A Cost Function for Joint Blind Equalization and Phase Recovery

Reza Berangi, Morteza Babaee, and Majid Soleimanipour

Abstract—In this paper a new cost function for blind equalization is proposed. The proposed cost function, referred to as the modified maximum normalized cumulant criterion (MMNC), is an extension of the previously proposed maximum normalized cumulant criterion (MNC). While the MNC requires a separate phase recovery system after blind equalization, the MMNC performs joint blind equalization and phase recovery. To achieve this, the proposed algorithm maximizes a cost function that considers both amplitude and phase of the equalizer output. The simulation results show that the proposed algorithm has an improved channel equalization effect than the MNC algorithm and simultaneously can correct the phase error that the MNC algorithm is unable to do. The simulation results also show that the MMNC algorithm has lower complexity than the MNC algorithm. Moreover, the MMNC algorithm outperforms the MNC algorithm particularly when the symbols block size is small.

Keywords—Blind equalization, maximum normalized cumulant criterion (MNC), intersymbol interference (ISI), modified MNC criterion (MMNC), phase recovery.

I. INTRODUCTION

HIGH-SPEED digital communication systems are very sensitive to inter-symbol interference (ISI) introduced from multi-path fading or other channel distortions. Blind equalization is a technique to remove ISI without employing training sequences used in the conventional equalizers. Blind equalization is desirable in such circumstances that sending training signals is either too expensive or impossible.

For a two-dimensional system a phase error in an output constellation after equalization is general. This type of phase error puts down the efficiency of equalizers significantly because it prevents a decision device from recovering the transmitted data from the output of the equalizer. Therefore, to combat the performance degradation caused by the phase error, a carrier tracking loop is used for recovering the carrier phase after the equalizer. Algorithms that perform both blind equalization and carrier recovery are composed of an equalizer and a separate loop for carrier tracking, like a phase-locked loop (PLL).

In this paper, by modifying the MNC, we introduce the new algorithm that accomplishes blind equalization and carrier

Reza Berangi is with Department of Computer Engineering, Iran University of Science and Technology, Tehran, Iran (e-mail: rberangi@iust.ac.ir).

Morteza Babaee is with Faculty of Electrical Engineering, Imam Hossein University, Tehran, Iran (e-mail: babaee.ma@gmail.com).

Majid Soleimanipour is with Faculty of Electrical Engineering, Sharif University of Technology, Tehran, Iran (e-mail: soleimanipour@sharif.edu).

phase recovery simultaneously without the use of a carrier recovery loop that compensates for the phase error.

A baseband model with a channel impulse response, channel input, additive white Gaussian noise (AWGN), and equalizer input are denoted by $h(n)$, $s(n)$, $w(n)$, and $x(n)$, respectively. The transmitted data symbols, $s(n)$, are assumed to consist of stationary independently and identically distributed (i.i.d.), real or complex non-Gaussian random variables. The equalizer input $x(n) = s(n)*h(n) + w(n)$ is then sent to a tap-delay-line blind equalizer with impulse response, intended to equalize the distortion caused by intersymbol interference (ISI) without a training signal. The output of the blind equalizer [6]

$$
\begin{aligned}
y(n) &= x(n)*v(n) \\
&= s(n)*g(n) + w(n)*v(n) \\
&= \sum_i g(i)s(n-i) + \sum_i v(i)w(n-i)
\end{aligned}
\tag{1}
$$

can be used to recover the transmitted data symbols, $s(n)$, where, $v(n)$ denotes the impulse response of the equalizer and $g(n)$ denotes the impulse response of the combined channel-equalizer system. The amount of intersymbol interference (ISI) defined as [7]:

$$
ISI\{g(n)\} = \frac{\sum_n |g(n)|^2 - \max\{|g(n)|^2, \forall n\}}{\max\{|g(n)|^2, \forall n\}}
\tag{2}
$$

has been used as a performance index of the designed v[n]. The smaller ISI implies the better performance. However, the ISI is not enough as a measure of performances. Thus, we used ΔSNR to evaluate the degree of noise enhancement [6]. "ΔSNR" represents the difference between the SNR before and after equalization. The larger ΔSNR values indicate a better performance for the equalization algorithm. The amount of SNR before equalization and after equalization is computed with the aid of the model which is introduced in the appendix.

Let $v = [v[l_1], v[l_1+1],...,v[l_1+L-1]]^T$ (associated with the equalizer $v[n]$) denotes the $L{\times}1$ unknown parameter vector to be determined.

The equalized signal $y[n]$ can be expressed in vector form as

$$
y[n] = v\,x[n]
\tag{3}
$$

where $x[n] = [x[l_1], x[l_1+1],...,x[l_1+L-1]]$ is an $1 \times L$ vector associated with the received signal $x[n]$.

The maximum normalized cumulant criterion (MNC) is one of the most widely used blind equalization algorithms. Since this cost function is invariant to a phase rotation in the constellation, the equalizer output signal constellation suffers from a random phase rotation. A rotator that can rotate the constellation back in the right position at the output of the equalizer is therefore needed in steady-state operation. The simulation results indicate that, channel equalization and phase recovery can be done simultaneously by maximizing the MMNC cost function, because it implicitly incorporates a phase-tracking loop, which automatically recovers the carrier phase.

A. Maximum Normalized Cumulant Criterion

The inverse filter criterion-based algorithm, ([1]-[5]) proposed by Wiggins, Donoho, Shalvi and Weinstein, Tugnait, and Chi and Wu, finds the optimum v by maximizing the following class of inverse filter criteria (also known as absolute normalized cumulants) using only two cumulants:

$$J_{MNC}(v) = \left|\gamma_{p,q}\{y(n)\}\right| = \frac{\left|C_{p,q}\{y(n)\}\right|}{[C_{1,1}\{y(n)\}]^{(p+q)/2}}$$
$$= \frac{\left|C_{p,q}\{y(n)\}\right|}{(\sigma_y^2)^{(p+q)/2}} \qquad (4)$$

where p and q are nonnegative integers, $p+q \geq 3$.

$\left|\gamma_{p,q}\{y(n)\}\right|$ is the normalized (p+q)th order cumulant of $y(n)$. In fact, $J_{MNC}(v)$ represents a general form of equalization criteria using absolute cumulants. In addition, the MNC $J_{p,q}$, with $p+q=3$ and $p+q=4$ are usually preferred to those for $p+q>4$ due to the fact that the larger cumulant orders increases the designed equalizer variance and its computational complexity. As stated in [5] a closed-form solution for determining the optimum v is almost difficult since $J_{p,q}$, is a highly nonlinear function of v. Accordingly, iterative gradient-type optimization algorithms such as the steepest descent algorithm and the Fletcher-Powell algorithm can be used to find the (local) maximum of $J_{p,q}$.

II. MMNC COST FUNCTION

The cost function of our proposed, modified maximum normalized cumulant criterion (MMNC) is [6]

$$J_{MMNC}(v) = J_R(v) + J_I(v)$$
$$= \left|\gamma_{p,q}\{y_R(n)\}\right| + \left|\gamma_{p,q}\{y_I(n)\}\right| \qquad (5)$$
$$= \frac{\left|C_{p,q}\{y_R(n)\}\right|}{[C_{1,1}\{y_R(n)\}]^{(p+q)/2}} + \frac{\left|C_{p,q}\{y_I(n)\}\right|}{[C_{1,1}\{y_I(n)\}]^{(p+q)/2}}$$

where $y_R(n)$ and $y_I(n)$ are the real and imaginary parts of the equalizer output, respectively.

Decomposing the MNC cost function into the real and imaginary parts allows both the *amplitude* and the *phase* of the equalizer output to be considered; therefore, joint blind *equalization* and *phase recovery* may be simultaneously accomplished, eliminating the need for a rotator to perform separate constellation-phase recovery in steady-state operation.

The tap-weight vector of the MMNC can be updated according to the gradient-type optimization methods that include the steepest descent method, the Newton and approximate Newton methods and the BFGS[1] and approximate BFGS methods.

The simulation results indicate that the MMNC alone can remove ISI and simultaneously correct the phase error, because it implicitly incorporates a phase-tracking loop, which automatically recovers the carrier phase [6].
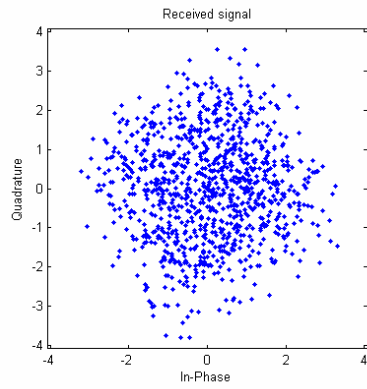
III. SIMULATION RESULTS

We carried out a numerical simulation that compares the MMNC with the MNC algorithm, using the equalizer update algorithm of BFGS. Both equalizers use a complex transversal filter structure having 30 tap weights. All the tap weights were initialized by setting the central tap weight to 1, and the others to zero. The source signals are uniformly distributed random sequences (16QAM modulated) and the channel is a Rayleigh fading multipath Channel. This channel has three paths with delays 0, 0.9 and 1.5 symbol period and the paths gain are 0dB, -3dB and -6dB respectively.
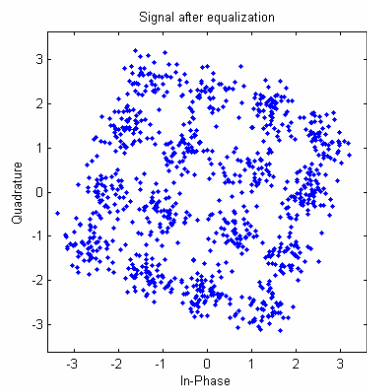
The maximum Doppler shift which specifies the amount of channel fading, is assumed 1Hz (The lager Doppler shifts are also used but not shown in the graphs). The white noise level in the signal is kept 35dB below the signal level. The original received signals are "eye closed" (see the constellation shown in Fig. 3a) due to the ISI introduced from the channel distortion. The two algorithms have similar convergence properties, as shown in Fig. 4, but the MNC keeps the phase error after convergence while the proposed algorithm can remove the phase error (see Fig. 3b-c).

Apart from that the proposed algorithm has a better ISI reduction than the MNC algorithm, as can be deduced when Fig. 4 is compared with Fig. 5. This reduction is much greater when the block size is small (see Fig. 6). Fig. 7 also shows the greater advantage of the MMNC in improving ΔSNR over the MNC particularly in smaller block sizes.

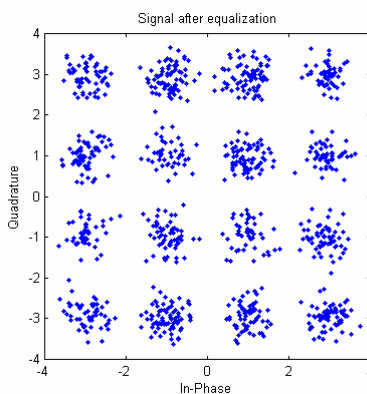[1] Broyden-Fletcher-Goldfarb-Shanno optimization algorithm

(a)

Fig. 4 ISI versue the number of iterations for the proposed and the MNC algorithms for 10000 symbols block

(b)

Fig. 5 ISI versue the number of iterations for the proposed and the MNC algorithms for 2000 symbols block

(c)

Fig. 3 Signal Constellations with 1000 symbols:
(a) Received signal, (b) Equalized with MNC algorithm,
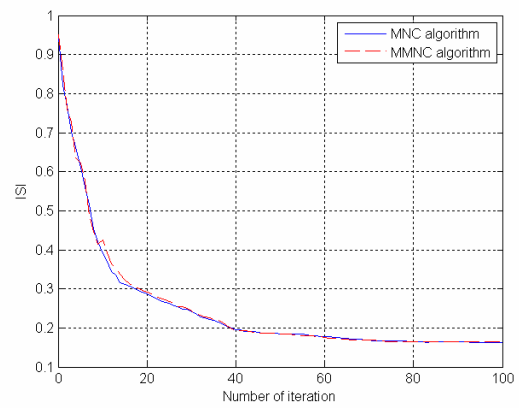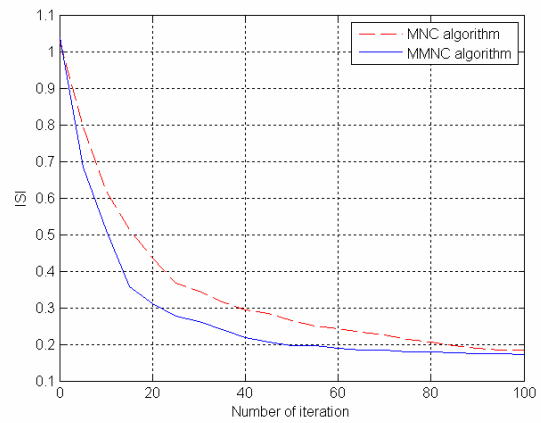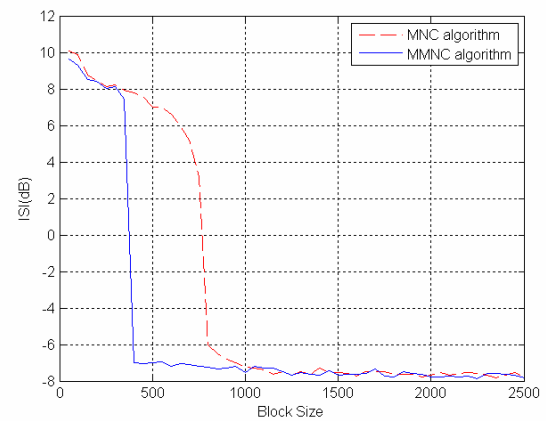(c) Equalized with our algorithm

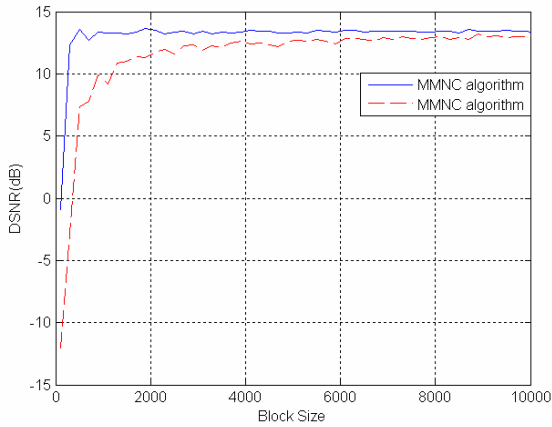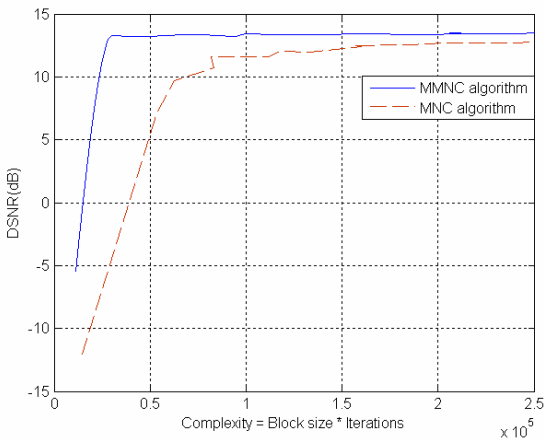Fig. 6 Residual ISI of the two algorithms for different block sizes

Fig. 7 ΔSNR of the two algorithms for different block sizes

Fig. 8 Computational complexity comparison of the two algorithms

TABLE I
PERFORMANCE COMPARISON OF THE ALGORITHMS WITH 1000 SYMBOLS

Criterion	MNC	MMNC
Number of iteration	54	46
ΔSNR (dB)	9.5327	13.2342

TABLE II
PERFORMANCE COMPARISON OF THE ALGORITHMS WITH 5000 SYMBOLS

Criterion	MNC	MMNC
Number of iteration	39	31
ΔSNR (dB)	12.5631	13.5337

Tables I, II show the ΔSNR and the number of iterations for the block sizes of 1000 and 5000. For both cases the MMNC algorithm needs a smaller iterations and reaches to a higher ΔSNR after convergence. For a better comparison between two algorithms, a computional complexity index is defined as the block size multiplied by the number of iterations. In Fig. 8, which shows the ΔSNR versus this computional complexity index for two algorithms, it is evident that the MMNC algorithm can reduce the complexity by a large ratio.

IV. CONCLUSION

In this paper, we presented a cost function that can be used for simultaneous blind equalization and phase recovery of the signals. A simple blind equalization algorithm based on this cost function is designed, which is very similar to the MNC algorithm but with a faster convergence and a greater ΔSNR especially for the smaller block sizes. Also, the MMNC algorithm has a complexity very less than the MNC algorithm.

APPENDIX

Fig. 9 shows a model of a system with distortion [6] in which the output signal, y, is composed of a distortion signal, n, and a linearly amplified signal αx :

$$y = \alpha x + n \qquad (6)$$

where the α is a gain, which gives the maximum non-distorted signal. It is assumed from the beginning that the distortion signal, n, is uncorrelated with the input signal, x. There might be a delay in y with respect to x which is assumed to be already compensated. The distortion signal power, N, can be found from:

$$N = \overline{nn^*} = \overline{(y - \alpha x)(y^* - \alpha^* x^*)} \qquad (7)$$

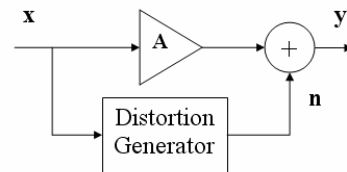where the star represents complex conjugate and the bar represent the time average.

Fig. 9 Model of a system with distortion

Equation (7) can be further expanded as:

$$N = \left| y \right|^2 + \left| \alpha \right|^2 \left| x \right|^2 - \alpha \overline{xy^*} - \alpha^* \overline{x^* y} \qquad (8)$$

To find the α that minimizes the N, derivatives of the N with respect to the real and imaginary parts of α can be set to zero:

$$\left. \frac{\partial N}{\partial \, real(\alpha)} \right|_{imag(\alpha)} = \left. \frac{\partial N}{\partial \, imag(\alpha)} \right|_{real(\alpha)} = 0 \qquad (9)$$

Substituting (8) into (9) and solving it gives:

$$real(\alpha) = \frac{\overline{x^* y} + \overline{xy^*}}{2|x|^2}, \ imag(\alpha) = \frac{\overline{x^* y} - \overline{xy^*}}{2j|x|^2} \quad (10)$$

Combining the equations in (10) gives the α as:

$$\alpha = \frac{\overline{x^* y}}{|x|^2} \quad (11)$$

The distortion power can be found by substituting (11) into (8):

$$N = |y|^2 - |\alpha|^2 |x|^2 \quad (12)$$

The undistorted signal power, from which the signal to distortion ratio can be calculated, is $S = |\alpha|^2 |x|^2$. It can be easily shown that: $\overline{xn^*} = \overline{x^* n} = 0$, which indicate that the distortion signal, n, is uncorrelated with the input signal, x at time 0.

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Reza Berangi received the B.Sc. degree in telecommunications and the M.Sc. degree in electronic engineering From Iran University of Science and Technology (IUST), Tehran, Iran, in 1985 and 1989 respectively, and the Ph.D. degree in mobile communications from Victoria University of Technology (VUT), Melbourne, Australia, in 1998. From 1986 to 1988, he worked with the Nasr Research and Development Centre, Tehran, Iran on signal processing hardware and software. From 1997 to 1999, he was with the Mobile Communications and Signal Processing Group at VUT, working on smart antenna mobile base stations. From 1999 to 2001, He was with the Australian Cooperative Telecommunication Research Centre (ATCRC), working on intelligent base stations for the third generation mobile communication systems at VUT. Since 2001, Dr. Berangi has been as an assistant professor at the computer department of IUST. His research interest is signal processing for mobile communications including adaptive algorithms, smart antenna, and peak to average minimization.

Morteza Babaee was born in Tehran, Iran, in 1982. From September 2001 to July 2007, he was with Imam Hossein University (IHU), Tehran, Iran, where he received the B.Sc. degree in electrical and electronic engineering in 2005, and the M.Sc. degree in communications engineering in 2007. Since 2005, he has been with DSP Research Institute where he has worked as a researcher. His research interests are in the areas of signal processing for wireless communications, statistical signal processing, blind signal processing, modulation and demodulation techniques, and communication theory.

Majid Soleimanipour received the B.Sc. degree in communications engineering from Sharif University of Technology, Tehran, in 1985, the M.Sc. degree in communications engineering from K.N. Toosi University of Technology, Tehran, in 1991, and the M.Sc. and Ph.D. degrees in electrical engineering from the University of Waterloo, Waterloo, Canada, in 1995 and 1999, respectively. From 1985 to 1990, he was with Nasr Electronic R&D Center where he worked as a researcher and project manager. He directed the center from 1987 to 1990 as the general manager. From 1990 to 1993, he was engaged in several projects in mobile communications. Since 1999, Dr. Soleimanipour has been with Imam Hossein University, Tehran, Iran, as an assistant professor and he was the Dean of the Faculty of Engineering from 2000 to 2006.