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A Hydro-Mechanical Model for Unsaturated Soils

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***Abstract*—**The hydro-mechanical model for unsaturated soils has been presented based on the effective stress principle taking into account effects of drying-wetting process. The elasto-plastic constitutive equations for stress-strain relations of the soil skeleton have been established. A plasticity model is modified from modified Cam-Clay model. The hardening rule has been established by considering the isotropic consolidation paths. The effect of drying-wetting process is introduced through the χ parameter. All model coefficients are identified in terms of measurable parameters. The simulations from the proposed model are compared with the experimental results. The model calibration was performed to extract the model parameter from the experimental results. Good agreement between the results predicted using proposed model and the experimental results was obtained.

***Keywords*—**Drying-wetting process, Effective stress, Elasto-plastic model, Unsaturated soils

I. Introduction

MOST hydro-mechanical constitutive models for unsaturated soils have been developed in a critical state framework using the two stress state variables approach. However, recently there has been greater interest in using the effective stress approach [1]-[4]. In the two stress state variables approach, total stress in excess of pore air pressure (referred to as net stress) and the difference between pore air pressure and pore water pressure (referred to as matric suction) are treated as independent variables in describing the mechanical behavior of an unsaturated soil. In the effective stress approach, matric suction and net stress combine to give a single effective stress, and only the effective stress is required for the complete characterization of the mechanical behavior of an unsaturated soil.

Advantages of the effective stress approach over the two stress state variables approach are that complete characterization of the stress-strain behavior of soil requires expressing of the behavior in terms of a single effective stress rather than two independent stresses. Also, the laboratory testing required for determining soil properties in terms of effective stress could be performed in any geotechnical engineering laboratory. It is substantially less cumbersome and time-consuming than that required to determine soil properties in terms of two independent stress variables.

This paper outlines the development of an elasto-plastic model for the behavior of unsaturated soils. The model requires specification of a single set of material parameters, χ. The effects of drying and wetting processes are also introduced through this parameter.

II. Model Development

A. Conceptual Model

The model is developed based on the effective stress principle taking into account suction effect. Both elastic as well as elasto-plastic straining of the solid skeleton are allowed for. The yield surface is defined in the effective stress space, but it is affected by plastic volumetric strain and matric suction. Plastic flow is defined using a generalized associated flow rule.

The constitutive relations for the fluid phases are derived based on two interactive continua: one representing the pore-liquid phase and the other the pore-gas phase. Thus, at every point in space two pressures are introduced, namely the average liquid pressure and the average gas pressure. The interaction between the two fluid phases is established through the soil water characteristic curve describing the dependency of the degree of saturation on suction. The coupling between the fluid phases and the matrix deformation is established through the effective stress parameters. All model coefficients are identified in terms of measurable parameters.

B. Notation

The standard notations for triaxial tests are used throughout. Two pairs of the work conjugate variables are mean effective stress and volumetric strain (p' and ε_v), and deviator stress and shear strain (q and ε_s). These variables are defined as,

$$
\begin{aligned}
p' &= \frac{\sigma'_a + 2\sigma'_r}{3} \\
q &= \sigma'_a - \sigma'_r \\
\varepsilon_v &= \varepsilon_a + 2\varepsilon_r \\
\varepsilon_s &= \frac{2}{3}\left(\varepsilon_a - \varepsilon_r\right)
\end{aligned}
\qquad (1)
$$

in which, σ'_a and σ'_r are the axial and radial effective stresses. ε_a and ε_r are the axial and radial strains. Compressive stresses and strains are assumed positive.

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C. Effective Stress

The effective stress equation for unsaturated soils is expressed as [5],

$$p' = p + \chi \ p_l + (1-\chi) p_g \tag{2}$$

where p_l is the pore liquid pressure, p_g is the pore gas pressure. The parameter χ can be quantified using the expression [6],

$$\chi = \begin{cases} [s/s_e]^{-\Omega} & for \quad s \geq s_e \\ 1 & for \quad s \leq s_e \end{cases} \tag{3}$$

in which, s_e is suction value marking the transition between saturated and unsaturated states, and Ω is a material parameter, with a best-fit value of 0.55. For the main wetting path $s_e = s_{ex}$, and for the main wetting path $s_e = s_{ae}$, in which s_{ex} is the air expulsion value and s_{ae} is the air entry value.

The incremental form of effective stress in (2) can be expressed as,

$$dp' = dp_{net} + \chi \ ds + s \ d\chi \tag{4}$$

in which, $p_{net} = p + p_g$ is the mean net stress and $s = -(p_g - p_l)$ is the matric suction.

D. Elastic Constitutive Equations

The stress -strain relations, for an elastic medium, is written as,

$$\begin{aligned} dp' &= K d\varepsilon_v^e \\ dq &= 3G d\varepsilon_s^e \end{aligned} \tag{5}$$

in which, $K = c^{-1}$ is the bulk modulus and $G = 3K(1-2\nu)/2(1+\nu)$ is the shear modulus. ε_v^e, ε_s^e are the elastic volumetric and shear strains of the soil skeleton. The compressibility coefficient of soil structure, c, can be determined from the slope of the unloading-reloading line [7],

$$c = \frac{\kappa}{v p'} \tag{6}$$

where, κ is the slope of the unloading-reloading line in the semi-logarithmic compression plane, v is the specific volume.

Now, combining (2) and (4) yields the stress-strain relation for unsaturated elastic soils as,

$$\begin{aligned} dp_{net} &= K d\varepsilon_v^e - \chi \ ds - s \ d\chi \\ dq &= 3G d\varepsilon_s^e \end{aligned} \tag{7}$$

E. Elasto-Plastic Constitutive Equations

To describe the plastic properties of the system, the yield function is defined in terms of effective stresses, but its size is assumed to be affected by suction as observed in the experiments by several investigators [8]-[10]. Within this context, the yield function takes the form,

$$f(p', q, s, \varepsilon_v^p) = 0 \tag{8}$$

where ε_v^p is a plastic volumetric strain. The function f is assumed to be smooth in the stress space at the point of interest, and the plastic behavior is assumed to emanate from the solid skeleton. However, all three phases (solid, liquid, and gas) are assumed to develop irreversible strains as in the case of elastic-plastic media with single porosity [11].

The plastic flow is defined through a generalized normality rule. Assuming the existence of a plastic potential where stresses act only through the effective stress and suction, then we have,

$$\begin{aligned} d\varepsilon_v^p &= d\Lambda \frac{\partial g}{\partial p'} \\ d\varepsilon_s^p &= d\Lambda \frac{\partial g}{\partial q} \end{aligned} \tag{9}$$

where $d\Lambda \geq 0$ is the plastic multiplier and the superscript p denotes the plastic response. The behavior is called associative, and the elasto–plastic stiffness of the underlying drained solid displays major symmetry when the yield surface and the plastic potential coincide.

Finally, assuming that the increments of strain consist of the elastic and plastic components, the following expressions can be established,

$$\begin{aligned} d\varepsilon_v &= d\varepsilon_v^e + d\varepsilon_v^p \\ d\varepsilon_s &= d\varepsilon_s^e + d\varepsilon_s^p \end{aligned} \tag{10}$$

To obtain the incremental elastic–plastic constitutive equations of the system, the consistency equation is first written in the usual way,

$$df = \frac{\partial f}{\partial p'} dp' + \frac{\partial f}{\partial q} dq + \frac{\partial f}{\partial s} ds + \frac{\partial f}{\partial \varepsilon_v^p} d\varepsilon_v^p = 0 \tag{11}$$

Substituting (5) and (9) into (11) yields an expression for the plastic multiplier $d\Lambda$,

$$d\Lambda = \frac{1}{H} \left[K \frac{\partial f}{\partial p'} d\varepsilon_v + 3G \frac{\partial f}{\partial q} d\varepsilon_s + \frac{\partial f}{\partial s} ds \right] \tag{12}$$

where the modulus $H \equiv H(s,T,X)$, which depends on suction is defined as,

$$H \equiv h + \frac{\partial f}{\partial p'} K \frac{\partial g}{\partial p'} + \frac{\partial f}{\partial q} 3G \frac{\partial g}{\partial q} > 0 \tag{13}$$

The hardening modulus $h \equiv h(s, X)$ is defined as,

$$h \equiv -\frac{\partial f}{\partial \varepsilon_v^p} \frac{\partial g}{\partial p'} = -\frac{\partial f}{\partial \varepsilon_v^p} \frac{\partial \varepsilon_v^p}{d\Lambda} \tag{14}$$

which describes the evolution of the size of the yield surface and depends on suction.

Making use of the constitutive equations (12) into the elastic equations in (7) yields the incremental elasto–plastic equations,

$$dp_{net} = D_{pp} d\varepsilon_v + D_{pq} d\varepsilon_s + a_{ps} ds + s d\chi \tag{15}$$
$$dq = D_{qp} d\varepsilon_v + D_{qq} d\varepsilon_s + a_{qs} ds$$

with

$$D_{pp} = K\left(1 - \frac{1}{H} K \frac{\partial g}{\partial p'} \frac{\partial f}{\partial p'}\right) \tag{16}$$
$$D_{pq} = K\left(-\frac{1}{H} 3G \frac{\partial g}{\partial p'} \frac{\partial f}{\partial q}\right)$$
$$D_{qq} = 3G\left(1 - \frac{1}{H} 3G \frac{\partial g}{\partial q} \frac{\partial f}{\partial q}\right)$$
$$D_{qp} = 3G\left(-\frac{1}{H} K \frac{\partial g}{\partial q} \frac{\partial f}{\partial p'}\right)$$
$$a_{ps} = -\chi - \frac{1}{H} K \frac{\partial g}{\partial p'} \frac{\partial f}{\partial s}$$
$$a_{qs} = -\frac{1}{H} K \frac{\partial f}{\partial p'} \frac{\partial g}{\partial s}$$

F. Plastic Driver

To define the essential features of the plastic framework described in the previous section, an appropriate plasticity driver needs to be invoked. Typically, the modified Cam-Clay model is used for this purpose [8], [9]. The yield surface is defined as a function of the mean effective stress, p', the deviator stress, q, the preconsolidation pressure, p'_c, and the slope of the critical state line, M. The shape of the yield surface is assumed to be elliptical in the $p' - q$ plane. Moreover, the associated flow rules are assumed to obtain the increment of plastic strains. The yield surface is defined as,

$$f = q^2 - M^2\left[p'\left(p'_c - p'\right)\right] = 0 \tag{17}$$

Differentiating the yield function with respect to p', q, s and ε_v^p, we can obtain

$$\frac{\partial f}{\partial p'} = M^2\left(2p' - p'_c\right) \tag{18}$$
$$\frac{\partial f}{\partial q} = 2q$$
$$\frac{\partial f}{\partial s} = -M^2\left[\left(p'_c - 2p'\right)\frac{\partial p'}{\partial s} + p' \frac{\partial p'_c}{\partial s}\right]$$
$$\frac{\partial f}{\partial \varepsilon_v^p} = -M^2 p' \frac{v_o p'_c}{\lambda - \kappa}$$

G. Hardening Rule

To derive the hardening function, we consider consolidation responses of two identical soil samples, which are at a constant suction, as shown in Fig. 1. Sample *A* is consolidated in a saturated state, whereas sample *B* is consolidated at suction, s, with $s > s_e$. Consider the stress path 1–2–3–4–5. The stress at point 1 is p'_{co}, and at point 5 it is p'_c. At point 1, the sample is unloaded to point 2. For the stress path 2–3, the suction within the sample is increased to s. From point 3 to point 5 the sample is loaded mechanically to effective stresses p'_4 and p'_c, at points 4 and 5, respectively.

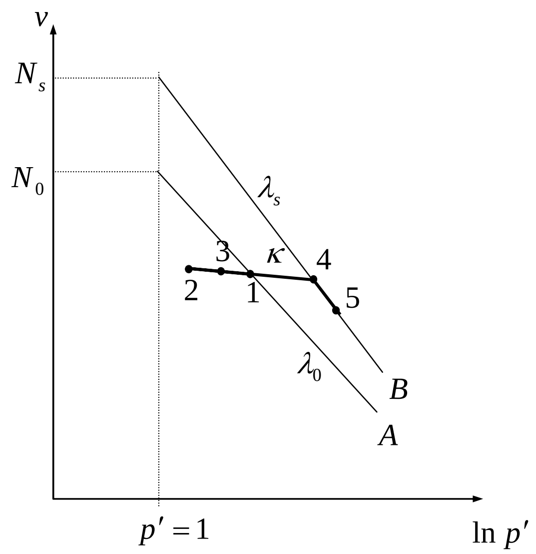

Fig. 1 hydro hardening in unsaturated soils

The plastic volumetric change from point 4 to point 5 can be written as,

$$\varepsilon_v^p = \varepsilon_v - \varepsilon_v^e = \frac{\left(\lambda_s - \kappa\right)}{v_o} \ln \frac{p'_c}{p'_4} \tag{19}$$

The specific volume, v_4, at point 4 is expressed as,

$$v_4 = N_s - \lambda_s \ln p'_4 = N_0 - \lambda_0 \ln p'_{co} - k \ln \frac{p'_4}{p'_{co}} \qquad (20)$$

Substituting for p'_4 in (19) from (20) and rearranging give,

$$\ln p'_c = \frac{N_s - N_0}{\lambda_s - \kappa} + \frac{\lambda_0 - \kappa}{\lambda_s - \kappa} \ln p'_{co} + \frac{v_o \varepsilon_v^p}{\lambda_s - \kappa} \qquad (21)$$

Differentiating (21) with respect to s and ε_v^p, we can obtain

$$\frac{\partial p'_c}{\partial s} = \frac{p'_c}{(\lambda_s - \kappa)^2}\left[(\lambda_s - \kappa)\frac{\partial N}{\partial s} - \left[(N_s - N_0) - (\lambda_0 - \kappa)\ln p'_{co} + v_o \varepsilon_v^p\right]\frac{\partial \lambda}{\partial s}\right] \qquad (22)$$

$$\frac{\partial p'_c}{\partial \varepsilon_v^p} = \frac{v_o p'_c}{\lambda_s - \kappa}$$

III. Verification and Application of Model

A. Model Parameters

In this section, simulations from the elastic-plastic model developed in this paper are compared with the experimental results from a series of suction-controlled shear tests ($ds = 0$) on a compacted kaolin sample. The effective cell pressure and overconsolidation ratio used in the experiments were 100 kPa and 2.0 respectively. The purpose of the comparisons was to investigate the validity of the constitutive relationships advocated in this paper, and to illustrate the predictive capabilities of the model by its ability to closely match the stress strain behavior of the test samples.

The model simulations were conducted by expressing the governing equations of the model and were then solved as a series of initial value problems.

The parameters used in the elasto-plastic model and the variation of preconsolidation pressures versus matric suction obtained from a series of suction-controlled isotropic loading tests are summarized in Table I and Fig. 2, respectively.

TABLE I
ELASTO-PLASTIC MODEL PARAMETERS

Parameters	Values
κ	0.019
λ	0.054
ν	0.20
M	0.88
s_{ex}	35kPa
s_{ae}	80kPa

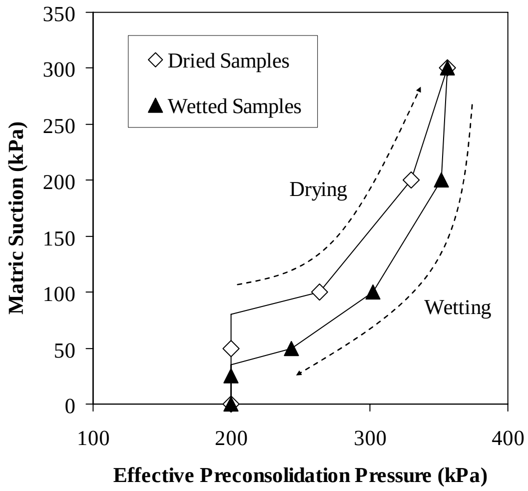

Fig. 2 loading collapse curve for compacted kaolin samples

B. Simulation of Test Results

In the case of suctioned-controlled shear tests ($ds = 0$) and following the normality rule, Equations (15) become,

$$dp_{net} = D_{pp} d\varepsilon_v + D_{pq} d\varepsilon_s \qquad (23)$$

$$dq = D_{qp} d\varepsilon_v + D_{qq} d\varepsilon_s$$

Fig. 3 and 4 show the comparison between the predicted stress-strain relations and the experimental results for each of the shear tests. For stress-strain curve, close comparisons between the model predictions and the shear test results are obtained. However, some small differences occur in the plot between shear strain and volumetric strain. This is probably due to errors in volume measurement of unsaturated soil samples which generally happens in this research area. In fact, the predictions would be accepted for practical purposes.

IV. Conclusion

A hydro-mechanical model for unsaturated soils including the suction effects on the yield surface is developed in a critical state framework based on the effective stress principle. Constitutive equations for stress-strain relations for unsaturated soils were established. The effects of drying and wetting processes are also introduced through the material parameters, χ. All model coefficients are identified in term of measurable parameters. The proposed model was applied to experimental data. The stress-shear strain relations and volume changes during shearing were modeled satisfactorily for practical purposes.

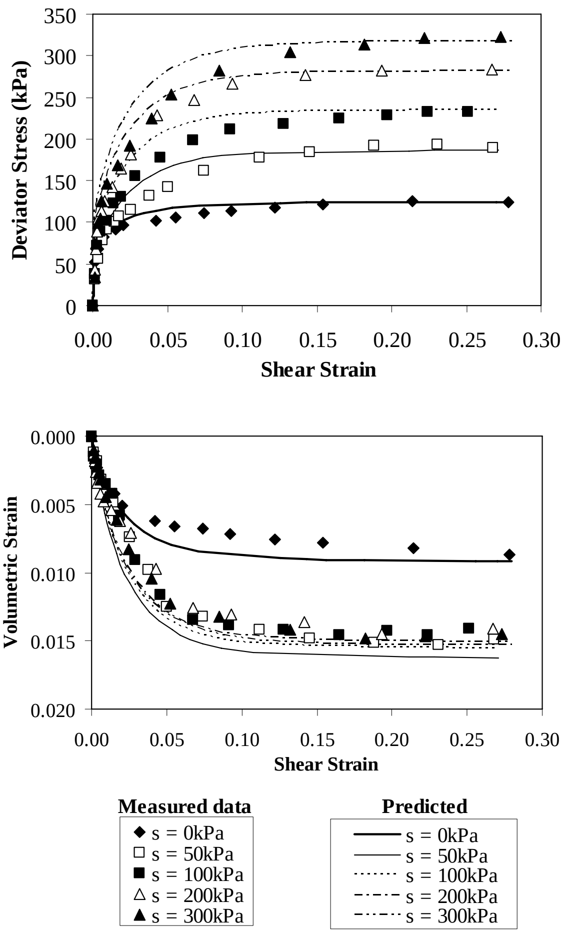

Fig. 3 Prediction of stress-strain relations obtained from shear tests on the dried samples

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REFERENCES

[1] R. W. Lewis, and B. A. Schrefler, *The Finite Element Method in the Deformation and Consolidation of Porous Media*, John Wiley and Sons, UK, 1987.

[2] D. C. Sheng, S. W. Sloan, and A. Gens, "Finite element formulation and algorithms for unsaturated soils. Part I: Theory," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 27, no. 9, 2003, pp. 745–765.

[3] B. Loret, and N. Khalili, "An Effective Stress Elastic-plastic Model for Unsaturated Porous Media," *Mechanics of Materials*, vol. 34, 2002, pp. 97–116.

[4] D. Gallipoli, A. Gens, R. Sharma, and J. Vaunat, "An elasto-plastic model for unsaturated soil incorporating the effects of suction and degree of saturation on mechanical behaviour," *Geotechnique*, vol. 53, no. 1, 2003, pp. 123–135.

[5] A. W. Bishop, *The Principle of Effective Stress*, Teknish Ukebladk, Norway, 1959, pp. 859–863.

[6] A. Uchaipichat, and N. Khalili, "Experimental investigation of thermo-hydro-mechanical behaviour of an unsaturated silt," *Geotechnique*, vol. 59, no. 4, 2009, pp. 339–353.

[7] D. M. Wood, *Soil Behaviour and Critical State Soil Mechanics*, Cambridge University Press, Cambridge, UK., 1990.

[8] E. E. Alonso, A. Gens, and A. Josa, "A constitutive model for partially saturated soils," *Geotechnique*, vol. 40, no. 3, 1990, pp. 405–430.

[9] S. J. Wheeler, and V. Sivakumar, "An elasto-plastic critical state framework for unsaturated soils," *Geotechnique*, vol. 45, no. 1, 1995, pp. 35–54.

[10] Y. J. Cui, and P. Delage, "Yielding and plastic behaviour of an unsaturated compacted silt," *Geotechnique*, vol. 46, no. 2, 1996, pp. 291–311.

[11] J. R. Rice, and M. P. Clearly, "Some basic stress diffusion solutions of fluid saturated elastic porous media with compressible constituents," *Review of Geophysics*, vol. 14, no. 2, 1976, pp. 227–241.

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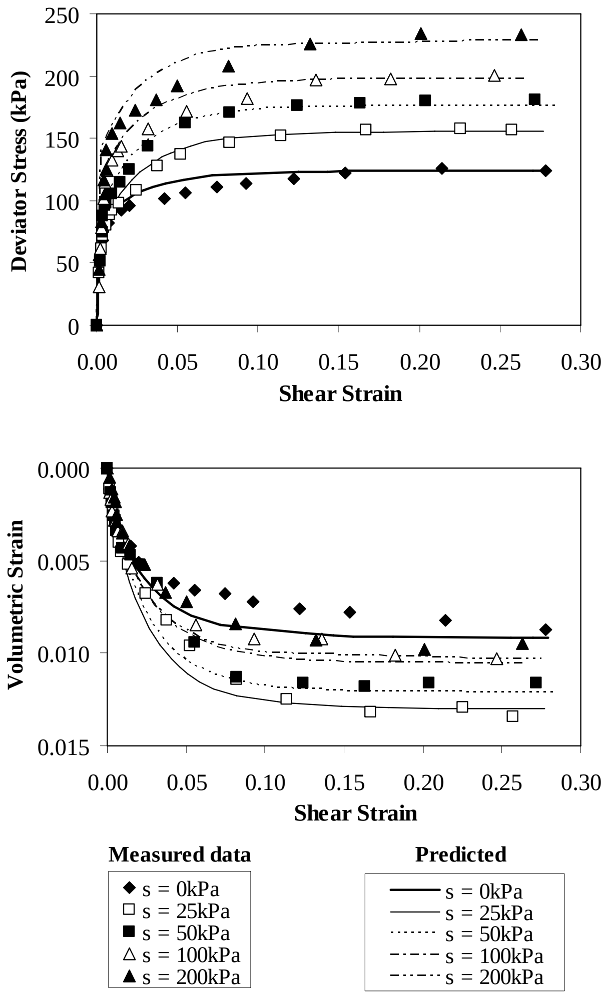

Fig. 4 Prediction of stress-strain relations obtained from shear tests on the wetted samples