers thereby introducing inter sub-carrier interference and significantly degrading the error performance. The current work deals with the problems of carrier frequency offset estimation and compensation for MB-OFDM systems.

The MB-OFDM system adopts preamble pattern to aid receiver algorithms related to timing and frequency synchronization as well as channel estimation. [5, 7] are classic preamble-based frequency offset estimation algorithms, which have exerted a profound influence in the study of OFDM systems. However, they are not available for UWB based systems. [8] derived the maximum likelihood (ML) estimator for frequency offset in MB-OFDM based UWB systems following [5] using the last two successive OFDM symbols of the packet synchronization sequence of preamble. [9] presented a coarse frequency offset estimation scheme considering different normalized carrier frequency offsets and different channel frequency responses in different bands. [10] proposed frequency offset estimators based on the best linear unbiased estimation(BLUE) principle. [11] addressed low-complexity, highly-accurate frequency offset estimation for time-invariant as well as time-variant channels.

In this paper a low-complexity frequency offset estimator based on [9] is suggested, which utilizes a half-cycle Constant Amplitude Zero Auto Correlation (CAZAC) sequence as the preamble sequence. The estimation is carried out under the mode of cross comparison for half period. The Cramer Rao Lower Bound (CRLB) and the complexity of algorithms are derived to value the efficiency of our scheme. Simulations are taken to verify the algorithm we proposed.

The rest of the paper is organized as follows. Section  $\Box$  presents the MB-OFDM system, characteristics of UWB channel with channel measurement parameters, and MB-OFDM signal model; Section  $\Box$  describes the proposed method; Section  $\Box$  derives the CRLB and algorithms complexity. Section  $\Box$  shows the Simulation results and performance comparison with [9]. Concluding and the summary are made in section  $\Box$ .

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#### II. THE MB-OFDM SYSTEM

## A. MB-OFDM Specifications

In the MB-OFDM-based UWB system, the carrier frequency is hopped with a pre-defined set of carrier frequencies according to a time-frequency code. ECMA Standard specifies three types of time-frequency codes (TFCs): TFI, TFI2 and FFI. Preamble patterns are associated with different time-frequency codes. Each preamble pattern is constructed by 23 synchronization sequences and 6 channel estimation sequences (CE). Fig.1 shows the structure of preamble pattern 1 and 2 according to TFI. The first 21 sequences of synchronization sequences are PS, and the other three are FS. Preamble pattern 3 and 4, which are defined according to TFI2, are interleaved.



Fig.1 Preamble structure for preamble pattern 1 and 2

#### B. UWB Channel Model

The IEEE 802.15 channel modeling sub-committee has adopted modified Saleh-Valenzuela (S-V) model that can distinguish clusters and rays arrival rates. The IEEE 802.15.3a UWB RF channel model described in [12] is given by

$$h(t) = X \sum_{l=0}^{L} \sum_{k=0}^{K} \alpha_{k,l} \delta(t - T_l - \tau_{k,l})$$
(1)

where  $\alpha_{k,l}$  is the channel coefficient for *kth* ray of *lth* cluster;  $T_l$  is the delay of *lth* cluster;  $\tau_{k,l}$  is the delay of *kth* ray related to *lth* cluster arrival time; X is the log-normal shadowing on the amplitude.

## C. Signal Model

In the MB-OFDM-based UWB systems, zero-padded (ZP) prefix are used instead of the conventional cyclic prefix (CP). Symbols are constructed by suffixing 32 ZP ( $N_{pre}$ ) and 5 guard ( $N_g$ ) samples to 128 (N) length IFFT sequence. The total number of samples in one OFDM symbol is  $d_0 = N + N_0$ ,  $N_0 = N_{pre} + N_g$ .

In order to reduce the complexity of the algorithm, a half-cycle CAZAC sequence c(k) is used as the preamble sequence.

$$c(k) = \exp\left(\frac{j\pi pk^2}{N_1}\right), 0 \le k \le \frac{N}{2} - 1$$
(2)

where *j* is the imaginary unit,  $j^2 = -1$ ; *p* is positive integer;  $N_1 = N/2$ ,  $N_1$  and *p* are coprime numbers.

Fig.2 is the autocorrelation of c(k), which shows that the sidelobes of autocorrelation operation are all zero. The unique characteristic of this function can be presented as

$$\sum_{k=0}^{N_1-1} c(k) c^* (k+\tau)_{\text{mod } N_1} = \begin{cases} N_1, \tau = 0\\ 0, \ \tau \neq 0 \end{cases}$$
(3)



Fig. 2 Autocorrelation characteristic of c(k)

Insert zero prefix and guard interval samples to the N-point IFFT of c(k) as

$$s_{q,l}(k) = \begin{cases} \frac{1}{N} \sum_{m=0}^{\frac{N}{2}-1} c_{q,l}(m) e^{j2\pi km/N}, & 0 \le n \le \frac{N}{2} - 1\\ \frac{1}{N} \sum_{m=\frac{N}{2}}^{N-1} c_{q,l}\left(m - \frac{N}{2}\right) e^{j2\pi km/N}, & \frac{N}{2} \le n < N\\ 0, & -N_{pre} \le n < 0; & N \le n < N + N_{0} \end{cases}$$
(4)

where  $s_{q,l}(k)$  denote the *lth* complex base band OFDM symbol in *qth* frequency band.

We assume perfect timing synchronization, and a normalized frequency offset  $\varepsilon_q = \Delta f_q / f$ , where  $\Delta f_q$  is the frequency offset in band q and f is the inter sub-carrier spacing. The *nth* sample received samples in *lth* symbol of *qth* frequency band can be represented as

$$r_{q,l}(n) = e^{j\varphi} e^{j2\pi\varepsilon_q(n+N_{pre}+lN_s)/N} x_{q,l}(n) + w_{q,l}(n)$$
(5)

where  $\varphi$  is an arbitrary carrier phase,  $w_{q,l}(n)$  is the *nth* time domain AWGN sample added to the *lth* symbol of *qth* frequency band with mean zero and variance  $\sigma^2 = E \left\{ |w_{q,l}(n)|^2 \right\}$ .  $N_s$  is the total number of samples in one OFDM symbol including ZP and guard intervals.  $x_{q,l}(n)$  is the channel output signal samples corresponding to the *qth* frequency band.

$$x_{q,l}(n) = \frac{1}{N} \sum_{k=0}^{N-1} C_{q,l}(k) H_{q,l}(k) e^{\frac{j2\pi kn}{N}}$$
(6)

where  $H_{q,l}(k)$  is the channel transfer function for *qth* frequency band.

# III. PROPOSED FREQUENCY OFFSET ESTIMATION SCHEME

As discussed in the previous section, the preamble sequence used in this paper achieves perfect autocorrelation characteristic, which will promote the estimation efficiency significantly. Meanwhile, the sequence is defined to be a half period one, so that the first half cycle of the received signal is same as the second part theoretically. The frequency offset could be got by performing complex conjugate multiplication of the two halves of one received OFDM symbol, and then repeating the same work over other symbols in and beyond one frequency band to get an average accumulated phase.

In order to fully consider the multipath effects, semi-cross contrast scheme is suggested. The estimator accumulates the phase offsets by comparing the first half cycle and the second one of two successive OFDM symbols in one band, and then extending the operation to all the symbols in this band. The final frequency offset estimation is achieved by averaging the values got in the previous steps.

Cross correlation between the first half cycle of the *lth* symbol and the second half of the (l + d)th symbol in *qth* frequency band is

 $R_{q,l}(n) = \sum_{(l,l+d \in L)} r_{q,l}^*(n) r_{q,l+d}\left(n + \frac{N}{2}\right)$ (7)

where

$$d = \begin{cases} 3d_0 \ , \ preamble \ 1 \& 2 \ (TFI) \\ 6d_0 \ , \ preamble \ 3 \& 4 \ (TFI2); \end{cases}$$

 $d_0$  is the frequency band width one symbol takes. *L* is the total number of symbols in one band. By substituting (5) into (7), we obtain

$$R_{q,l}(n) = e^{j2\pi\varepsilon_q dN_s/N} x_{q,l}^*(n) x_{q,l+d}(n+N/2) + G_{q,l} + W_{q,l}$$

$$= e^{j(2\pi\varepsilon_q N_s d/N + \pi\varepsilon_q)} \left\{ \left| x_{q,l}(n) \right|^2 + G'_{q,l} + W'_{q,l} \right\}$$
(8)

where

$$G_{q,l} = e^{-j2\pi\varepsilon_q(n+N_{pre}+lN_s)/N} x_{q,l}^*(n) w_{q,l+d}(n+\frac{N}{2})$$
(9)  
+  $e^{j2\pi\varepsilon_q(n+N_{pre}+(l+d)N_s)/N} x_{q,l+d}(n+\frac{N}{2}) w_{q,l}(n)$   
$$G'_{q,l} = e^{-j2\pi\varepsilon_q(n+N_{pre}+lN_s)/N} x_{q,l}^*(n) w_{q,l+d}(n)$$
(10)  
+  $e^{j2\pi\varepsilon_q(n+N_{pre}+(l+d)N_s)/N} x_{q,l+d}(n) w_{q,l}(n)$ 

$$W_{q,l} = w_{q,l}^*(n)w_{q,l+d}(n+\frac{N}{2})$$
(11)

$$W'_{q,l} = w^*_{q,l}(n)w_{q,l+d}(n)$$
(12)

The phase offset estimated from the *nth* sample pair of the *lth* symbol and (l + d)th OFDM symbol could be derived as

$$\phi_{q,l}(n) = \left(\frac{2\pi N_s d}{N} + \pi\right)\varepsilon_q + angle \left\| x_{q,l}(n) \right\|^2 + G'_{q,l} + W'_{q,l} \right] (13)$$
  
Define  $N_{q,l} = \left| x_{q,l}(n) \right|^2 + G'_{q,l} + W'_{q,l}$ . Since the proposed algorithm adopts a semi-cross contrast scheme, the number of samples compared is limited to  $M/2$ . The phase offset

samples compared is limited to M/2. The phase offset estimated of M/2 of the first half of *lth* OFDM symbol and the second half of (l + d)th symbol in *qth* frequency band could be represented as

$$\phi_{q,l}(n) = (\frac{2\pi N_s d}{N} + \pi) \varepsilon_q + \frac{1}{M/2} \sum_{n=0}^{\frac{M}{2}-1} angle(N_{q,l}) \quad (14)$$

The frequency offset estimation of lth OFDM symbol is derived as

$$\hat{\varepsilon}_{q,l} = \varepsilon_{q,l} + \frac{\frac{1}{M/2} \sum_{n=0}^{M/2-1} angle(N_{q,l})}{2\pi \frac{N_s}{N} d + \pi}$$
(15)

For there are totally L OFDM symbols in one band, the number used for symbols comparison used within one frequency band is L-1. The frequency offset of band q could be calculated by the L-1 operations, which is represented as

$$\hat{\varepsilon}_{q} = \varepsilon_{q} + \frac{\frac{1}{(L-1)} \left( \sum_{l=1}^{L-1} \frac{1}{M/2} \sum_{n=0}^{M/2-1} angle(N_{q,l}) \right)}{2\pi \frac{N_{s}}{N} d + \pi}$$
(16)

The noise here is Gaussian distributed and may introduce a uniformly distributed phase offset of  $\pm \pi$  with zero mean and variance of  $\pi^2/3$ . Thus, the variance of the estimation within the *lth* OFDM symbol is derived as

$$Var(\hat{\varepsilon}_{q,l}) = \frac{2N^2}{3M(2dN_s + N)^2}$$
(17)

On a further step, variance from L OFDM symbols in one band could be achieved as

$$Var(\hat{\varepsilon}_{q}) = \frac{2N^{2}}{3(L-1)M(2dN_{s}+N)^{2}}$$
(18)

#### IV. CRLB AND ALGORITHMS COMPLEXITY

In order to evaluate the efficiency of the proposed algorithm, the CRLBs and algorithm complexities are derived respectively.

# A. CRLB for estimated frequency offset

Following the method of [9], (7) can be rewritten as

$$R_{q,l}(n) = B_{q,l}(n) + jD_{q,l}(n)$$
 (19)

where

$$B_{q,l}(n) = A\cos(2\pi\varepsilon_q dN_s / N + \pi\varepsilon_q) + w'_{q,l}(l,n)$$
(20)  
$$D_{q,l}(n) = A\sin(2\pi\varepsilon_q dN_s / N + \pi\varepsilon_q) + w''_{q,l}(n)$$
(21)

and  $A = |x_{q,l}(n)|^2$ ,  $w'_{q,l}(l,n)$  and  $w''_{q,l}(n)$  are the in-phase and q-phase components of the effective noise of the complex conjugate product.

The joint probability density function can be written as

$$f(R_{q,l}(n),\chi) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{M}{2}} e^{-\frac{1}{2\sigma^2}\left[\sum_{n=0}^{M/2} \left(B_{q,l}(n) - B(n)\right)^2 + \left(D_{q,l}(n - D(n))^2\right)\right]} (22)$$

where  $\chi$  is the joint function of A and  $\mathcal{E}_{q}$ ,

$$B(n) = A\cos(2\pi\varepsilon_q dN_s / N + \pi\varepsilon_q), \qquad (23)$$

$$D(n) = A\cos(2\pi\varepsilon_q dN_s / N + \pi\varepsilon_q).$$
(24)

The fisher information matrix is

$$I(\chi) = \begin{bmatrix} -E\left\{\frac{\partial^2 \ln(f(R_{q,l}(n),\chi))}{\partial A^2}\right\} - E\left\{\frac{\partial^2 \ln(f(R_{q,l}(n),\chi))}{\partial a\partial \varepsilon_q}\right\} \\ -E\left\{\frac{\partial^2 \ln(f(R_{q,l}(n),\chi))}{\partial \varepsilon_q \partial A}\right\} - E\left\{\frac{\partial^2 \ln(f(R_{q,l}(n),\chi))}{\partial \varepsilon_q^2}\right\} \end{bmatrix}$$
(25)

from which we get the CRLBs for variance of frequency estimations within one symbol as

$$Var(\hat{\varepsilon}_{q,l}) = \frac{2M\sigma^2}{A^2 M^2 \left(2\pi d \,\frac{N_s}{N} + \pi\right)^2} \tag{26}$$

And that within one band is

$$Var(\hat{\varepsilon}_q) = \frac{2N^2}{SNR \cdot M(L-1)(2\pi dN_s + N\pi)^2} \quad (27)$$

where SNR= $A^2 / \sigma^2$ .

# B. Algorithm complexity

The proposed algorithm implements the estimation by half cycle semi-cross contrast, which reduces the time of add and shift operation significantly. We derive the operation complexity by counting the operation time. Firstly the reducing rate of addition can be expressed as

$$\eta_{1} = \frac{\Delta AOT}{\sum AOT} = \frac{\frac{M}{2}(L-1)}{M(L-1)+1} \approx \frac{M(L-1)+1}{2} 50\%$$
(28)

where AOT represents the addition operation time.

Similarly the reducing rate of arithmetic shift may be represented as

$$\eta_2 = \frac{\Delta AST}{\sum AST} = \frac{\left(\frac{M}{2} - 1\right)(L-1)}{(M-1)(L-1)} = \frac{1}{2} - \frac{1}{2(M-1)} \approx 50\% (29)$$

where AST means arithmetic shift time.

#### V. SIMULATION AND DISCUSSIONS

The simulations of the proposed frequency offset estimation are carried out to study the performance contrasting with [9]. We use the simulation parameters as specified in [2]: N = 128,  $N_{pre} = 32$ ,  $N_g = 5$ , carrier frequencies f = 4.125MHz.

We consider the CRLB performances of different symbol length firstly. There are 24 synchronization sequences in one OFDM frame, including 21 packet synchronization sequences and 3 frame sequences, and 6 channel estimation sequence which is used to do fine frequency estimation . For convenience, we get L=2,4,6 to simulate and analyze the method. Fig 3 shows the CRLBs of the proposed method and [9] method as a function of SNR and L. With the increase of L, the algorithm performances better. The minimum could be lower than  $10^{-7}$ , which is rather small. Meanwhile, the CRLB decreases with the increase of SNR. The algorithm performs well even in the environment of SNR=0.

As there are 2 TFI patterns mapping with 4 preambles, the algorithm applicability needs to be considered. In Fig.4 we simulate the CRLB on TFI and TFI2 respectively. It is shown that it works well especially suitable for TFI2. With the increase of SNR, the CRLB could be lower than  $10^{-8}$ .

We can see from Fig.3 and Fig.4 that the CRLB of the proposed algorithm is a little bigger than that of [9]. So a third simulation is done to evaluate the difference. Fig.5 shows the difference between the proposed one and [9], from which we know that the difference is in the magnitude of  $10^{-7}$ , and decreases rapidly with the SNR increasing. When the SNR is above 15, the difference is close to zero.

Another phenomenon existed in the simulation is that the proposed algorithm of L = 6 performances almost the same with [9] of L = 4. So the capability of the proposed could be promoted by increasing the symbols used for estimation, which will somewhat increases the complexity but it is much less than the complexity we have decreased

#### VI. CONCLUSION

We have presented a low complexity frequency offset estimation algorithm for MB-OFDM based UWB systems. The proposed estimator utilizes a half-cycle CAZAC sequence as the preamble sequence, puts forward the concept of semi-cross contrast, incorporates the effects of different carrier frequency offsets, different preamble structures in different bands and achieves approximate 50% complexity decreasing comparing with [9]. The CRLBs of our proposed method is very close to that of [9] for moderate to high SNRs. The proposed estimator is of significant applicability to all preamble patterns.

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Fig. 3 CRLB analysis for different symbols length





Fig. 5 Difference between the proposed method and the reference

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