

# A Generator from Cascade Markov Model for Packet Loss and Subsequent Bit Error Description

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**Abstract**—In this paper we present a novel error model for packet loss and subsequent error description. The proposed model simulates the error performance of wireless communication link. The model is designed as two independent Markov chains, where the first one is used for packet generation and the second one generates correctly and incorrectly transmitted bits for received packets from the first chain. The statistical analyses of real communication on the wireless link are used for determination of model's parameters. Using the obtained parameters and the implementation of the generator, we collected generated traffic. The obtained results generated by proposed model are compared with the real data collection.

**Keywords**—Wireless channel, error model, Markov chain, Elliot model, Gilbert model, generator, IEEE 802.11.

## I. INTRODUCTION

THE significant advance in the networks design led to the need of optimizing the future communication protocols with the aim to evaluate the network performance. The most common techniques include simulation, analytical models and analysis of empirical data. Accurate error modeling process is necessary for understanding the network behavior and is essential in designing of error control protocols or real time applications.

Discrete Markov models, mainly the Gilbert model, are commonly used for modeling of the network error characteristics. This approach is based on the analysis of a communication on link.

In [2] is published the way for the construction of the generalized Markov model in the special cascade for solving probabilities of partly dependent events. It is not limited to solve data error problems and it is very probable, that this model can be also useful for modeling many other types of probability cases. This model is Gilbert's cascade model.

In this paper we present the novel cascade Markov model and results from designed and programmed generator for

received traffic. First part of this generator is a generator for packet loss. It is based on a simplified Gilbert model [2]. Second part is a generator for bit error rate. This is based on a complete Elliot model [4]. Using the obtained parameters and the implementation of the generator, we collected generated traffic. Two generators need to work independently. Therefore, in the process of generation, the error-bit generator must continue to produce numbers also in the packet loss state (determined by the upper generator), when its output is not recorded. The obtained results generated by proposed model are compared with the real data collection.

## II. THEORETICAL BASIS AND MODEL

The wireless network simulation depends on the statistical analysis of the wireless link performance. For this purpose, we analyzed the network traces from the view of error bursts and error gaps (error-free interval) together with various bit/packet error rate stats. An example of error burst and error gap can be seen at fig. 1. Then we calculate statistics of the lost and received packets and of the received packets with and without error. We define a packet cluster as a group of consecutive packets of defined length. It is an error trace with length equal to multiples of the original packet bit length used in the measurement.

The following analysis of network traces was performed:

- Error burst length distribution
- Error gap length distribution
- Burst length distribution of lost packets
- Burst length distribution of received packets
- Probability of receiving an error-free packet cluster of defined size
- Probability of receiving a defective packet cluster of the defined size
- Loss probability of the whole packet cluster of the defined size in two variations:
  - The whole packet cluster is lost, when every packet in the cluster is lost.
  - The whole packet cluster is lost, when at least one packet in the cluster is lost.

The average length of inter-burst intervals (gaps) in a packet of length  $n$  and the average number of gaps are defined in [5].

As stated in [5], the last "1" in the error trace of the received packet right before the lost packet and the first "1" of the error trace of this lost packet are not a part of the same bit error

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burst. To simplify the model, we also assume that error bursts don't run over the packet borders, as expressed by (1).

$$\bar{G}(n) = \sum_{j=0}^{\infty} (j+1) \cdot \frac{\text{gaps in n-bit packet with length of } (j+1) \text{ bits}}{\text{all gaps in n-bit packet}} \quad (1)$$

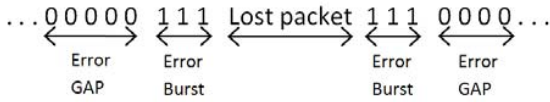


Fig. 1 Example of error gap and error burst in error trace.

*A. Packet Loss-Receive Model*

For the packet loss modeling, we have chosen the simple Markov probability scheme. Its basic parameters are set according to [3]:

Lemma: If we assume whole packet length as an element of the error sequence, then it is possible to find an equivalent model of the block error rate with matrices  $\hat{P}$  and  $\hat{H}$ , which is determined by:

$$\hat{\pi} \cdot (\hat{P} \cdot \hat{H})^{k_1} \cdot [\hat{P} - (\hat{P} \cdot \hat{H})]^{k_2} \dots (\hat{P} \cdot \hat{H})^{k_n} \cdot \underline{1} = \pi \cdot (P \cdot H)^{(n-k_1)} \cdot [P^n - (P \cdot H)^n]^{k_2} \dots (P \cdot H)^{(n-k_n)} \cdot \underline{1} \quad (2)$$

Mathematical variables used in this article are compatible with [3], [4] – symbols describing the packet error model use “^” sign and symbols without “^” sign describe a bit error model.

Final probability state vector is

$$\hat{\pi} = (\hat{\pi}_1, \hat{\pi}_2) \quad (3)$$

Generator matrix of received packets is

$$\hat{H} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (4)$$

Transition probability matrix is as in Fig. 2.

$$\hat{P} = \begin{pmatrix} 1 - \hat{p}_{12} & \hat{p}_{12} \\ \hat{p}_{21} & 1 - \hat{p}_{21} \end{pmatrix} \quad (5)$$

Then according to [6]

$$\hat{\pi} = \left( \frac{\hat{p}_{21}}{\hat{p}_{12} + \hat{p}_{21}}, \frac{\hat{p}_{12}}{\hat{p}_{12} + \hat{p}_{21}} \right) \quad (6)$$

Probability of  $k$  packets with the constant length  $n$  is

$$\hat{p}(k \cdot n) = \hat{\pi} \cdot (\hat{P} \cdot \hat{H})^k \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (7)$$

From (3) – (7), we obtain:

$$\hat{p}(k \cdot n) = \begin{pmatrix} \hat{p}_{21} & \hat{p}_{12} \\ \hat{p}_{12} + \hat{p}_{21} & \hat{p}_{12} + \hat{p}_{21} \end{pmatrix} \cdot \begin{pmatrix} 1 - \hat{p}_{12} & 0 \\ \hat{p}_{21} & 0 \end{pmatrix}^k \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (8)$$

And then by mathematical induction, we obtain:

$$\hat{p}(k \cdot n) = \frac{\hat{p}_{21} \cdot (1 - \hat{p}_{12})^k + \hat{p}_{12} \cdot \hat{p}_{21} \cdot (1 - \hat{p}_{12})^{k-1}}{\hat{p}_{12} + \hat{p}_{21}} \quad (9)$$

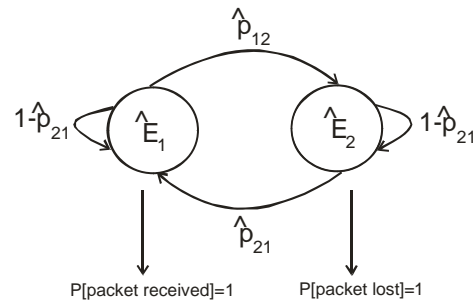


Fig. 2 Model of packet loss generator

*B. Error Model in Received Packets*

For the bit error modeling, we have chosen the Gilbert model [5]. Its basic parameters are set according to [3].

Final probability state vector is

$$\pi = (\pi_1, \pi_2) \quad (10)$$

Generator matrix of correctly received bits is

$$H = \begin{pmatrix} h_1 & 0 \\ 0 & h_2 \end{pmatrix} \quad (11)$$

Transition probability matrix is as in Fig. 3

$$P = \begin{pmatrix} 1 - p_{12} & p_{12} \\ p_{21} & 1 - p_{21} \end{pmatrix} \quad (12)$$

Then according to [5]

$$\pi = \left( \frac{p_{21}}{p_{12} + p_{21}}, \frac{p_{12}}{p_{12} + p_{21}} \right) \quad (13)$$

Probability that received packet of length  $m$  will be

decoded correctly is

$$p(m) = \pi \cdot (\mathbf{P} \cdot \mathbf{H})^m \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (14)$$

Finally, probability that packet of length  $l \cdot n$  from all sent data is received and decoded correctly is

$$r(l \cdot n) = \hat{\pi}_1 \cdot p(l \cdot n) = \hat{\pi}_1 \cdot \left[ \pi \cdot (\mathbf{P} \cdot \mathbf{H})^{l \cdot n} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right] \quad (15)$$

Important condition is independent work of two generators. Therefore, in the process of generation, the error-bit generator must continue to produce numbers also in the packet loss state (determined by the upper generator), when its output is not recorded.

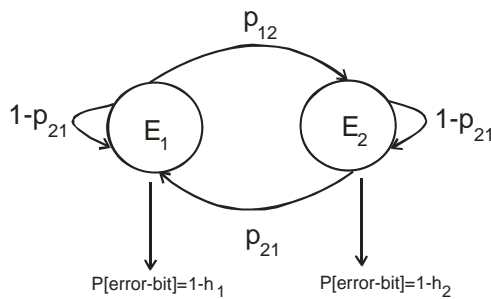


Fig. 3 Model of the bit error generator

### III. RESULTS

#### A. Data Collection

In our analysis, we recognize 3 types of transmission errors: bit error, packet error, packet loss.

Bit error as the basic error representation of digital channel denotes a single bit altered by the channel impairment during the transmission. Packet error occurs, when there is at least one bit error in the given received packet. Packet loss is a special case of packet error, where the loss of the whole packet has occurred (due to the transmission cross-talk, the failure to decode the packet at physical level, or various other reasons). This kind of error is the most difficult to detect because the upper transmission layers are unaware of the loss, as well as of its reason. Therefore, a packet sequence number must be employed in the measurement to detect this type of error.

As you will see later, it is very important to distinguish between the packet error caused by a defective bit and the packet error caused by a lost packet.

From the real world measured data, we are creating a bit error trace. The bit error trace contains information whether a particular bit was transmitted correctly. "0" represents the correctly received bit, whilst "1" says the bit was decoded incorrectly. The packet loss is represented as a burst of consecutive 1's of the length of the entire lost packet.

From the bit error trace, with knowledge of the packet length, several stats can be derived.

A bit error burst is run of consecutive 1's between two 0's providing the length of incorrectly decoded (or lost) data. The burst analysis can be used to create a graph showing the occurrence of error runs for every burst length.

We have collected the data by using IEEE 802.11b/g wireless protocols. We have set up a single hop wireless network using two PCs attached with 802.11 Wi-Fi network cards and running OS Linux. The infrastructure network type was created with one station acting as Access point (AP) and the second one joining to the network.

Measurement was done in the presence of other wireless networks to provide results as close to the real-world indoor scenario as possible. The communicating antennas were stationary, in non-line of sight layout through one wall.

We used a program written in C to generate the defined traffic. On the receiver side, modified open-source wireless card driver [1] was used to dump the received packets directly coming from hardware to avoid further packet changes by the operating system. The dump was then analyzed for bit errors and packet losses (checking the packet sequence number).

The configuration presented herein along with an evaluation of its error performance can be helpful for analyzing various aspects of wireless networks simulated in various scenarios (e.g. IEEE 802.11b/g simulation in [10]).

Note, that measured characteristics are only valid for the corresponding wireless protocol used (IEEE 802.11b/g). Error traces obtained using different MAC layer specifications are very likely to have different characteristics.

The packet loss parameters have to be computed from the error traces of packet clusters of defined sizes ( $k \cdot 8480$  in our case). The error trace length in the error-bit parameter computation has to be an integer fraction (with no remainder from the division) of the packet length used in the measurement.

#### B. Results

We calculated parameters for 1 model representing the packet loss generator from two pairs of packet length multiples  $k$ . Two Elliot models from two foursome values representing error packets of length  $m$  were calculated for errors bit generator. To compare the accuracy compared to the Gilbert model errors generator, we calculated parameters from one pair of values representing error packets with length  $m$  in an error trace.

Solving the system of nonlinear equations (9) and (14) for input values according to the Table I, II, and III, we get these parameters:

TABLE I  
PARAMETERS FOR CASCADE GILBERT MODEL

Parameter	Value
$\hat{p}_{12}$	0,0118059714786
$\hat{p}_{21}$	0,8094790008428
$\hat{\pi}_1$	0,9856249999982
$\hat{\pi}_2$	0,0143750000018
$p_{12}$	0,0000030996857
$p_{21}$	0,0003230000000
$h_1$	1,0000000000000
$h_2$	0,9990000000000
$\pi_1$	0,9904946682066
$\pi_2$	0,0095053317933

TABLE II  
PARAMETERS FOR CASCADE ELLIOT 1 MODEL

Parameter	Value
$\hat{p}_{12}$	0,0118059714786
$\hat{p}_{21}$	0,8094790008428
$\hat{\pi}_1$	0,9856249999982
$\hat{\pi}_2$	0,0143750000018
$p_{12}$	0,0000021976076
$p_{21}$	0,0006328260163
$h_1$	0,9999996204228
$h_2$	0,5146542725194
$\pi_1$	0,9965393292860
$\pi_2$	0,3460670713939

TABLE III  
PARAMETERS FOR CASCADE ELLIOT 2 MODEL

Parameter	Value
$\hat{p}_{12}$	0,0118059714786
$\hat{p}_{21}$	0,8094790008428
$\hat{\pi}_1$	0,9856249999982
$\hat{\pi}_2$	0,0143750000018
$p_{12}$	0,0000001096854
$p_{21}$	0,0000325693071
$h_1$	0,9999975239649
$h_2$	0,5002063841629
$\pi_1$	0,9966435500111
$\pi_2$	0,0033564499888

also displayed square error for every measurement and corresponding model.

TABLE IV  
PACKET LOSS MODEL

$k$	$1-p(k,n) = P[\text{packet loss}]$	
	real data	Model from $k=1, k=8$
1	0,014375	0,014375
2	0,026000	0,026011
3	0,037884	0,037510
4	0,048000	0,048873
5	0,059375	0,060102
6	0,071268	0,071198
7	0,081436	0,082164
8	0,093000	0,093000
9	0,105856	0,103708
10	0,112500	0,114290
15	0,159475	0,165353

TABLE V  
BIT ERROR MODELS

$m$	$p(m) = P[\text{errorless packet of length } m]$			
	real data	Elliot 1 Model from $m=1, m=424, m=1060, m=8480$	Elliot 2 Model from $m=1, m=212, m=4240, m=8480$	Gilbert Model from $m=106, m=848, m=8480$
106	0,996430	0,996274	0,996370	0,999042
212	0,996088	0,996002	0,996097	0,998179
265	0,995950	0,995866	0,995961	0,997778
424	0,995498	0,995458	0,995551	0,996683
530	0,995212	0,995186	0,995279	0,996031
848	0,994369	0,994371	0,994460	0,994368
1060	0,993786	0,993829	0,993916	0,993443
1696	0,992238	0,992202	0,992282	0,991208
2120	0,990900	0,991119	0,991195	0,989978
4240	0,985796	0,985722	0,985776	0,984764
8480	0,975016	0,975015	0,975028	0,975018

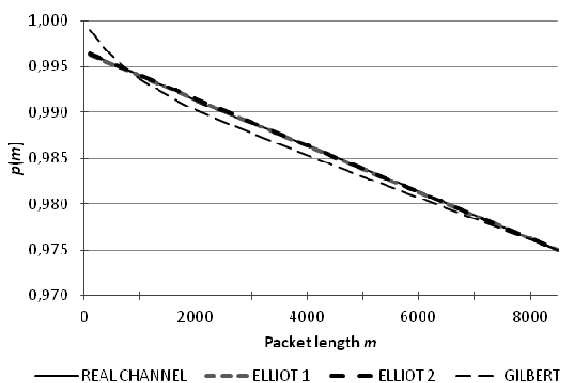


Fig. 4 The dependency of the real frequency of occurrence and computed probability of packet errorless to their length for Gilbert and Elliot models

At the Figs. 4 and 5 there are displayed the dependencies of the probabilities of receiving errorless packets to packet's lengths for two models and for data from generator. There is

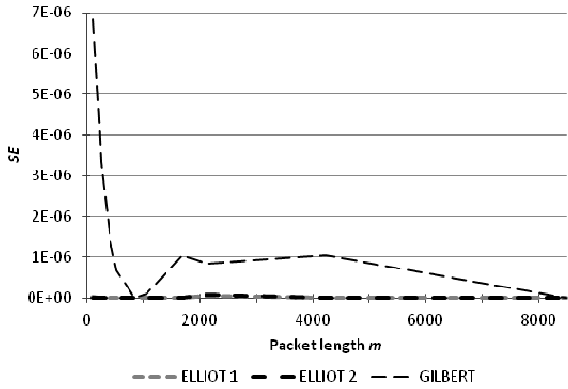


Fig. 5 SE of the real frequency of occurrence and computed probability of packet errorless for different packet lengths for Gilbert and Elliot models

Fig. 6 - 11 show cluster analysis of designed and implemented errors generators. The generator with called Elliot 1-2 is realized by alternation of generators Elliot 1 and Elliot 2 every 8000 packets with length 8480 bits. The numbers of clusters are expressed on average just 8000 such packets. Lost packets are expressed by generator with sequence 8480 units in the error word. Each horizontal axe is logarithmic. In fig. 9 – 11 also vertical axes are logarithmic.

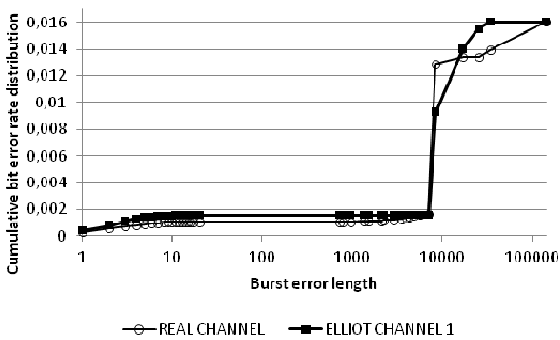


Fig. 6 Cumulative bit error rate distribution dependency of burst error length

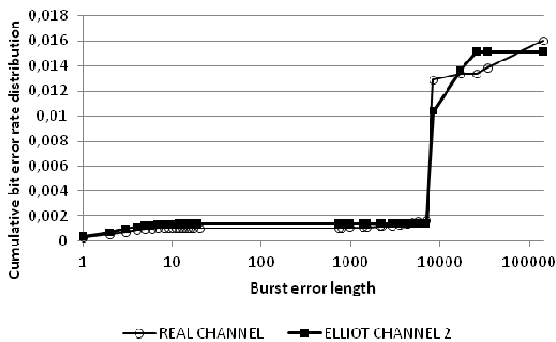


Fig. 7 Cumulative bit error rate distribution dependency of burst error length

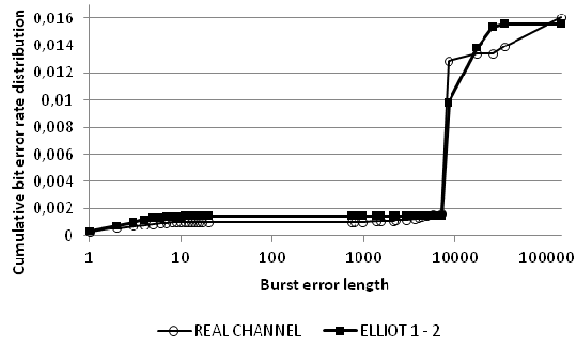


Fig. 8 Cumulative bit error rate distribution dependency of burst error length

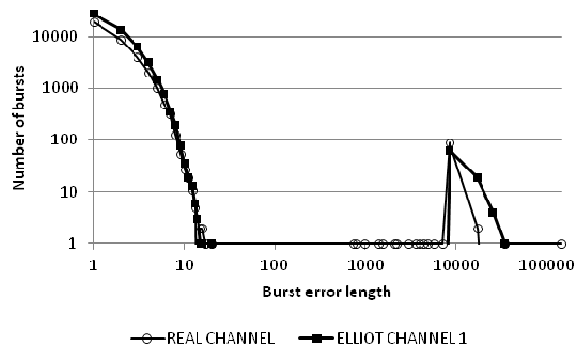


Fig. 9 Burst error length histogram

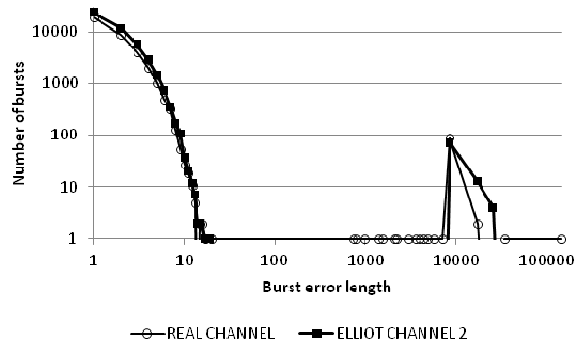


Fig. 10 Burst error length histogram

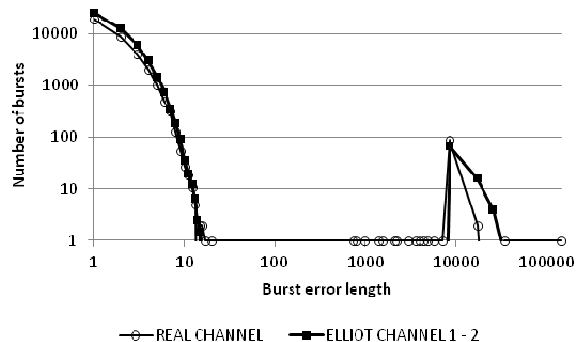


Fig. 11 Burst error length histogram

## IV. CONCLUSION

From the proposed generator design, it is apparent that it allows many degrees of freedom. A bit error model contains a huge square matrix exponent (8480 in our work), therefore, solving a system of nonlinear equations from (14) gives us too many right results and choosing the best result is not a trivial decision. It is necessary to analyze which model is suitable to generate a similar sequence of gaps and bursts. The level of credibility of the data from this generator is determined by the expected needs in their following application. For example, in the data transmission throughput optimization of the forward error correction, it is necessary to know whether of the analyzed data errors are independent or dependent. If they are dependent, then we need to know the average and maximal error burst lengths [6].

This model allows us to design a nearly unrestrained number of generators fulfilling the defined requirements (because of the huge number of results in the system of nonlinear equations from (14)). It is defined separately for every transmission condition in the channel (every SNR level in our case), therefore, it's not as limited as e.g. Rayleigh's channel description [7] and gives more possibilities than Ricean model.

Already published Markov models do not divide packets into lost or received (with or without errors). These models are mainly based on simplified Gilbert model [8] or on increasing the number of states of Elliot model [9]. Mentioned models usually model only error rate of received packets, or if they cover also lost packets, it rapidly enlarges the complexity of model. In [2] is published the way for the construction of the generalized Markov model in the special cascade for solving probabilities of partly dependent events. It is not limited to solve data error problems and it is very probable, that this model can be also useful for modeling many other types of probability cases. This model is Gilbert cascade model.

In this paper, model for the generation of erroneous bits is extended to Elliott model and also the generator is implemented. Based on the results of the experiments it is shown, that Elliott cascade model gives better results than Gilbert cascade model.

## ACKNOWLEDGMENT

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## REFERENCES

- [1] MadWifi project. Available: <http://madwifi-project.org/>.
- [2] Pavlovič, J., Polec, J., Poctavek, J. and Kotuliaková, K., "Markov Chain Error Generator for Wireless Channel". In: *Proceedings of SPIE*. Vol. 8008: Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2011, Wilga, Poland, 23-29 May 2011. Bellingham: SPIE - The International Society for Optical Engineering, 2011.
- [3] Polec, J. and Popovcová, K., "An generalization of the Markov's model of errors appearance at data transfer", in *Proceedings of FEP'25 Conference on Electronic Computers and Informatics*, Košice, sept. 1994, pp. 98-102.
- [4] Malý, O. and Širáň, J., "Markov model for error occurrence in discrete data transmission channels" (in Slovak), *Informačnysystémy*, č.5, Bratislava, 1984, pp. 333-345.
- [5] Kanal, L. and Sastry, A.R.K., "Models for Channels with Memory and their Applications to Error Control", in *IEEE Proceedings*, Vol.66, No.8, 1978, pp. 724-744.
- [6] Kotuliaková, K. "Hybrid ARQ methods in wireless communication channels", PhD thesis, Dept. of Telecommunications, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology, Bratislava, 2006.
- [7] Aguiar, A. and Gross, J., "Wireless channel models", Technical Report, TU Berlin, Telecommunication Networks Group, April 2003.
- [8] Lemmon, J.J., "Wireless link statistical bit error model", NTIA Report 02-394, U.S. Department of Commerce, Boulder, June 2002.
- [9] Ebert, J.P. and Willig, A., "A Gilbert-Elliott bit error model and the efficient use in packet level simulation", TKN Technical Reports Series, TU Berlin, March 1999.
- [10] Sestrienka, M., Oravec, M. and Pavlovičová, J., Simulation of Physical Layer and Medium Access Control Sublayer in IEEE 802.11x Networks, *Proc. ELMAR-2010*, 52nd International Symposium ELMAR-2010, 15-17 September 2010, Zadar, Croatia, ISSN 1334-2630, ISBN 978-953-7044-11-4, pp. 261-264.