

# A General Stochastic Spatial MIMO Channel Model for Evaluating Various MIMO Techniques

Fang Shu, Li Lihua, and Zhang Ping

**Abstract**—A general stochastic spatial MIMO channel model is proposed for evaluating various MIMO techniques in this paper. It can generate MIMO channels complying with various MIMO configurations such as smart antenna, spatial diversity and spatial multiplexing. The modeling method produces the stochastic fading involving delay spread, Doppler spread, DOA (direction of arrival), AS (angle spread), PAS (power azimuth Spectrum) of the scatterers, antenna spacing and the wavelength. It can be applied in various MIMO technique researches flexibly with low computing complexity.

**Keywords**—MIMO channel, Spatial Correlation, DOA, AS, PAS.

## I. INTRODUCTION

IN recent years, as the increase demand of transmitting high data rates, the research of MIMO (Multiple-Input Multiple-Output) techniques which have the ability of achieving extraordinary bit rates became a potential technique. Among which, smart antenna, spatial diversity and spatial multiplexing are the emerging MIMO techniques.

Smart antenna (SA) utilizes the strong spatial correlation among antennas. The received signals on adjacent antennas has just a phase offset, compensating the phase offset before transmitting is able to control the signal direction and obtain considerable beamforming gain. When researching on the smart antenna technique, it mainly considers the AS and DOA of the signal.

For spatial diversity (for example: STBC) and spatial multiplexing (for example: V-BLAST) techniques, the MIMO channel capacity grows linearly with the number of antennas if the fading between each antenna pair is independent and identically Rayleigh-distributed. But in realistic environment, because of the antenna configuration and the distribution of the scatterers around the antennas, there is spatial correlation

among subchannels. The existence of the spatial correlation will degrade MIMO channel capacity severely. Therefore, when researching on the spatial diversity and spatial multiplexing techniques, the spatial correlation among antennas is mainly considered.

There are several kinds of spatial channel models in [1]-[3] which are mainly for the research on adaptive array processing, such as the Lee's model, the Geometrically Based Single Bounce model and the Gaussian Wide-sense Stationary Uncorrelated Scattering model. These models were intended to provide information about certain MIMO channel characteristics such as the AS or DOA, but without description of full channel properties. The correlation matrix modeling method or the kronecker modeling method in [4] is able to establish spatial correlated MIMO channel, but without information about the DOA and AS. So this model is mainly for the research of the spatial diversity and spatial multiplexing techniques. Besides, there are geometrical MIMO channel models [5]-[9] which are able to give full characteristics of the MIMO channel, but with high complexity and usually cannot reflex the realistic MIMO channel well.

This article investigates full MIMO channel characteristics. On integrating and generalizing full channel characteristics, a general stochastic spatial MIMO channel model is proposed in this paper. The general spatial MIMO channel model can be used for research on various MIMO techniques. The modeling method introduces full parameters to MIMO channel, such as delay spread, Doppler spread, DOA, AS, PAS, antenna spacing and wavelength. The modeling method produces stochastic spatial MIMO channel fading with low computing complexity, which is able to simulate various MIMO channel environments.

This paper established the stochastic spatial MIMO channel model under Matlab environment, which contains three steps. First step is to generate the stochastic independent MIMO channel, the second step is to compensate the signal phase shift on each antenna and the last step is to multiply spatial correlated matrixes with kronecker method. The block diagram is as follows:

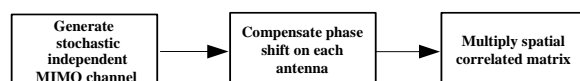


Fig. 1 The modeling flow chart

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## II. GENERATE STOCHASTIC INDEPENDENT MIMO CHANNEL

The transmit signal is

$$x(t) = s(t)e^{j(f_c + \phi_0)} \quad (1)$$

Where  $s(t)$  is the base band transmit signal,  $f_c$  is the carrier frequency,  $\phi_0$  is the initial phase of transmit signal.

Assuming that the initial phase  $\phi_0 = 0$  and the moving speed of mobile station is  $v$ . When there is no noise existed, through the single path propagation, the received signal is

$$y(t) = c(t)s(t)e^{j f_c t + 2 j t \pi f_m \cos \alpha} \quad (2)$$

Where  $c(t)$  is fading coefficient of the channel,  $f_m = v / \lambda$  is the maximal Doppler shift,  $\alpha$  is the angle between the moving direction of the mobile station and the signal. The received signal can be simplified as

$$y(t) = s(t)r e^{j\theta(t)} e^{j f_c t} \quad (3)$$

If removing the carrier frequency, the received base band signal is

$$\tilde{y}(t) = s(t) \cdot r(t) e^{j\theta(t)} = s(t) \cdot (T_c + jT_s) \quad (4)$$

Obviously, the base band signal under single path fading can be regard as the transmitted base band signal multiplies a complex number  $r e^{j\theta}$ ,  $r e^{j\theta} = T_c + jT_s$ . The single path channel fading  $r e^{j\theta}$  can be obtained by letting the Gauss noise passing through the linear Doppler shift filter [10] as follows:

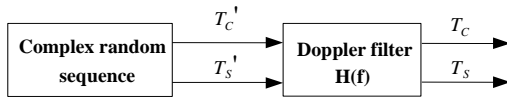


Fig. 2 Single path channel fading block

Selecting proper parameters can control the envelope of the fading, so as to be complied with certain distribution and possessing time coherence property. The Doppler shift filter usually adopts the Jakes [10] mode. After establishing the independent single path fading, the independent multipath fading can be established based on it and the independent MIMO channel can be established.

## III. COMPENSATING SIGNAL PHASE SHIFT

When researching on spatial diversity and spatial multiplexing, the spatial correlation of the channel is usually considered, whereas for smart antenna mainly focusing on the DOA and AS of the signal. In order to exhibit the phase derivative across the antenna arrays and the DOA information of the signal, we can compensate a phase shift on each antenna according to their time delay as (5). Thus, the MIMO channel is able to behave directional property [11].

$$w_m(\phi) = f_m(\phi) e^{\frac{-j2\pi(m-1)d \sin(\phi)}{\lambda}} \quad (5)$$

Where  $w_m(\phi)$  is the average phase shift on the  $m$  th antenna, the DOA of the signal is  $\phi$ . For a linear array,  $f_m(\phi)$  is the complex radiation pattern of antenna  $m$ . For the full radiation antennas

$$f_m(\phi) = 1 \quad (6)$$

Assuming that the system has  $M_T$  transmit antennas and  $M_R$  receive antennas, we should compensate the phase shift on the transmit antennas and receive antennas separately. After compensating the phase shift on transmitter, the new MIMO channel matrix  $\tilde{\mathbf{H}}$  is generated as follows:

$$\tilde{\mathbf{H}} = \mathbf{H} \begin{bmatrix} w_1(\phi_T) & & & \\ & w_2(\phi_T) & & \\ & & \dots & \\ & & & w_{M_T}(\phi_T) \end{bmatrix} \quad (7)$$

Where  $\mathbf{H}$  is the MIMO channel matrix without compensating,  $\phi_T$  is AOD (Angle Of Departure) of the transmitted signal. Similarly, compensating the phase shift on receiver as follows:

$$\tilde{\mathbf{H}} = \begin{bmatrix} w_1(\phi_R) & & & \\ & w_2(\phi_R) & & \\ & & \dots & \\ & & & w_{M_R}(\phi_R) \end{bmatrix} \mathbf{H} \quad (8)$$

where  $\phi_R$  is AOA (Angle of Arrival) of the received signal.

## IV. MULTIPLY SPATIAL CORRELATED MATRIXES WITH KRONECKER METHOD

The general stochastic spatial MIMO channel can be produced on the previous generated MIMO channel with correlation matrix method [4]. Assuming that the system has  $M_T$  transmit antennas and  $M_R$  receive antennas, the generated stochastic independent multipath MIMO channel impulse response is

$$\tilde{\mathbf{H}} = \begin{bmatrix} H_{11} & H_{21} & \dots & H_{1M_T} \\ H_{21} & H_{22} & \dots & H_{2M_T} \\ \dots & & & \\ H_{M_R 1} & H_{M_R 2} & \dots & H_{M_R M_T} \end{bmatrix} \quad (9)$$

Where  $H_{i,j}$  indicates the channel impulse response coupling the  $j$  th transmitter to the  $i$  th receiver element.

$$H_{i,j}(\tau) = \sum_{n=1}^L A_n \delta(\tau - \tau_n) \quad (10)$$

Where  $L$  indicates the number of multi-path,  $A_n$  indicates the amplitude of the  $n$  th path,  $\delta(\bullet)$  indicates the impulse

function and  $\tau_n$  indicates the time delay of the  $n$ th path. Then the general stochastic spatial MIMO channel  $\mathbf{H}_G$  can be modeled as

$$\text{vec}(\mathbf{H}_G) = \mathbf{R}^{1/2} \text{vec}(\tilde{\mathbf{H}}) \quad (11)$$

Where  $\text{vec}(\mathbf{H}_G)$  is the operator stacking the columns of  $\tilde{\mathbf{H}}$  under each other,  $\mathbf{R}^{1/2}$  indicates the square-root decomposition of  $\mathbf{R}$ .

$$\mathbf{R} = \mathbf{R}_t \otimes \mathbf{R}_r \quad (12)$$

$\mathbf{R}$  is the correlation control matrix, which can be generated by kronecking the transmit correlation matrix  $\mathbf{R}_t$  and the receive correlation matrix  $\mathbf{R}_r$ ;  $\otimes$  designates the Kronecker product.

$$\mathbf{R}_t(\mathbf{R}_r) = \begin{bmatrix} 1 & \rho_{12} & \dots & \rho_{1M_t} \\ \rho_{21} & 1 & \dots & \rho_{2M_t} \\ \dots & & 1 & \\ \rho_{M_t 1} & \rho_{M_t 2} & & 1 \end{bmatrix} \quad (13)$$

Where  $\rho_{i,j}$  indicates the transmit correlation coefficient (or the receive correlation coefficient) between the  $i$ th transmit (or receive) antenna and the  $j$ th transmit (or receive) antenna.

$$\rho_{i,j} = \rho_{j,i}^* \quad (14)$$

Where  $(\bullet)^*$  indicates the complex conjugation. Then the general stochastic spatial correlated MIMO channel matrix  $\mathbf{H}_G$  can be alternatively formulated as

$$\mathbf{H}_G = \mathbf{R}_r^{1/2} \tilde{\mathbf{H}} \mathbf{R}_t^{1/2} \quad (15)$$

In order to reflect the comprehensive spatial MIMO channel, we should consider various parameters of the spatial MIMO channel, such as the DOA and the AS of the signal and PAS distribution around the antennas. Thus, the generated stochastic MIMO channel is able to reflect full properties of the propagation environment.

Assuming that the PAS distribution is  $p(\phi)$ , the AS of the received signal is  $\sigma$ , the plane wave arrives at the two antennas whose antenna spacing is  $D$ . The cross correlation between the two signals can be obtained as follow [12]

$$\rho_{xx} = \int_{-\pi+\phi}^{\pi+\phi} \cos[2\pi D / \lambda \sin(\phi)] p(\phi) d\phi \quad (16)$$

$$\rho_{xy} = \int_{-\pi+\phi}^{\pi+\phi} \sin[2\pi D / \lambda \sin(\phi)] p(\phi) d\phi \quad (17)$$

Where,  $\rho_{xx}$  and  $\rho_{xy}$  are the real part and the image part of the cross correlation between two antennas.

Therefore, according to the PAS, the antenna spacing, the DOA and the AS of the signal, the spatial correlation coefficient  $\rho_{i,j}$  could be established. For multiple antennas at

the transmitter and the receiver, we should compute  $\rho_{i,j}$  of every antenna pair and construct the correlation matrix as (13).

Experiments prove that the PAS distribution obeys Laplacian distribution as (18). Its probability density function of the average received power as follows:

$$P(\phi, \sigma, \bar{\phi}) = N_0 \exp\left[-\frac{\sqrt{2}|\phi - \bar{\phi}|}{\sigma}\right] \quad (18)$$

Where the DOA of the signal is  $\bar{\phi}$ , the AS of the signal is  $\sigma$ .

$$N_0^{-1} = \int_{-\pi+\bar{\phi}}^{\pi+\bar{\phi}} \exp\left[-\frac{\sqrt{2}|\phi - \bar{\phi}|}{\sigma}\right] d\phi \quad (19)$$

Where  $p$  is the average received power,  $N_0$  is the unitary constant.

## V. SIMULATION RESULTS

We can take any row of the MIMO channel matrix  $\mathbf{H}_G$  as  $\mathbf{H}_t'$  here to verify the generated transmit correlation matrix [13] as follow

$$\mathbf{R}_t = E[(\mathbf{H}_t')^H \mathbf{H}_t'] \quad (20)$$

Where  $E$  indicates expectation,  $(\bullet)^H$  indicates the conjugation transpose.

As the same way, we can take any column of the MIMO channel matrix, as  $\mathbf{H}_r'$  to verify the generated receive correlation matrix as follows:

$$\mathbf{R}_r = E[\mathbf{H}_r'(\mathbf{H}_r')^H] \quad (21)$$

Assuming that the transmitter has 2 antennas, the antenna spacing is  $0.5\lambda$ , PAS obeys Laplacian distribution, the AS is 10 degrees, and DOA is 30 degrees. The theoretical value and the generated value of the transmitted correlation matrixes are separately as following (22) and (23).

$$\begin{bmatrix} 1 & 0.0124 + 0.9026i \\ 0.0124 - 0.9026i & 1 \end{bmatrix} \quad (22)$$

$$\begin{bmatrix} 1 & 0.0142 + 0.9110i \\ 0.0142 - 0.9110i & 1 \end{bmatrix} \quad (23)$$

It is proved that the generated stochastic spatial MIMO channel can precisely reflex the spatial MIMO channel correlation.

Figs. 3 and 4 are the eigenvalues's CDF of the generated stochastic independent  $4 \times 4$  MIMO channel and the spatial correlated  $4 \times 4$  MIMO channel.

From Fig. 3, it is clear that when there is no spatial correlation, the  $4 \times 4$  MIMO channel can be viewed as 4 parallel independent subchannels. The MIMO channel capacity can be viewed as the sum of the capacity of 4 independent subchannels.

Fig. 4 shows the eigenvalues's CDF of the generated stochastic spatial correlated  $4 \times 4$  MIMO channel when the AS

at the transmitter and receiver are both 2 degrees. And the antenna spacing is  $0.5\lambda$ , PAS obeys Laplacian distribution. When the AS is small which result in high spatial correlation, as Fig. 4 shows, the  $4 \times 4$  MIMO channel just can be decomposed as one subchannel. The MIMO capacity degrades severely when it can only provide one subchannel.

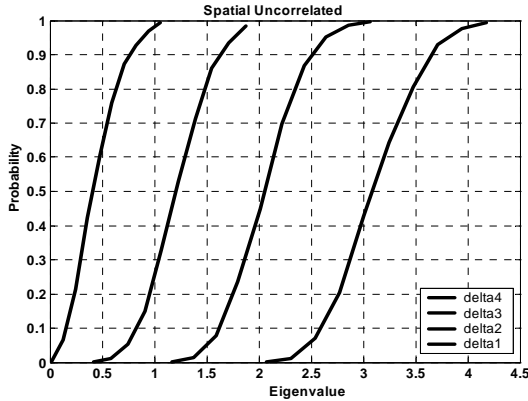


Fig. 3 CDF of eigenvalues of the generated independent  $4 \times 4$  MIMO channel

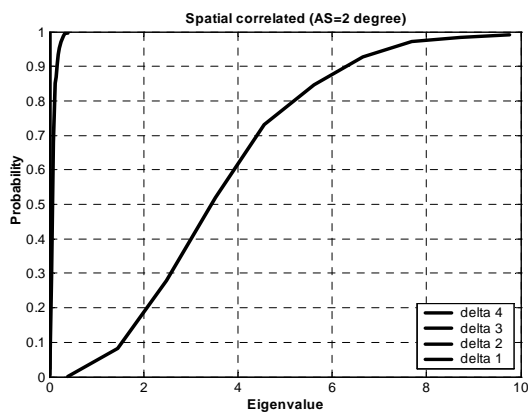


Fig. 4 CDF of eigenvalues of the generated spatial correlated  $4 \times 4$  MIMO channel

Fig. 5 shows the directional gain of the generated stochastic  $4 \times 1$  MIMO channel with 2 degrees AS and 20 degrees AS. The DOA of the signal is 30 degrees and the antenna spacing is  $0.5\lambda$ , PAS obeys Laplacian distribution. The simulation results show that when the antenna number is 4, the beam width is comparatively broad.

Fig. 6 shows the directional gain of the generated  $8 \times 1$  stochastic MIMO channel with 2 degrees AS and 20 degrees AS. The DOA is 30 degrees and the antenna spacing is  $0.5\lambda$ , PAS obeys Laplacian distribution. When the antenna number increases to 8, the beam width becomes narrower.

From the simulation results, it is obviously that as the antenna number increases, the beam width decreases; as the AS increases, the directional gain becomes to distort and the power of the side lobe becomes stronger.

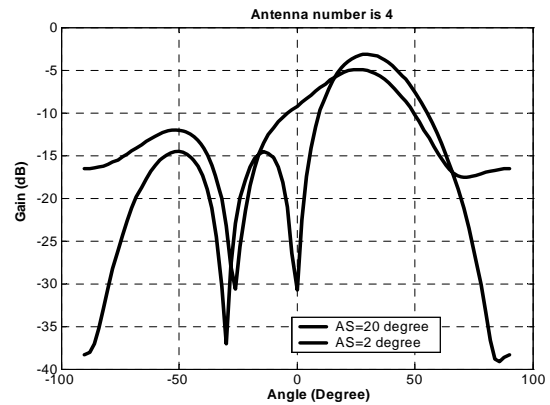


Fig. 5 Directional gain of the  $4 \times 1$  MIMO channel with 2 degree AS and 20 degree AS

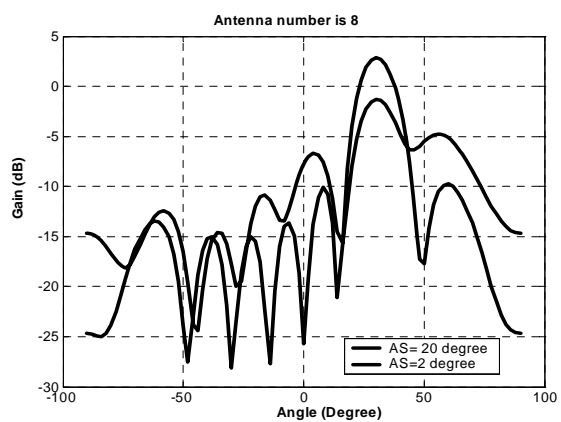


Fig. 6 Directional gain of the  $8 \times 1$  MIMO channel with 2 degree AS

Fig. 7 shows the spatial correlation coefficient between 2 antennas with the same DOA and different AS. The DOA is 10 degrees, the AS are 2 degree, 10 degree and 40 degree separately. From the simulation results, it is obviously that the spatial correlation coefficients decrease rapidly when the AS is large.

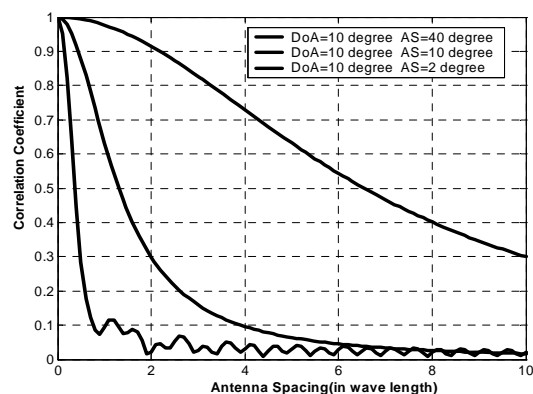


Fig. 7 Spatial correlation coefficient between 2 antennas

Fig. 8 is the statistical CDF of the channel capacity. The figure shows the results under uncorrelated, 30 degrees AS and 50 degrees AS of the MIMO channel. The DOA is 30 degrees and the antenna spacing is  $0.5 \lambda$ , PAS obeys Laplacian distribution. The transmit antenna number and the receive antenna number are both 4, the SNR of the channel is 10dB. Simulation results show that the MIMO channel capacity degrades as the AS decreases.

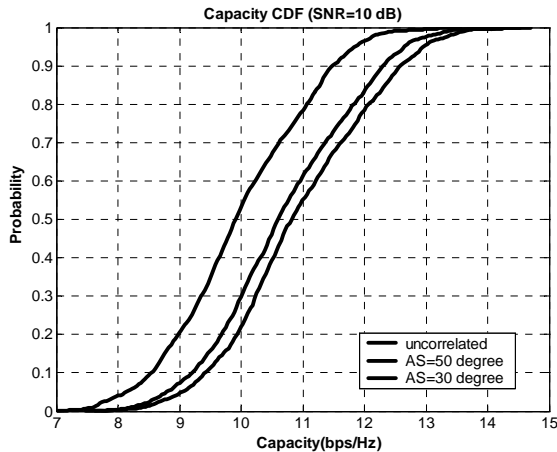


Fig. 8 CDF of MIMO channel capacity

## VI. CONCLUSION

This paper provides a method to establish a general stochastic spatial MIMO channel model. This model can reflect various characteristics of the spatial correlated MIMO channel with low computation and high flexibility. This general stochastic MIMO channel model is suitable for evaluating various MIMO techniques.

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