

# A Local Invariant Generalized Hough Transform Method for Integrated Circuit Visual Positioning

Fei Long Wei, Hua Yang, Hai Tao Zhang, Zhou Ping Yin

**Abstract**—In this study, an local invariant generalized Houghtransform (LI-GHT) method is proposed for integrated circuit (IC) visual positioning. The original generalized Hough transform (GHT) is robust to external noise; however, it is not suitable for visual positioning of IC chips due to the four-dimensionality (4D) of parameter space which leads to the substantial storage requirement and high computational complexity. The proposed LI-GHT method can reduce the dimensionality of parameter space to 2D thanks to the rotational invariance of local invariant geometric feature and it can estimate the accuracy position and rotation angle of IC chips in real-time under noise and blur influence. The experiment results show that the proposed LI-GHT can estimate position and rotation angle of IC chips with high accuracy and fast speed. The proposed LI-GHT algorithm was implemented in IC visual positioning system of radio frequency identification (RFID) packaging equipment.

**Keywords**—Integrated Circuit Visual Positioning, Generalized Hough Transform, Local invariant Generalized Hough Transform, ICpacking equipment.

## I. INTRODUCTION

VISUAL positioning is one of the most important technologies in the integrated circuit (IC) packaging equipment, and it can be used to pick or place IC chips with high accuracy in real time. There are many visual positioning algorithms proposed for 2D shape extraction under similarity transformation [1], [2], such as gray-based methods, feature-based methods, texture-based methods, and so on. The generalized Houghtransform (GHT) [3], as one classic representation of feature-based algorithms, is commonly used and fairly successful method due to its strong robustness to external influences, such as noise, occlusion, and illumination change and so on. The GHT is extended from Hough transform [4] for detection of arbitrary shape and has proved to be an extremely robust algorithm in the presence of occlusion and noise. However, in order to accurately estimate the position, angle and scale of 2D shape objects, the dimensionality of GHT parameter space should be set to 4D, which means that the robustness of GHT is at the expense of substantial storage requirement and high computational load. Therefore, the GHT algorithm is not suitable for real-time IC visual positioning due to high computation complexity.

To overcome the limitations of GHT algorithm, many research groups paid attention to improve the GHT algorithm by using the reduction of the storage requirement or the

dimensionality of GHT parameter space. Lee et al. [5] introduced a novel method for reducing the storage requirement of GHT, by using a boundary point as the reference point. However, the selection of the reference point leads to less robustness especially when the occlusion exists, and the resolution of accumulator space also greatly affects the performance of algorithm [6]. Ulrich et al. [7], [8] suggested that the model and the object images of GHT algorithm should be treated in hierarchical manners by using the image pyramid to reduce the size of accumulator array and speed up the GHT algorithm. The top pyramid level of object image is employed to quickly estimate the approximate position, angle and scale information, and the low pyramid level of object image is used for the accurate visual positioning, respectively. The recognition process of Ulrich's algorithm starts on the top pyramid level to low one by using the conventional GHT, and its performance is greatly increased by hierarchical search strategy. However, the GHT algorithm employs the gradient angle of edge point as the geometric feature, which is not invariant to the rotation of object. Tsai et al. [9] utilized the concavity, radius and normal angle of the edge points to establish the features of object for visual positioning instead of the gradient angle, and the performance of this algorithm is greatly affected by the curvature estimator. Especially, it cannot work well for the object image which is mainly composed of straight line. Ser et al. [10] selected the difference of gradient direction between the pair of edge points as the features of object, it reduces the dimensionality of parameter space to 2D, and it is a much faster and more efficient improved GHT. However, it is limited for the occlude object visual positioning, and it is also invalid when there are some broken boundaries in the object image. Aguado et al. [11] obtained the invariant geometric feature by using a pair of edge points, which are selected in the direction of straight line. This method is useful and efficient only for one object visual position and not suitable for many objects, due that the pair edge points should belong to the same object. These improved GHT algorithms are not suitable for multi-objects visual positioning under occlusion and noise in IC packaging equipment.

Thus, in this study, we proposed a novel improved local invariant generalized Hough transform (LI-GHT) algorithm for IC chips visual positioning by using the angle between the gradient of edge point pair as the local invariant geometric feature instead of gradient angle. The remainder of this paper organized as follows. In the Section II, the conventional GHT is briefly introduced and the novel LI-GHT is proposed and discussed in detail. In the Section III, many experiments are used to demonstrate the performance of new LI-GHT algorithm

F. L. Wei, H. Yang\*, H. T. Zhang, and Z. P. Yin are with State Key Laboratory of Digital manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wu Han, 430074, China (\*Corresponding author: Tel: +86-27-87559725; e-mail: huayang@hust.edu.cn).

for IC visual positioning. Last, the Section IV shows the conclusions of this study.

## II. LOCAL INVARIANT GENERALIZED HOUGH TRANSFORM (LI-GHT)

### A. Conventional GHT

The conventional GHT is briefly consists of two parts: the offline construction of a reference-table (R-Table) in the model image, and the online visual positioning in the object image by using R-Table.

In the offline part, the edges of model are detected by using Sobel or Canny operations firstly, and then an R-Table described the relationship between edge point and reference point is constructed in the model image and used for visual positioning in object image. The centroid of all edge points in the model image is commonly employed as the reference point for the construction of GHT R-Table. Fig. 1 shows that the gradient angles  $\theta_i$  of each edge point  $P_i$  in the model image is used as the index of GHT R-Table, and the corresponding vectors  $\vec{r}_i$  from edge point to reference point  $P_r$  are stored in the GHT R-Table, shown in Table I.

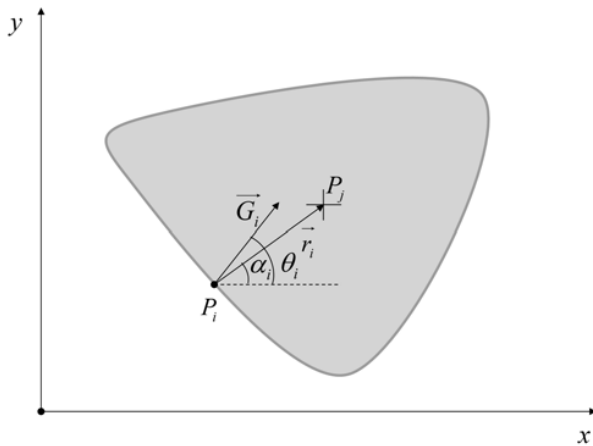


Fig. 1 The principle of constructing R-Table in GHT

TABLE I  
THE R-TABLE OF GHT

Index( $\theta$ )	Entries( $\vec{r}_n$ )
0	$\{\vec{r}_n   \theta = 0\} n = 1, 2, 3 \dots$
$\Delta\theta_\theta$	$\{\vec{r}_n   \theta = \Delta\theta_\theta\} n = 1, 2, 3 \dots$
$2\Delta\theta_\theta$	$\{\vec{r}_n   \theta = 2\Delta\theta_\theta\} n = 1, 2, 3 \dots$
$3\Delta\theta_\theta$	$\{\vec{r}_n   \theta = 3\Delta\theta_\theta\} n = 1, 2, 3 \dots$
...	...

In the online part, only considering the translation and rotation of object, a three-dimensionality (3D) accumulator array  $ACC$  is initialized according to the parameter space for voting process; secondly, the gradient angle  $\theta_i$  of each edge points  $P_i(x_i, y_i)$  in the object image is calculated by using the image difference method; thirdly, the rotation gradient angle  $\theta = \theta_i + \phi_k$  is calculated based on  $\theta_i$  and the quantized angle  $\phi_k$ , which denotes the object rotation angle between the model image and the object image; fourthly, the corresponding entries

$\vec{r}_n$  of each edge points  $P_i(x_i, y_i)$  are selected based on  $\theta$ ; fifthly, the possible positions of each edge points  $P_i(x_i, y_i)$  in the object image can be estimated by using (1), and the cell  $ACC(x_{io}, y_{io}, \phi_k)$  receives a vote and is increased by one; lastly, the accurate position and rotation angle  $(x_o, y_o, \phi_o)$  in the object image are directly determined as the maximum vote index in  $ACC$ .

$$\begin{aligned} x_{io} &= x_i + |\vec{r}_n| \cos(\alpha_n) \\ y_{io} &= y_i + |\vec{r}_n| \sin(\alpha_n) \end{aligned} \quad (1)$$

where  $\alpha_n$  is the angle between  $\vec{r}_n$  and  $x$  orientation.

### B. LI-GHT

In order to estimate the accurate position and rotation angle of object, the dimensionality of parameter space of conventional GHT should be set to 3D, and it leads high computation load for conventional GHT. To overcome this limitation of GHT, in this part, a novel LI-GHT algorithm is introduced in more detail, it is based on local invariant geometric feature, and LI-GHT algorithm also consists of an offline part and an online part like the conventional GHT. In the offline part, the index of R-Table of LI-GHT is constructed by using the local invariant geometric features, while the index of GHT R-table is based on the gradient angle directly. The detail procedure is described as follows:

- A1). Detect the object edges in the model image by using Canny detection method [12].
- A2). For each edge point  $P_i$  in the model image, it is necessary to search the corresponding point  $P_j$  for the construction of local invariant geometric feature. The second point  $P_j$  is selected as one point of intersection between the edge of object and a circle, the center point and radius of which are the current point  $P_i$  and  $\lambda$ , respectively. The points  $P_i^n (n = 0, 1, \dots, N)$  denote the  $N$  intersection points. The rotation angles  $\mathbb{R}_i^n$  between the gradient direction of  $P_i$  and the vectors direction from  $P_i$  to  $P_i^n$  along anti-clock-wise are calculated. The second point can be determined as the intersection point with the minimum rotation angle  $\min \mathbb{R}_i^n$ , shown in the Fig. 2. In this study, the radius of circle  $\lambda$  is set to be 5 pixels.
- A3). The index angle  $\beta_i$  is defined by these two edge points  $(P_i, P_j)$  as the following (2):

$$\beta_i = \arccos\left(\frac{\vec{G}_i \cdot \vec{G}_j}{|\vec{G}_i| |\vec{G}_j|}\right) \quad (2)$$

where  $\beta_i$  denotes the angle between vector  $\vec{G}_i$  and  $\vec{G}_j$ .  $\vec{G}_i = (G_{ix}, G_{iy})$  and  $\vec{G}_j = (G_{jx}, G_{jy})$  vectors are the gradient vectors of  $P_i, P_j$  in  $x$  and  $y$  orientation, respectively. The  $G_{ix}, G_{iy}$  components of  $G_i$  are calculated by using the convolution of the grey value with the mask of  $[1/12, -8/12, 0, 8/12, -1/12]$  in  $x$  and  $y$  orientation respectively, and the truncation error is much smaller than Sobel and Prewitt gradient operator [13]. The angle  $\beta_i$  is a local invariant geometric feature and treated as the

index of R-Table of LI-GHT. Considering the calculation error of gradient angle of  $P_i$ ,  $\beta_i \pm \Delta$  is used in the index of the R-Table instead of  $\beta_i$ , where  $\Delta$  denotes the calculation error of  $\beta_i$ .

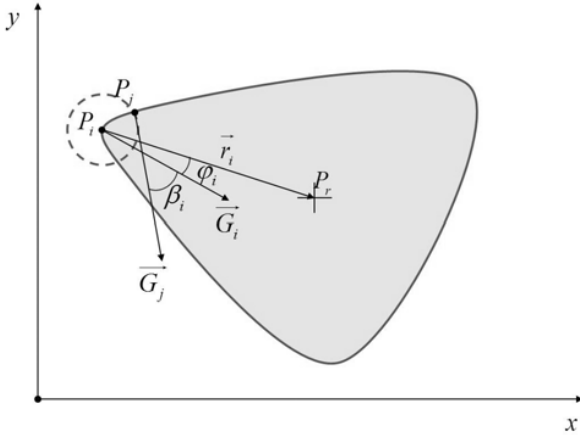


Fig. 2 The principle of constructing R-Table in LI-MGHT

Index( $\beta$ )	Entries( $\varphi, r$ )
0	
$\Delta\theta_\beta$	$\{(\varphi_n, r_n)   \theta = \Delta\theta_\beta\} n = 1, 2, 3 \dots$
$2\Delta\theta_\beta$	$\{(\varphi_n, r_n)   \theta = 2\Delta\theta_\beta\} n = 1, 2, 3 \dots$
$3\Delta\theta_\beta$	$\{(\varphi_n, r_n)   \theta = 3\Delta\theta_\beta\} n = 1, 2, 3 \dots$
...	...

Index( $\varphi$ )	Entries( $\theta$ )
0	
$\Delta\theta_\varphi$	$\{\theta_n   \theta = \Delta\theta_\varphi\} n = 1, 2, 3 \dots$
$2\Delta\theta_\varphi$	$\{\theta_n   \theta = 2\Delta\theta_\varphi\} n = 1, 2, 3 \dots$
$3\Delta\theta_\varphi$	$\{\theta_n   \theta = 3\Delta\theta_\varphi\} n = 1, 2, 3 \dots$
...	...

A4). For each edge point  $P_i$ , the angle  $\varphi_i$  between the gradient vector  $\vec{G}_i$  and the vector  $\vec{P}_i\vec{P}_r$ , and the distance  $d_i$  between  $P_i$  and  $P_r$  are calculated as the following (3):

$$\varphi_i = \arccos\left(\frac{\vec{G}_i \cdot \vec{P}_i\vec{P}_r}{|\vec{G}_i| \cdot |\vec{P}_i\vec{P}_r|}\right) \quad (3)$$

$$d_i = |\vec{P}_i\vec{P}_r|$$

where  $P_r$  is the centroid of all edge points in the model image, shown in Fig. 2 The  $\varphi_i$  and  $d_i$  as the function of  $\beta_i$ , and treated as the entries in the R-Table of LI-GHT, shown in Table II.

A5). To determine the rotation angle of object between the model image and the object image, another reference table (A-Table) is constructed in the proposed LI-GHT algorithm. Table III shows that  $\varphi_i$  is the index, the gradient angle  $\theta_i$  of  $P_i$  is treated as the entry of A-Table, and  $\theta_i$  can be derived as the following equation:

$$\theta_i = \arctan\left(\frac{G_{iy}}{G_{ix}}\right) \quad (4)$$

After the R-Table and A-Table are constructed, an online and effective voting procedure for visual positioning is carried out as follows:

- B1). A 2D accumulator array  $ACC_{loc}$  and a 1D accumulator array  $ACC_{rot}$  are initialized for accumulating the vote of all possible position and angle of object in the object image, respectively.
- B1). A 2D accumulator array  $ACC_{loc}$  and a 1D accumulator array  $ACC_{rot}$  are initialized for accumulating the vote of all possible position and angle of object in the object image, respectively.
- B2). Detect the object edges in the object image by using Canny detection method [12], and then determine the second corresponding point  $P_j$  for the construction of local invariant geometric feature via the method as described in A2).
- B3). The angle  $\beta_i$  is calculated for each edge point  $P_i$  as (2), and then the index in the R-Table of LI-GHT is determined based on the angle  $\beta_i$ . For all entries  $(\varphi_n, r_n)$  corresponding to  $\beta_i$ , all the possible vectors  $\vec{r}_i$  from  $P_i$  and the reference point can be derived as the following (6):

$$\begin{bmatrix} r_{ix}(1) \\ r_{iy} \end{bmatrix} = \begin{bmatrix} \cos(\varphi_n) & \sin(\varphi_n) \\ -\sin(\varphi_n) & \cos(\varphi_n) \end{bmatrix} \begin{bmatrix} G_{ix} \\ G_{iy} \end{bmatrix} \quad (5)$$

$$\vec{r}_i = (r_{ix}, r_{iy}) \quad (6)$$

Then, all the possible positions are calculated from the vector  $\vec{r}_i$  as the following (7):

$$x_{io} = x_i + \frac{d_n r_{ix}}{|\vec{r}_i|}$$

$$y_{io} = y_i + \frac{d_n r_{iy}}{|\vec{r}_i|} \quad (7)$$

Then the cell  $ACC_{loc}(x_{io}, y_{io})$  receives a vote:

$$ACC_{loc}(x_{io}, y_{io}) = ACC_{loc}(x_{io}, y_{io}) + 1$$

In this study, a fuzzy voting strategy is employed to extend the robustness of LI-GHT algorithm. For one possible position  $(x_{io}, y_{io})$  determined by above steps, both the cell  $ACC_{loc}(x_{io}, y_{io})$  and the neighbors  $\omega \times \omega$  around it are voted. In this paper, a Gaussian kernel neighbors is used and  $\omega$  is set to be 5, shown in the following:

$$\begin{bmatrix} 2 & 4 & 5 & 4 & 2 \\ 4 & 9 & 12 & 9 & 4 \\ 5 & 12 & 15 & 12 & 5 \\ 4 & 9 & 12 & 9 & 4 \\ 2 & 4 & 5 & 4 & 2 \end{bmatrix}$$

B4). After all edge points have been processed from procedure B2) to B3), the pixel-level position  $P_o(x_o, y_o)$  can be

easily determined when  $ACC_{loc}(x_o, y_o)$  is the maximum of  $ACC_{loc}$ .

B5). After the position  $(x_o, y_o)$  is located by procedure B4), the angle  $\varphi_i$  (see (3)) between the gradient vector  $\vec{G}_i$  of  $P_i$  and  $\vec{P}_i\vec{P}_o$  is estimated. Afterwards, the index of the A-Table is determined based on the angle  $\varphi_i$ . For all entries  $\theta_n$  corresponding to the angle  $\varphi_i$ , the possible rotation angle is estimated as the following (8):

$$\phi = \theta_n - \theta_i \quad (8)$$

where  $\theta_i$  is the gradient angle of  $P_i$ . Then, the possible rotation angle  $\phi$  receives a vote and is increased by one as the following:

$$ACC_{rot}(\phi) = ACC_{rot}(\phi) + 1$$

B6). The rotation angle is determined as the angle  $\phi_o$  when  $ACC_{rot}(\phi_o)$  is the maximum of  $ACC_{rot}$ .

The accuracy of the position and rotation angle is refined to sub-pixel level via the least square fit [8] in the proposed algorithm. After obtaining the position and rotation angle  $(x_o, y_o, \phi_o)$ , the similarities between the model image and one part of object image are calculated in the  $3 \times 3 \times 3$  neighborhood of  $(x_o, y_o, \phi_o)$ . And a second-order polynomial, the coefficients of which are calculated via the least square fit method, is employed to fit the discrete similarities, shown in (9). Then, the refined position and rotation angle is  $(x', y', \phi')$  when  $f(x', y', \phi')$  is the maxima of  $f(x, y, \phi)$  in the  $3 \times 3 \times 3$  neighborhood of  $(x_o, y_o, \phi_o)$ .

$$f(x, y, \phi) = a_0 + a_1x + a_2y + a_3\phi + a_4x^2 + a_5xy + a_6x\phi + a_7y^2 + a_8y\phi + a_9\phi^2 \quad (9)$$

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### III. EXPERIMENT

To verify the performance of proposed LI-GHT algorithm, the IC chips were rotated with various arbitrary known angles, and the chip images as the object images were captured by using vision system. The model image can be extracted from one object image. The proposed LI-GHT was implemented to search the position and rotation angle for all object images. The experiment results are shown in Fig. 3.

Fig. 3 shows that the proposed algorithm is still robustness, even if noise, occlusion, illumination changes exist in object images. The profiles of accumulator  $ACC_{loc}$  and  $ACC_{rot}$  of another chip (see Fig. 4) are shown in Fig. 5. Fig. 5 shows the proposed LI-GHT algorithm produce a sharp peak in  $ACC_{loc}$  and  $ACC_{rot}$  via the procedures B1) ~ B5), which means the proposed LI-GHT algorithm performs very well for the object image with translation and rotation.

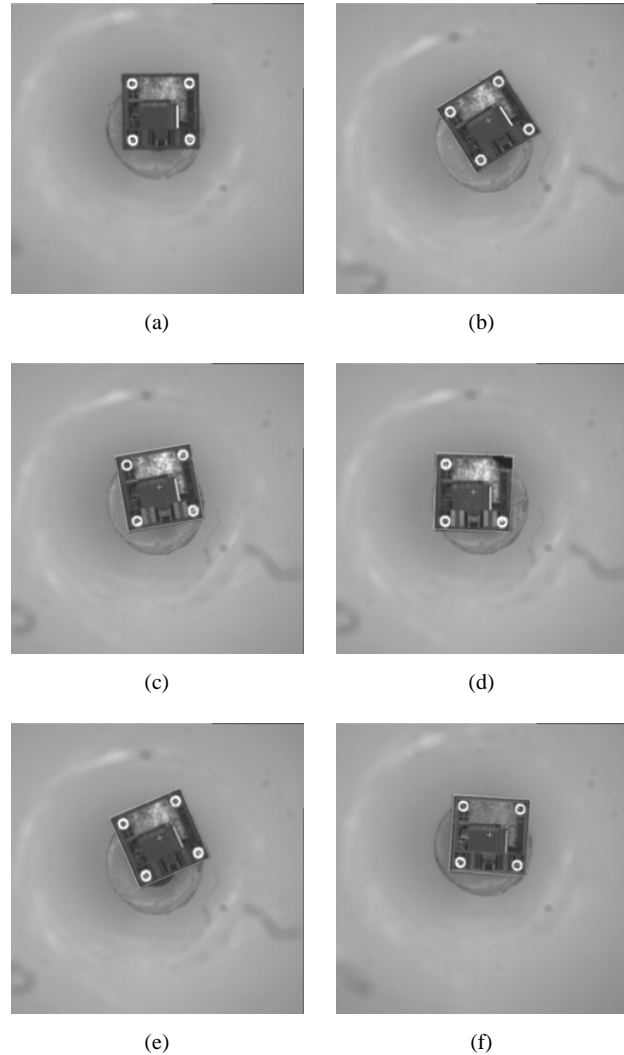


Fig. 3 Object image size= $800 \times 800$ , model image size= $212 \times 212$ .

(a) Image for the model extraction; (b) Position **(404. 162, 345. 419)**, rotation: 9.574, similarity: 0.856; (c) Position **(373. 935, 359. 307)**, rotation: 357.294, similarity: 0.737; (d) Position **(406. 175, 319. 089)**, rotation: 22.984, similarity: 0.962; (e) position **(418. 058, 310. 604)**, rotation: 356.354, similarity: 0.905; (f) Position **(417. 678, 324. 021)**, rotation: 31.636, similarity: 0.908

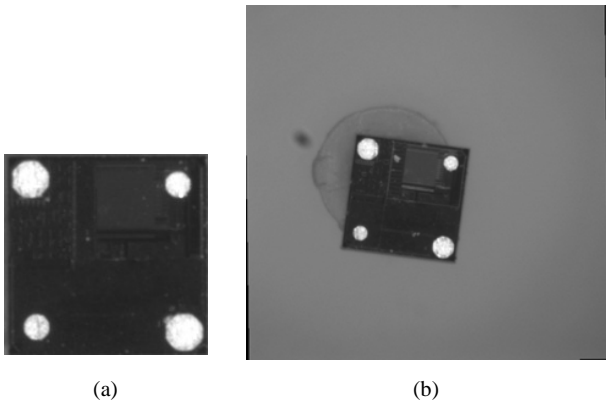


Fig. 4 Chip (a) Model image; (b) Object image

To verify the accuracy of the proposed LI-GHT algorithm, an IC chip image, shown in Fig. 3 (a), is translated for 10 times with each 0.1 pixels interval in both  $X$  and  $Y$  orientation, and rotated 10 times with each 1 degrees interval via the bilinear extrapolation. Afterwards, we selected the translated and rotated images as object images and the chip in Fig. 3 (a) as model image. Then, the position and rotation angle  $(x_o, y_o, \phi_o)$  of IC chip is estimated by implementing the proposed LI-GHT algorithm.  $(x_o + \Delta x, y_o + \Delta y, \phi_o + \Delta \phi)$  are defined as the precise values, where  $\Delta x, \Delta y$  and  $\Delta \phi$  denote the translation and rotation interval.  $(x_t, y_t, \phi_t)$  are defined as the test values and were estimated by implementing the proposed LI-GHT algorithm for these translational and rotational object images.

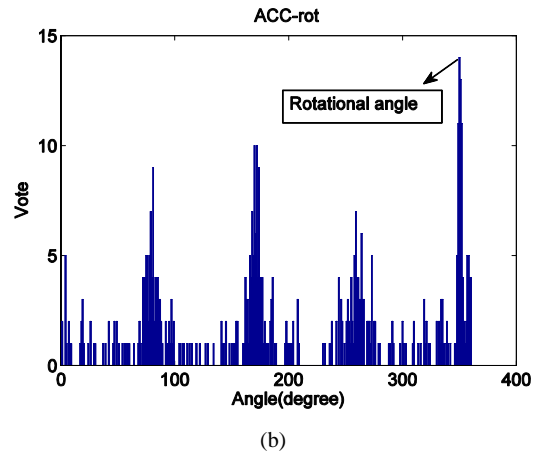
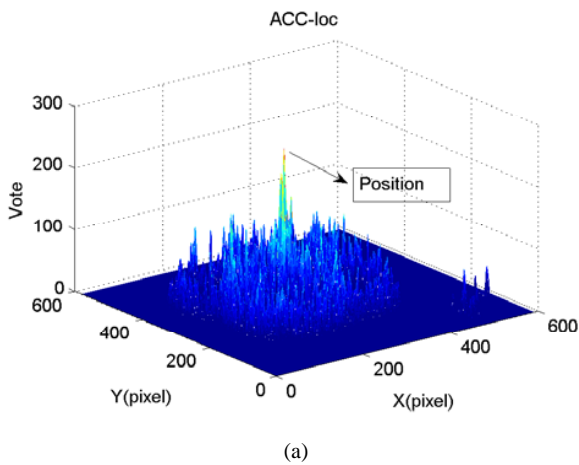


Fig. 5 (a) The profile of  $ACC_{loc}$ ; (b) The profile of  $ACC_{rot}$

Fig. 6 shows that estimation errors of visual positioning between the precise values and the test values. It can be seen the accuracy of position are less than 0.15 pixels, and the accuracy of rotation angle are less than 0.2 degree. This means that the proposed LI-GHT algorithm can estimate the accurate position and rotation angle of object under noise.

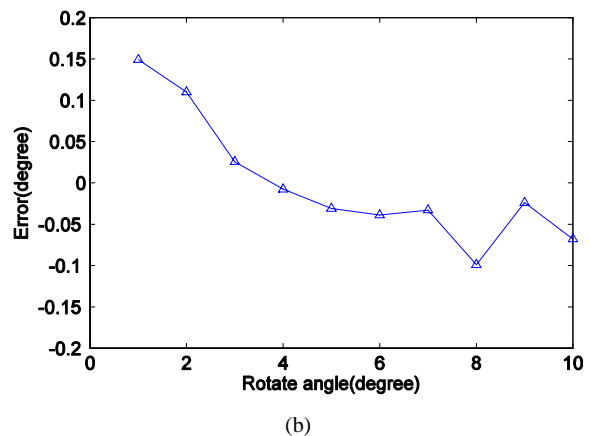
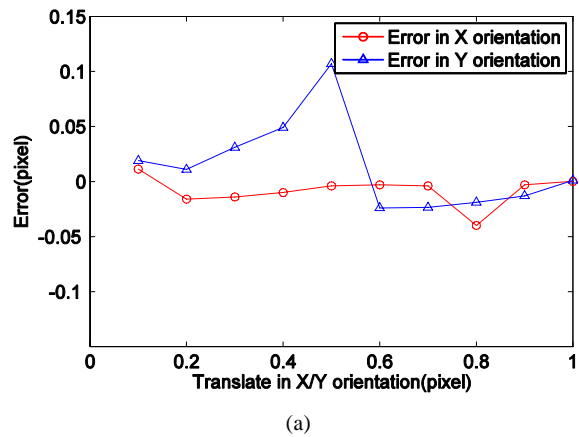


Fig. 6 (a) Error in X/Y orientation; (b) Error in rotation

The computational time of the proposed LI-GHT algorithm was also compared with the conventional GHT. Fig.7. shows the experiment results of implemented time for both LI-GHT and GHT.

From the experiment results, it can be seen that the computation time of the conventional GHT is increasing with the angle searching range, while the computation time of LI-GHT is remain unchanged due to the rotational invariance of the local geometric feature. Furthermore, the computation time of LI-GHT is much smaller than conventional GHT when there is a large rotation angle for object. This means that the proposed LI-GHT algorithm can fast accurately estimate the position and rotation angle at any angle of rotation.

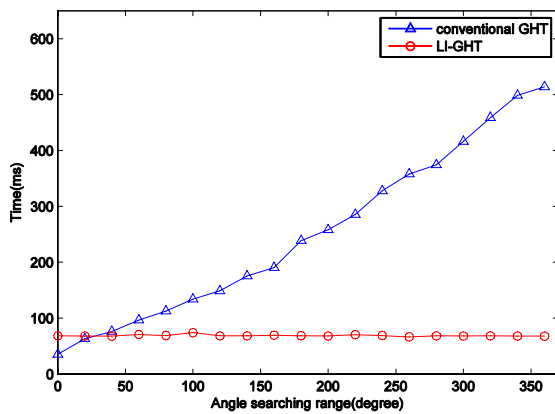
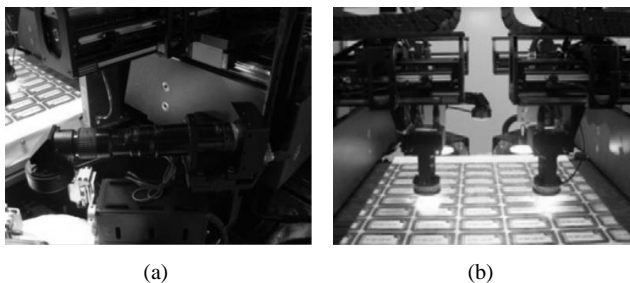


Fig. 7 The computation time of the conventional GHT and LI-GHT in different angle searching ranges

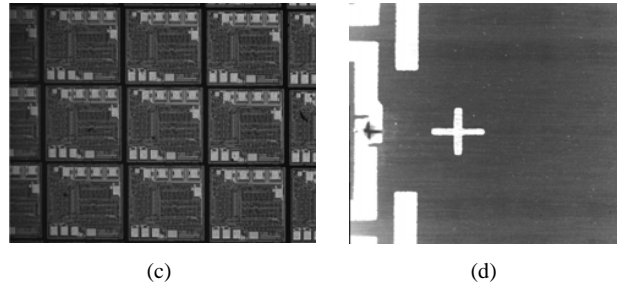


Fig. 8 RFID packaging equipment



(a)

(b)



(c)

(d)

Fig. 9 (a) The machine vision system of the chip picking process; (b) The machine vision system of the chip placing process; (c) Chips on wafer; (d) The cross on circuit

The proposed LI-GHT algorithm was successfully implemented in the visual positioning system of the chip picking and placing processes in RFID packaging equipment, shown in Fig. 8. Figs. 9 (a) and (b) show the chip picking and placing processes of visual positioning system in RFID packaging equipment. The visual positioning system of the chip picking process search the IC chips in the object images, afterward, the position and rotation angle of IC chips are sent to the motion control system, then manipulator picks up the chip from the wafer, shown in Fig. 9 (c). After the position of the cross on the circuit is located by using the visual positioning system of the chip placing process, the manipulator places the chip on the cross, shown in Fig. 9 (d).

#### IV. CONCLUSION

In this study, a novel improved generalized Hough transform LI-GHT based on local geometric feature is proposed for IC chip visual positioning. In order to determine position and rotation angle, the dimensionality of parameter space of conventional GHT should be set as 3D, and it leads high computation load. The proposed LI-GHT algorithm employs the local geometric feature of a pair of points as the index of LI-GHT R-Table, while the conventional GHT uses the gradient angle as the index of R-Table; the dimensionality of parameter space of LI-GHT is only 2D, while the one of GHT is 3D. Thus, the computational time of LI-GHT is much less than GHT and is suitable for IC chip visual position in real time. The performance of the proposed LI-GHT algorithm was demonstrated by implementing in RFID packaging equipment and compared with the conventional GHT algorithm. The experiment results show that the accuracy of LI-GHT algorithm is very high, and the implemented time is much less than the conventional GHT algorithm. The proposed LI-GHT algorithm is also robust in spite of noise, occlusion, illumination changes, and has been applied to the chip picking and placing processes of RFID packaging equipment successfully.

#### ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundation of China under Grant 91023034 Grant 51120155001, and the Open-Project of the State Key Laboratory of Digital Manufacturing Equipment and Technology of China under Grant DMETKF2013002.

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