

A Contribution to the Application of the Structural Analysis Method in Entrepreneurial Practice

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Abstract—Quantitative methods of economic decision-making as the methodological base of the so called operational research represent an important set of tools for managing complex economic systems, both at the microeconomic level and on the macroeconomic scale. Mathematical models of controlled and controlling processes allow, by means of artificial experiments, obtaining information for optimal or optimum approaching managerial decision-making. The quantitative methods of economic decision-making usually include a methodology known as structural analysis - an analysis of interdisciplinary production-consumption relations.

Keywords—economic decision-making, mathematical methods, structural analysis, technical coefficient

I. INTRODUCTION

At present, the methods of operational research represent a set of tools for the management of complex economic systems, both at the microeconomic level and on the macroeconomic scale. According to Zimola [1], the operational research provides an opportunity to improve the economic decision-making, particularly in terms of speed and competency, using quantitative methods. That is why the mathematical modelling is more and more often used as an integral part of economic theory and economic practice.

II. STRUCTURAL ANALYSIS

Each production - consumption system (national economy, department, field, branch, and company) consists of certain elements (fields, branches, technologies, products), among which there are links such as supplier-customer relations. Any change in one element may show itself in those elements which the given element is directly or indirectly linked with. When examining the conditions of economic balance of the

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production - consumer systems, during analysis and operational reflection of changes in production-consumption relations of the production - consumer systems we can bear on an existing set of general theoretical knowledge of mathematical modelling and analysis of production-consumption links, usually known as *the structural analysis*¹.

The structural models as products of this methodology display endogenous and exogenous production-consumption relations of the production-consumption system in various conditions. According to Řimánek [2], the structural model can be understood as an instrument enabling us to capture the mutual relations of the individual elements of the economic system and the links of these elements to the surrounding environment. It represents a certain quantitative way of displaying the production - consumption processes, where the creation and utilization of production is captured not only as a whole, but in its structure as well.

The presumption of equality between the resources and needs of each element of the production - consumption system is an essential feature of the structural model. The structural model represents a system of balances which describe the resources and also the distribution of consumptions of all the model components (elements) of the production - consumption system in a single complex of equations. From the mathematical apparatus, the structural models use mainly linear algebra. The mathematical essence of the basic types of the structural models is represented by a set of linearly independent equations. The number of linearly independent equations depends on the type of the model, for example in case of Leontief's and Pichler's type; it equals the number of variables, so the system of equations has a clear solution (if a solution exists at all).

Using the mathematically defined models balance of factory-consumer relations are prevalent and the historically oldest models in the economy and these had originally view relationships in the reproductive process at the macroeconomic level [3],[4] gradually expanded to the level of business entities and has been the one of the important tools of economic governance. Professional literature gives numerous examples dedicated to real applications, for ex. [5], [6].

¹There are various names used in various literary sources, for ex. input-output models, models of relations among the branches, balance models, Leontief's models or matrix planning models

III. LEONTIEF'S MODELS

Wassily W. Leontief is generally regarded as the founder of the structural analysis. The basic methods W. W. Leontief called „input – output analysis“ are explained in his book „The Structure of American Economy, 1919-1929[3].

Leontief's static models are the most common representatives of the structural models. The first model assembled by Leontief was the so called closed static model used for analysis of the structure of the U.S. economy in the years 1919 – 1929[3]. The closed model does not include any autonomous input or output; all variables are fully interconnected by their feedbacks. On the other hand, the models where one group of branches acts as producers and consumers at the same time, while the second group acts only as consumers, are open models. As Leontief himself[4]states, when justifying the transition from a closed to an open model, the open model is more useful in terms of active economic policy. In an open model, we can analyze not only the existing situation, but if there are the technical coefficients, we can also calculate the extent of the overall production of the individual branches to any specified size and structure of the final production. The application of these models or variations has been extensively used in studies spanning a broad variety of disciplines.

The static open Leontief's structural model is clearly schematically illustrated by a staggered table that graphically illustrates the flows of goods, raw materials, energies and other production factors, both within any random production and consumption system such as a company, and between them and the surrounding environment in the period under consideration, Fig. 1.

Total production of n-branches		Production consumption of branch				Output	Total production
Total consumption of m-branches		1	2	...	j	...	
consumption of energy, raw materials and semi-finished products coming from branches	1	x ₁₁	...		x _{1n}	y ₁	x ₁
	2

	i	(I. quadrant)				II. quadrant	.

raw material input into the system	n	x _{n1}	...		x _{nn}	y _n	x _n

Input	.	s _{1,1}	...		s _{1,n}	.	.
	.	s _{2,1}	.	.	s _{2,n}	.	.
	.	(III. a quadrant)				.	.
.	s _{d,1}	...		s _{d,n}	.	.	
depreciations, wages, profit and other costs	.	z _{1,1}	...		z _{1,n}	.	.
	.	z _{2,1}	.	.	z _{2,n}	.	.
	.	(III. b quadrant)				.	.
.	z _{k,1}	...		z _{k,n}	.	.	

Fig. 1 Basic scheme of the staggered table
Revised from: [7]

As stated by Janovská [8] if we call the products with varying degrees of aggregation branches, then the staggered table for any random production - consumer system with n-branches can be divided into several "fields", known as "quadrants".

The first "quadrant" of the staggered table shows the consumption of production - the flow of materials, intermediate products and energies between the production branches.

The second "quadrant" represents the so called final consumption (in practice, it means especially sales - sales of products of the given branch, but also its "non-productive" consumption in the system).

The third a (III.a) "quadrant" captures the material - energy input into the system (in practice, it means the consumption of the purchased raw materials, materials and energies).

The third b (III.b) "quadrant" shows the newly created value (part of the value of production created by material inputs processing; in practice, they are depreciations, labour costs, the profit from sales of production). [8]

The schematic staggered table can be used to derive a basic set of equations for the production-consumption system with n-manufacturing branches and k-supplying branches - a system of balance (distribution) equations:

system (n + k) of distribution equations, showing the presumption of economic balance of the system:

$$x_i = \sum_{j=1}^n x_{ij} + y_i \tag{1}$$

when i = 1,2,...,n j = 1,2,...,n

$$s_d = \sum_{j=1}^n s_{dj} \tag{2}$$

when d = 1,2,...,k

where:

- x_i ...total production volume of the i- th branch
- x_{ij}....consumption of production of the i-th branch in the j-th branch of the given production-consumption system
- y_i total „final consumption“ of production of the i-th branch
- s_dtotal consumption of production of the d-th supplying branch
- s_{d,j} ...consumption of production of the d-th supplying branch in the j-th branch

As stated by Vepřek [9]simplifying presumption of linearity between the manufacturing consumption of production of the i-th branch in the j-th branch x_{ij} and the volume of production of the j-th branch x_jexpresses the relation that is known as theLeontief's cost function:

$$x_{ij} = a_{ij} x_j \tag{3}$$

where

a_{ij} coefficient of direct specific consumption of production of the i – th branch in the j –th branch; expresses production volume of the i -th branch necessary to manufacture a production unit of the j – th branch

x_{ij}consumption of production of the i -th branch in the j -th branch of the given production-consumption system

x_jthe volume of production of the j -th branch

Should we introduce the expression (3) into the system of distribution equations (1), (2), then their matrix expression is

$$\underline{x} = \underline{Ax} + \underline{y} \quad (4)$$

$$\underline{s} = \underline{Cx} \quad (5)$$

where

\underline{x} vector of total production volume x_i ; $i = 1, 2, \dots, n$

\underline{y} vector of final consumption of production – sales of production y_i ; $i = 1, 2, \dots, n$

\underline{A} (n,n) ..matrix of direct consumption coefficients a_{ij} ; $i, j = 1, 2, \dots, n$

\underline{s} vector of total consumption of production of supplying branches s_d $d = 1, 2, \dots, k$

\underline{C} (k,n) ..matrix of direct consumption coefficients s_{dj} ; $d = 1, 2, \dots, k$, $j = 1, 2, \dots, n$

After modification:

$$\underline{y} = (\underline{E} - \underline{A})\underline{x} \quad (6)$$

$$\underline{x} = (\underline{E} - \underline{A})^{-1} * \underline{y} \quad (7)$$

$$(\underline{E} - \underline{A})^{-1} = \underline{B} \rightarrow \underline{x} = \underline{B} * \underline{y} \quad (8)$$

$$\underline{s} = \underline{C} * \underline{B} * \underline{y} \quad (9)$$

\underline{E} type unit matrix /n,n/

\underline{B} (n,n) ..matrix of coefficients of complex (direct and indirect) consumption of production of the i -th branch in the j -th branch

The elements b_{ij} of matrix B represent the total amount of production of the i -th branch needed to manufacture a production unit of the j -th branch intended for final consumption - sales.

The indirect consumption is a mediated consumption, given by the fact that the relevant j -th branch production needs include not only the given i -th branch but also the production

of other branches that also consume the production of the relevant i -th branch.

IV. TECHNICAL COEFFICIENT - PROBLEMS OF ITS CONSTRUCTION

The basic and pivotal presumption in construction of the Leontief's static models is the proportional relationship

between production and consumption $x_{ij} = a_{ij}x_j$.

It means that the coefficient a_{ij} expresses what amount of product of the i -th branch is necessary in the given system to manufacture a production unit of the j -th branch, and that is why it represents a fundamental quantity of the structural model. The relevance it is determined with decides about the demonstrative power of the structural analysis.

Depending on whether the starting balance (staggered table) has been prepared using natural or value expression, the coefficients a_{ij} have the matching character.

Quantification of flows between the individual branches in natural units (for ex. tons, MJ, ...) makes up the basis of the natural structural analysis. This analysis can also be supplemented by information about the flows between the economic sectors and the environment, such as production of waste [10], consumption of resources [11], utilization of land [21].

As stated by Weinzettel in [13], this type of analysis is too demanding for data, which is why it is not applied very often.

If we start from the balance in natural units, the value of coefficient a_{ij} is primarily determined by the production technology and that is why the coefficient a_{ij} is known as the technical coefficient. The technical coefficients have, in this case, the nature of a consumption standard which is characteristic for the given production technology. As stated by Vepřek [9], when using the natural units, it is a disadvantage that the technical coefficients generally do not meet the usually sufficient condition of enabling the economic interpretation of the matrix elements $[E-A]^{-1}$, i.e. that $a_{ij} \leq 1$ ($i, j = 1, \dots, n$), and therefore it is required that the character of the elements of the inversion matrix must be closely examined in such cases.

The value expression of the coefficient a_{ij} is more common especially at higher management levels. In this case, the elements a_{ij} of matrix A express the consumption of production of the i -th branch required to manufacture a production unit of the j - th branch in monetary units. The basis of the monetary structural analysis is the quantification of the flows between the individual branches in monetary units (for ex.: Kč, EUR, etc..).

When using the monetary units, there is a problem of uniform appreciation of products. If it is impossible to keep the principle of uniform appreciation, it leads to distortion of material links displayed in a model and it reduces its accuracy, or even puts its usability in jeopardy. When using the monetary units, the technical coefficients usually meet the sufficient condition for economic interpretation ability of the matrix elements $[E-A]^{-1}$, i.e. that $a_{ij} \leq 1$ ($i, j = 1, \dots, n$).

As stated by Weinzettel in [13], when the monetary units are replaced by natural units for selected products, we are talking about the so-called *hybridanalysis*. The outcomes of this kind of analysis are directly the physical flows of the selected materials (for ex. tons of coal, tons of oil, and others). This method is described in more details in the literature [14],[15], and it was later used in the literature [16].

Habr [17] claims that the validity of the hypothesis of direct proportionality between production and production consumption *should be limited in time*. The technical coefficients can not be considered as constant in longer-term; they are gradually adjusting to reflect the prices of the inputs and they reflect the new technologies. Experience shows that in the national economy as a whole, the changes do not usually occur in steps, but they are gradual. *The effect of the introduction of the new technologies* into operation depends not only on the technical parameters but also on the relative importance in the production branch in question. If the new technology takes up only a small part of the capacity of the given branch, then its effect on the technical coefficients can usually be neglected, otherwise it is necessary to examine the technical coefficients.

Janovská [8] points out that this assumption (a direct proportion between production and production consumption) does not describe the reality accurately enough - the rate at which it approximates to reality is different for different branches and may not be acceptable for each type of the production-consumption system or for each type of production. It can be an expression of long-term production experience. When the same manufacturing procedure, and the same technology of production is used to manufacture a certain product, the consumption of used raw materials, energies, materials and semi-finished products depends above all on the size of production of the given product and is only marginally influenced by other factors.

In cases where structural analysis is used as a cognitive instrument of detailed relations in economic groups, *the heterogeneity of the individual branches* is a major factor that has significant impact on the technical coefficients. The technical coefficients are in fact average values (weighted by the value of production) influenced by structural changes. It is obvious that the size of the technical coefficients is also affected by the degree of aggregation of the individual branches. As Rojíček claims [18], the more homogeneous the individual branches are, the more stable the technical coefficients will be, because the lower the possibility of structural changes is.

The question of *defining the branch* is one of the most important issues which the informative value not only of the inter-branch balance, but also the mathematical models derived on its basis depend on. In the designed tables, the number of branches practically varies from several tens to several hundreds, while, as noted by Habr [17], the stability of the technical coefficients requires the individual branches to be as homogenous as possible, i.e. the classification of branches should be as delicate as possible. However, too large number of branches is not only unsustainable in terms of data

collection and their processing, but also in terms of their analysis. The balance matrices used in practice for the entire national economy tend to be large, where n may reach the value of hundreds or thousands. That is why further processing makes use of the aggregation principle that enables adjusting the entire balance to size which allows easier subsequent processing.

The changes of the technical coefficients are then essentially influenced by two circumstances:

- *changes in technology*
- *changes in the structure of a branch*

It means that the usefulness of construction of the structural model and the relevance of the provided information for the decision-making processes depend on the correct determination of the coefficients a_{ij} themselves as the technical coefficients or the consumption standards.

As already mentioned before, the basic presumption of the Leontief's static models is the linearity of relations between the production consumption and the production of the individual branches. The inclusion of nonlinear dependencies (production and production consumption) is not so much interfering with the mathematical problems, but it is mainly the difficulty of construction of the nonlinear models, and the issue of feeding such a model with sufficiently accurate input data.

V. THE INPUT DATA - SUITABLE AND CONSISTENT DATA

The solution of the issue of feeding the model with the basic data - the acquisition of *suitable and consistent data* is of fundamental importance during the construction but, above all, in practical application of the structural models. Many models have not seen any wider practical use just because this issue had been underestimated. According to Mikušová [19] the enterprise and government executives has revealed that one of the most frequently reported common disappointments is a lack of data integrity and system inability to produce useful information to support decision-making.

During the assembly of the structural models of the production-consumption system, there was a question how to obtain such basic data, to avoid the technical coefficients being affected (or to have them affected as little as possible) by the method of the data collecting and processing.

There are a number of views of a quality dates [20]. In a simplified way it can be said that quality dates are as follows [21]:

- they are in harmony with company's strategic priorities and customers' priorities,
- they are appropriately chosen for company's operation as well as for work of individuals,
- it is easy to develop and adjust them,
- they can be quickly implemented,
- they are easily understandable,
- they lead to the improvement,
- they are not an inviolable thing,
- they are not left to exist forever.

Given that all the matrix models are relatively highly sensitive to changes in the numerical values, it is important for the technical coefficients of the model to describe the modelled reality with most precision, because in case of matrix inversion operations the inaccuracies can expand out of control. In his work [22], Rutrle indicates that experiments have proved that 5% deviation in the matrix of direct consumption may cause up to 30% deviation in some elements of the inverse matrix, i.e. the matrix complex consumption coefficients. The issue of the matrix inversion is also described by Simonovits in his work [23]. It means that in extensive computational work even small errors can accumulate and they can significantly debase the results, even if the basic hypotheses are accurate.

Obtaining relevant data, especially at company level is then the usual fundamental problem arising during composition of the structural model. Workers outside the company sphere in question often come across barriers when it comes to obtaining relevant data, in particular with reference to a "trade secret".

The source of knowledge may also come from the structural models based on the use of the available retrospective data of a technical nature (capacity characteristics, consumer numbers) that are published and widely available for example in annual reports and bulletins, and in many cases these files of aggregated data do not change significantly year on year, and they can form the basis for establishing the basic matrix of production and the consumer relations and, by means of a simple software transformation, they can be used to calculate the matrices of the so called coefficients of complex (both direct and indirect) consumption.

VI. CONCLUSION

In conclusion, we can say that the use of the economic-mathematical methods in entrepreneurial practice is completely justified and it gives the opportunity to speed up, refine and improve the decision-making processes of managers and executives of companies, but it should be noted that the excessive application of mathematics has a paradox tendency of obscuring the reality.

As the Nobel prize winner in economics W. Leontief himself writes, "*uncritical enthusiasm for mathematical formulation tends often to conceal the ephemeral substantive content of the argument behind the formidable front of algebraic signs*" (...) in no other field of empirical inquiry [as in economics] has so massive and sophisticated a statistical machinery been used with such indifferent results. (...) .. most of these [mathematical models] are relegated to a stockpile without any practical application.² [24]

As stated by a Czech economist and main macroeconomic strategist of ČSOB Sedláček [24], "mathematics is an incredibly powerful tool, but you must understand that the mathematical apparatus can never yield more than what you put into it. And if mathematics is the basic language of

economics, then the economists must put into its formulas some economic content of their own, otherwise their theory becomes an empty exercise, when the after some mathematical "abracadabra" brings the desired result from the axiomatic presumptions."

In any case, the structural models represent a compromise between the pursuit for the most precise and the most comprehensive picture of the economic reality and the request for availability of adequate input data and parameters for the completion of the model. When using any structural model, it is necessary to keep in mind the limitations and simplifications it results from, and to take them into account when interpreting the outcomes acquired from a model.

ACKNOWLEDGMENT

The work was supported by the specific university research of Ministry of Education, Youth and Sports of the Czech Republic No. SP/ 2011/27.

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