

# A Dynamic Filter for Removal DC - Offset In Current and Voltage Waveforms

Khaled M.EL-Naggar

**Abstract**—In power systems, protective relays must filter their inputs to remove undesirable quantities and retain signal quantities of interest. This job must be performed accurate and fast. A new method for filtering the undesirable components such as DC and harmonic components associated with the fundamental system signals. The method is based on a dynamic filtering algorithm. The filtering algorithm has many advantages over some other classical methods. It can be used as dynamic on-line filter without the need of parameters readjusting as in the case of classic filters. The proposed filter is tested using different signals. Effects of number of samples and sampling window size are discussed. Results obtained are presented and discussed to show the algorithm capabilities.

**Keywords**—Protection, DC-offset, Dynamic Filter, Estimation.

## I. INTRODUCTION

IN power system protection, the fundamental frequency voltage or current signals must be decomposed by relays to enable fast response. One of important protection applications is distance protection. In such protection system, to locate the fault accurately and fast the fundamental voltage and current signals must be extracted on-line. Therefore digital filtering in distance relays includes some form of signal parameter estimation algorithm. The resistance-inductance behavior of the power system produces dc- offset components in the fault current which are exponentially decaying. It is known that distance relays have the tendency to overreach in the presence of dc- offset component in the faulted current waveform.

A number of techniques has been proposed to remove the dc-offsets from the fault currents. Digital filters, which eliminate the effect of an exponentially decaying component, were proposed in many references. Some of these filters are based on static and dynamic techniques [1], [2], [3]. Sachdev et-al [1] proposed the application of the well known least error square estimation technique.

For decomposing the fault current and removing the dc offset components [1]. The least error squares is an accurate estimation technique but in case of the data set is contaminated with bad measurements, the solution may not be accurate unless extra filters are used for detecting the bad measurements. A digital mimic filter for the removal of DC-offset current waveforms is presented in reference 2. The proposed filter in this reference is used in conjunction with other classic estimators such as least error square to detect the whole system signals components for relaying purposes. Reference 3 proposed the use of Kalman filtering

algorithm to remove the dc-offset. Kalman filter is a dynamic estimator that can track states on-line. It is suitable for using with the non-stationary signals. Other techniques based on domain transformations were presented in many references [4]. Fourier Walsh transformations were presented in references [4],[5]. Reference [6] proposed a method based on wavelet transformations to measure the dc offset of the signal during short duration variations in distribution systems. A modified discrete wavelet transform based techniques have been applied in many references [7], [8]. Composed algorithms were also presented [9]. In this reference, an analytical method for measuring the accurate fundamental frequency component of a fault current signal distorted with dc- offset is presented. The proposed algorithm is composed of four stages: sine filter, linear filter, Prony's method and measurement. The Prony's method is used to estimate the parameters of the dc- offset and the characteristic frequency component. Genetic algorithms have been applied also to various power system protection problems with promising results [10].

This paper introduces a new method based on dynamic Least Absolute Value filtering algorithm (LAVF) for the measurement of the dc- offset in current and voltage waveforms. The problem is formulated as an estimation problem. The goal is to minimize the error square in estimated state parameters. The LAVF is used to find the optimum parameter estimation of the formulated problem. The results show that the proposed algorithm can identify and measure the dc- offset contents of any distorted wave at power system buses. The algorithm can track the dc-offset contents on digital bases. This gives the algorithm the advantage over other classical methods that need to readjust its parameters when dealing with non stationary waveform [11].

## II. PROBLEM FORMULATION

The waveform given by equation (1a) is used to perform the study [5].

$$Z(t) = A_{dc1} + \sum_{n=1}^N \frac{A_{ac}}{n} \cos\left(\frac{nt\pi}{8} + n\theta^o\right) + A_{dc2}e^{-t/\tau_1} + A_{dc3}e^{-t/\tau_2} + A_{dc4}e^{-t/\tau_3} + A_{dc5}e^{-t/\tau_4} \quad (1a)$$

In a discrete form this equation will be:

Khaled M. EL-Naggar is with the College of Technological Studies, Kuwait, (email: knaggar@ieee.org)

$$Z(k) = A_{dc1} + \sum_{n=1}^N \frac{A_{ac}}{n} \cos\left(\frac{nk\pi}{8} + n\theta^o\right) + A_{dc2}e^{-k/t_1} + A_{dc3}e^{-k/t_2} + A_{dc4}e^{-k/t_3} + A_{dc5}e^{-k/t_4} \quad (1b)$$

This complicated waveform has constant dc value ( $A_{dc1}$ ) as well as many dc components ( $A_{dc2}, \dots, A_{dc5}$ ) with different decaying time constants ( $t_1, \dots, t_4$ ). Harmonics noise are also appear in this equation. This makes the equation suitable for demonstrating the ability of the algorithm in estimating all signal components and separating them.

By choosing suitable sampling frequency and data window size, equation 1 can be used to generate  $N$  samples. The  $N$  equations representing the actual instantaneous voltage values form the measurement matrix in a discrete form. The parameters to be estimated here are: the DC values ( $A_{dc}$ ), the exponential decaying time constants ( $t_i$ ), the fundamental and harmonic magnitudes ( $A_{ac}$ ) and the phase angle ( $\theta$ ) if existed. It is important to mention that the algorithm can estimate the fundamental component parameters. This means that the algorithm has a dual function. It can filter and reject undesired components and estimate the fundamental magnitude, frequency and phase angle for protection purposes.

Expanding the *cos* and exponential terms as following:

$$B \sin(\omega t + \theta) = B[\sin(\omega t) \cos \theta + \cos(\omega t) \sin \theta]$$

$$Ae^{-Ct} = A\{1 - Ct + 0.5C^2t^2 - (1/6)C^3t^3\} + \dots$$

equation 1 can be rewritten as:

$$Z(k) = H(k)X(k) + e(k), \quad k=1, 2, \dots, N \quad (2)$$

Where  $Z(k)$  is the signal sample at step  $k$ ,  $X$  represents the unknowns mentioned above,  $H(x)$  is the connection matrix that relates the unknowns to the measurement, while  $e(k)$  represents the error in measurement  $k$ . It is clear that the described system of equations (2) is a highly over-determined system. The main objective now is to find the best estimate of the vector  $X(k)$ . The problem is an optimization one. The LAVF approach presented in this work is employed to find the optimum values of the state vector  $X(k)$  that minimizes the absolute error vector  $e(k)$ , using the following objective function:

$$F = \sum_{i=1}^N |e_i(k)| \quad (3)$$

where:

$e_i(k)$  is the individual error square for each measurement.

### III. THE PROPOSED LAVF ALGORITHM

In this part, the on-line estimation process of the parameters described in section 2 is performed using the least absolute value filtering algorithm (LAVF). Although the complete derivation of the proposed filter equations is beyond the scope of this paper and given in reference [11], a short description is given next. The dynamic filter works on the discrete state space model described by the measurement equation and the state transition equation in the following form.

$$Z(k) = H(k)X(k) + e(k) \quad (4)$$

$$X(k+1) = \Phi(k)X(k) + \varpi(k) \quad (5)$$

The state transition formulation depends on the type of reference chosen. Either stationary reference or rotating reference can be used. The measurement error vector  $e(k)$  and the state error  $\varpi(k)$  are assumed to be white sequence with known covariance as,

$$E\{e(k)e(j)^T\} = \begin{cases} 0 & ; \quad j \neq k \\ R(k) & ; \quad j = k \end{cases} \quad (6)$$

$$E\{\varpi(k)\varpi(j)^T\} = \begin{cases} 0 & ; \quad j \neq k \\ Q(k) & ; \quad j = k \end{cases}$$

The initial condition of  $X(0)$  is a Gaussian random vector with the following statistics,

$$E\{X(0)\} = \bar{X}(0) \quad (7)$$

$$E\{[X(0) - \bar{X}(0)][X(0) - \bar{X}(0)]^T\} = \bar{P}(0) \quad (8)$$

where  $\bar{P}(0)$  is the initial error covariance matrix of the states, with dimensions  $u \times u$  where  $u$  is the number of unknowns. The covariance of the error at any step ( $k$ ) can be obtained by replacing  $X(0)$  with  $X(k)$  in equation (7).

The algorithm starts with an initial estimate for the system parameter vector  $\bar{X}(0)$  and its error covariance matrix ( $\bar{P}(0)$ ) at some point  $k=0$ . These estimates are denoted as  $\bar{X}, \bar{P}$ , where  $(\bar{\quad})$  means that these are the best estimations at this point, prior to assimilating the measurement at instant  $k$ . With such initial values, of both parameters and error co-variances, filter gain matrix  $K(k)$  at this step is calculated as follows,

$$K(k) = \left[ H(k) + R(k)Ly^T \bar{P}^{-1}(k) \right]^{-1} \quad (9)$$

Assuming that the state vector dimension is  $u \times 1$ , the vectors  $L$  and  $y$  are defined as:  $L$  is  $u \times 1$  column vector  $(1, 1, \dots, 1)^T$ ; and  $y^T$  is  $1 \times u$  row vector  $(1, 1)$  [11]. Using the filter gains, estimates are updated with measurements  $Z(k)$  through equation (10), and error co-variances for update estimates are computed from equation (11).

$$\hat{X}(k) = \bar{X}(k) + K(k) \{i(k) - H(k)\bar{X}(k)\} \quad (10)$$

$$P(k) = [I - K(k)H(k)]P(k) [I - K(k)H(k)]^T + K(k)R(k)K^T(K) \quad (11)$$

Finally, error co-variances and estimates are projected ahead to repeat with  $k=2$ .

$$\bar{P}(k+1) = \Phi(k)P(k)\Phi^T(k) + Q(k) \quad (12)$$

$$\bar{X}(k+1) = \Phi(k)\hat{X}(k) + R(k) \quad (13)$$

The process is repeated until the last sample is reached. It is assumed that the co-variances and the transition matrices are known.

#### IV. TESTING OF ALGORITHM

The performance of the proposed method is evaluated using different generated waveforms. Different factors that affect the estimation process such as the number of samples and data window size are considered.

Results presented here are for the waveform given by equation (1) as it includes all kind of noise that needs to be filtered. This complicated waveform has constant dc value as well as many dc components, with different decaying time constants. Harmonic noise also appear in this equation. This makes the equation suitable for demonstrating the ability of the algorithm in filtering all other components from the fundamental one. The exact values for the parameters are  $A_{dc1}=A_{dc2}=100$ ,  $A_{dc3}=90$ ,  $A_{dc4}=75$ ,  $A_{dc5}=50$ ,  $t_1=25$ ,  $t_2=80$ ,  $t_3=50$ ,  $t_4=100$  micro sec. The fundamental ac amplitude is 100 and the phase angle equals  $5^\circ$  or 0.087266 radians. Using these values, equation 1 becomes

$$f(k) = 100 + \sum_{n=1}^4 \frac{100}{n} \cos\left(\frac{nk\pi}{8} + n5^\circ\right) + 100e^{-k/25} + 90e^{-k/80} + 75e^{-k/50} - 50e^{-k/100}$$

This equation is sampled at different sampling rates and different window sizes. Results are used to fed the algorithm. The estimated parameters are obtained and er results are presented in figures.

Figures 1, 2 and 3 show the effect of varying the number of samples at constant data window size (1 cycle) on some of the parameters  $A_{dc1}$ ,  $A_{dc2}$ ,  $A_{dc3}$ , and  $A_{dc4}$ . In figure 1, the first dc component, is shown, while figure 2 shows the second dc components variation. The third and fourth DC components are extracted as well and shown in figures 3 & 4. It is clear that the algorithm detected the various dc components of the signal at a very high degree of accuracy. The maximum resulted error with the dc values is found to

be less than 1.0%. Figure 4 shows sample of results obtained for the decaying time constants of the signal, dc components. In this figure, the variations of the second time constant with the exact value of 80 micro second are shown. The maximum error resulted in estimating the time constant  $t_2$  is found to be about 0.7%. All other time constants are estimated with higher degree of accuracy.

Figures 5 and 6 clarify very important advantage of the proposed algorithm. These figures show the estimated values of the fundamental magnitude and phase angle. Figure 5 shows the variation of the fundamental phase angle in radians. It is clear that the estimated values are very close to the exact value, which is 5.0 degree. The maximum error is found to be less than 0.07 %. The fundamental magnitude (100) is also estimated at very high degree of accuracy as shown in figure 5. The important thing here is that the algorithm not only can detect and separate the dc components, but also it can estimate the fundamental component which is the needed one for the protection purposes. This means that the algorithm can be used as a one integrated unit in the power system protection instead of using two separate filters. To illustrate the effect of varying the data window size on the estimated parameter values, the number of samples is held constant at 100 samples per cycle while the data window size is varied as 1, 2, and 3. All parameters are estimated. Results obtained for all the parameters reveal that 1 cycle of data window size is sufficient. Increasing the window after that will not improve the accuracy more. For all voltages and time constants, this is the conclusion. Therefore it is concluded that there is no need to spend more calculation time using more than one cycle. The forgoing results confirm that the proposed filter model can be used as a very useful tool for power system protection. It can perform many functions simultaneously. It can serve as a dc-offset detector, fundamental component estimator and it can even be used as a harmonic filter.

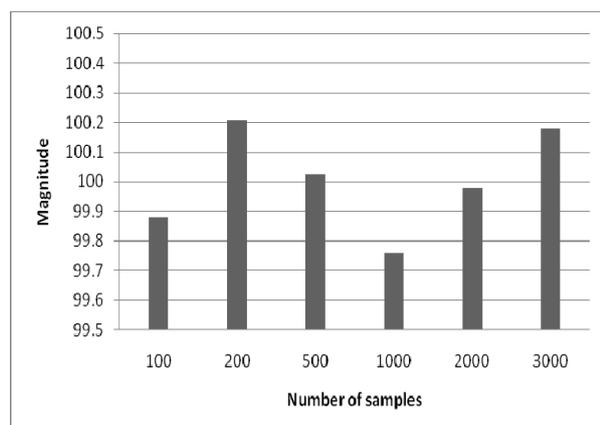


Fig. 1 Variations of the DC1 component magnitude with the number of samples at constant window size(1 cycle)

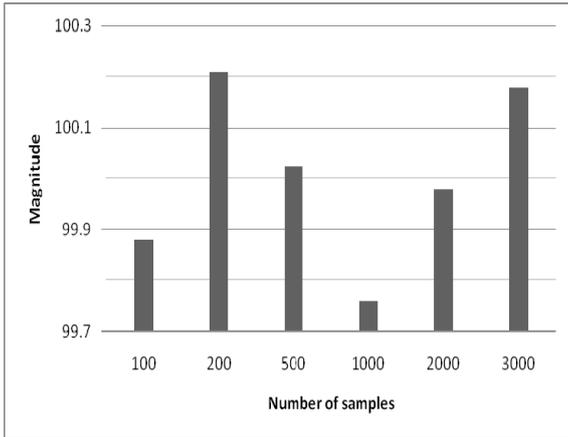


Fig. 2 Variations of the DC2 component magnitude with the number of samples at constant window size(1 cycle)

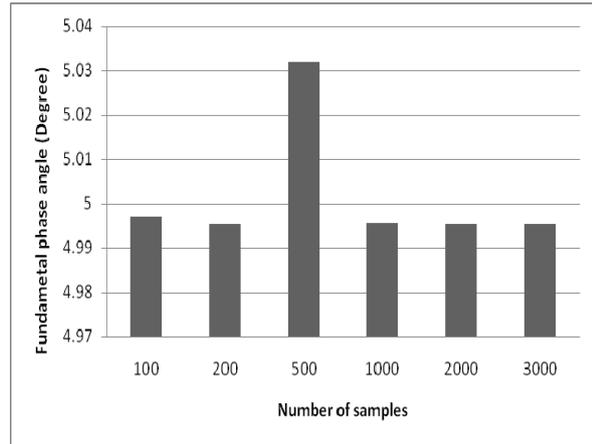


Fig. 5 Variations of the fundamental phase angle with the number of samples at constant window size(1 cycle)

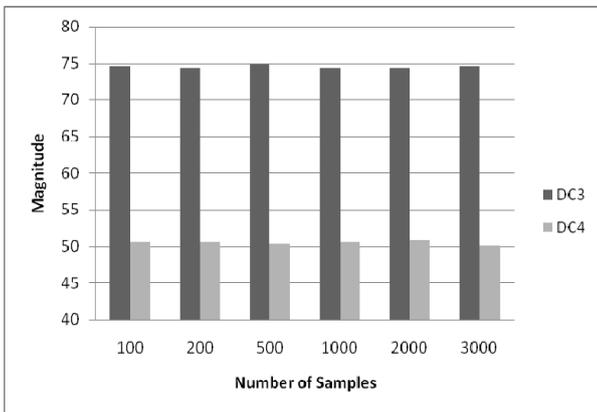


Fig. 3 Variations of the DC4 & DC5 components with the number of samples at constant window size (1 cycle)

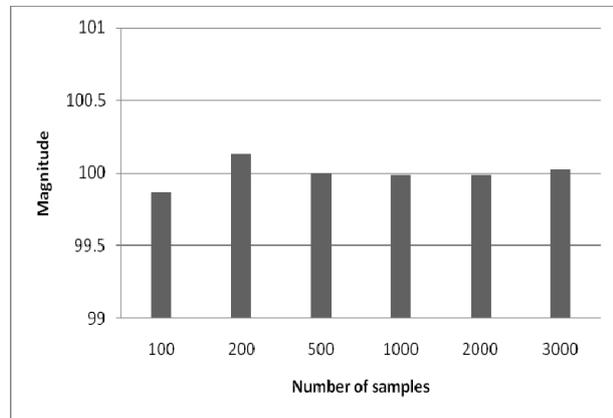


Fig. 6 Variations of the fundamental magnitude with the number of samples at constant window size(1 cycle)

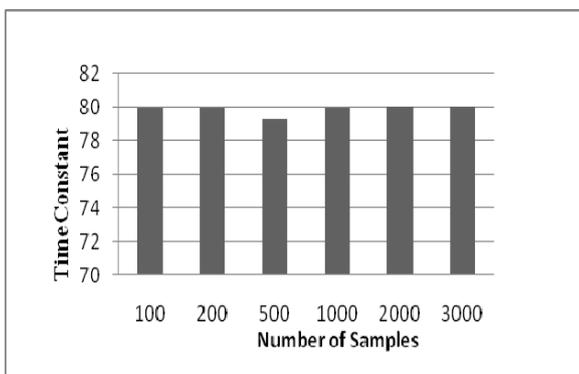


Fig. 4 Variations of the second dc time constant ( $T_2$ ) with the number of samples at constant window size (1 cycle)

## V. CONCLUSIONS

This paper presents a powerful method for filtering the dc components associated with the power system signals for protection purposes. The method is based on least absolute filtering algorithm. The problem is formulated as an estimation problem. The goal is to minimize the sum of absolute error associated with the measurement equations. The proposed technique is then used to solve the formulated minimization problem. The method is tested using simulated data that presents a fundamental signal contaminated with both dc and harmonic components. The results of this paper show that the proposed algorithm can identify and measure the dc-offset parameters in current and voltage signals in a power system. The algorithm estimates the parameters on digital bases. It can detect and identify the parameters of any superimposed noise. It can also estimate the fundamental wave parameters. The method has also the advantage of self tuning when dealing with non stationary waveforms. The

method can be considered as a very accurate reliable tool for power system protection applications.

#### REFERENCES

- [1] M.S. Sachdev and M.A. Baribeau, A new algorithm for digital impedance relays, IEEE Trans. Power App. & Syst. Vol. Pas-98, pp.253-260, Dec.
- [2] G. Benmouyal, Removal Of Dc- Offset In Current Waveforms Using Digital Mimic Filtering, Ieee Trans. On Power Delivery, Vol. 10, No. 2, April 1995.
- [3] A.A.Girgis, R.G.Brown, Application Of Kalman Filtering In Computer Relaying, Ieee.Trans. Pas-101, No.9, Sept.1981, Pp.3387-3397.
- [4] H.J. Altuve F, I Diaz V., Ernesto Vazquez M, Fourier And Walsh Digital Filtering Algorithms For Distance Protection, Ieee Trans. On Power Systems, Vol. 11, No. 1, Feb. 1996.
- [5] Jyh- Cheng Gu, Removal Of Dc Offset In Current And Voltage Signals Using A Novel Fourier Filter Algorithm, Ieee Trans. On Power Delivery, Vol. 15, No. 1, Jan. 2000.
- [6] J A. M. Gaouda, M.M.A.Salama, M.R.Sultan And A.Y.Chikhani, Application Of Multiresolution Signal Decomposition For Monitoring Short- Duration VariatioIn Distribution Systems, Ieee Trans. On Power Delivery, Vol. 15, No. 2, April 2000.
- [7] P. Suresh Babu and S. V. Jayaram Kumar, "A Novel Algorithm To Extract Exact Fundamental Frequency Components During Faults For Digital Protection Of Power System", ARPN Journal Of Engineering And Applied Sciences, vol. 4, no. 10, December 2009, pp. 78-82.
- [8] Eisa, A.A.A.; Ramar, K., Removal of decaying DC offset in current signals for power system phasor estimation, Proc. of 43rd International Universities Power Engineering Conference, Padova, UPEC 2008 , pp.1-4
- [9] Stoon- Ryul Nam, Sang- Hee Kang, And Jong- Keun Park, An Analytic Method For Measuring Accurate Fundamental Frequency Components, IEEE Trans. On Power Delivery, Vol. 17, No. 2, April 2002.
- [10] Wael M.Al-Hasawi, Khaled M.El-Naggar, "A Genetic Based Filter Algorithm for Removal of DC Offset In Current And Voltage Signals Using ", The 4th IASTED International Conference On Power And Energy Systems (Europes 2004), Rhodes, Greece, June 2004.
- [11] G.S. Christensen And S.A. Soliman, "Optimal Filtering Of Linear Discrete Dynamic Systems Based On Least Absolute Value Approximations", Automatica, 1990, Vol. 26, No. 2.