

# A Computationally Efficient Design for Prototype Filters of an M-Channel Cosine Modulated Filter Bank

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**Abstract**—The paper discusses a computationally efficient method for the design of prototype filters required for the implementation of an M-band cosine modulated filter bank. The prototype filter is formulated as an optimum interpolated FIR filter. The optimum interpolation factor requiring minimum number of multipliers is used. The model filter as well as the image suppressor will be designed using the Kaiser window. The method will seek to optimize a single parameter namely cutoff frequency to minimize the distortion in the overlapping passband.

**Keywords**—Cosine modulated filter bank, interpolated FIR filter, optimum interpolation factor, prototype filter.

## I. INTRODUCTION

OVER the years several techniques have been developed for the design of near-perfect reconstruction filter banks. In wideband audio applications stopband attenuation in nonadjacent channels is required to be greater than  $-100\text{dB}$ . Hence we relax the perfect reconstruction condition in favour of high stopband attenuation, in these filter banks the aliasing is cancelled only approximately and the distortion function is only approximately a delay. These systems are called pseudo-qmf filter banks. When the analysis and synthesis filters of the M-channel pseudo-qmf filter bank are cosine-modulated versions of the prototype filter we have a cosine-modulated filter bank. The advantage of the cosine modulated filter bank is that the cost of the analysis filter bank is the cost of one filter plus modulation overheads. Also during the design phase we are required to optimize the coefficients of the prototype filter only. For approximate reconstruction the following two conditions are required to be satisfied as nearly as possible.

$$|H(\omega)|^2 + |H(\omega - \pi)|^2 = 1 \quad \text{for } 0 < \omega < \pi/M \quad (1)$$

$$|H(\omega)| = 0 \quad \text{for } \omega > \pi/M \quad (2)$$

Manuscript received April 30, 2006.

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Earlier methods of design of the prototype filter involved nonlinear optimization. Creusere and Mitra [2] proposed an efficient method using the Parks McClellan Algorithm not involving nonlinear optimisation. The cost function in this case was the maximum ripple in the overlapping passbands. Vaidyanathan [3] proposed an efficient method based on the Kaiser window approach. We will discuss here the interpolated FIR based approach suggested by Zijing[4]. Whereas in the above the interpolation factor is chosen arbitrarily as  $M/2$  we will use in our design the optimum value of  $L$  which is calculated as that value of  $L$  that yields minimum number of multipliers which in turn depends on filter order as suggested in [5].

## II. DESIGN TECHNIQUE FOR IFIR FILTER

Consider the design of the prototype filter  $H(z)$  having order  $N$  and a transition width of  $\Delta f$ . If the frequency response of this filter is stretched by a factor of  $L$  to obtain a filter  $G(z)$  then  $G(z)$  will have a transition width of  $L\Delta f$ . The order of  $G(z)$  will be  $1/L$  times that of  $H(z)$ . That is multiplications and addition will now be decreased by a factor of  $L$ . Now  $G(z^L)$  will be a filter having a response similar to  $H(z)$ , except for the fact that  $G(z^L)$  will have a copy centered at  $2\pi/L$ . If we cascade  $G(z^L)$  with a filter  $I(z)$  centered at  $2\pi/L$ , with stopband edge at  $(2\pi/L - \omega_s)$  the resultant filter will be  $H(z)$ .  $G(z)$  is called the model filter and will have an order much less than that of  $H(z)$ .  $I(z)$  is called the image suppressor and will be of very low order. The optimum value of  $L$  is based on the method proposed by Alireza and Wilson [5] and is given by

$$L_{\text{opt}} = 2\pi / (\omega_p + \omega_s + \sqrt{2\pi(\omega_s - \omega_p)}) \quad (3)$$

which is to be rounded to the nearest integer.

The model filter  $G(z)$  will have a transition width  $L$  times that of the prototype filter. So the order will be  $1/L$  times that of  $H(z)$ . The transition width of  $I(z)$  is still larger therefore it will have a very small order. Let  $\omega_p^H$  and  $\omega_s^H$  denote the passband and stopband edge of  $H(z)$ .

$$\omega_s^H = \pi/M \quad (4)$$

$$\text{and } \omega_p^H = \pi/2M \quad (5)$$

Then the passband and stopband edge of the model filter  $G(z)$  will be  $L\omega_p^H$  and  $L\omega_s^H$  respectively.

### III. OPTIMISATION PROCEDURE

The objective function to be minimized is chosen to be  $\phi$ . Define a filter  $P(e^{j\omega})$  given by  $P(e^{j\omega}) = |H(e^{j\omega})|^2$ . This means that  $P(e^{j\omega})$  is approximately a Nyquist (2M) filter. The impulse response  $p(n)$  of  $P(e^{j\omega})$  will have the property

$$\begin{aligned} g(2Mn) &= 0, \text{ for } n = \pm 1, \pm 2, \dots \\ &= 1/(2M) \text{ for } n = 0 \\ &= \max |g(2Mn)| \text{ for all } n \text{ except } n=0. \end{aligned} \quad (6)$$

The cut off frequency is varied to obtain the smallest value of  $\phi$ . As shown in the Fig. 1 the objective function is a convex function of the cutoff frequency  $\omega_c$ .

### IV. DESIGN OF PROTOTYPE FILTER

The image suppressor  $I(z)$  will be designed using the Kaiser window method before the optimization of the model filter  $G(z)$ . The passband edge of  $I(z)$  is chosen to be slightly less than  $\pi/2M$  and the stopband edge is chosen to be slightly greater than  $2\pi/L - \pi/M$ .  $I(z)$  must have such an order so that  $H(z)$  has the desired stopband attenuation. The model filter  $G(z)$  is also designed using the Kaiser window approach. Choose the order of  $G(z)$  such that  $H(z)$  will have desired attenuation. This filter will have peak ripples in passband and stopband similar to that of  $H(z)$ . The filter order is estimated approximately as

$$N = (As - 7.95) / (14.6 \Delta f). \quad (7)$$

$$\Delta f = (\omega_s - \omega_p) / 2\pi \quad (8)$$

The order of the model filter  $G(z)$  and the stopband edge are fixed before the start of the optimization procedure. Compared to the direct design of the prototype filter, here we need to optimize only the coefficients of the model filter  $G(z)$ , which will have an order less than that of  $H(z)$ . Also the coefficients of image suppressor filter need not be optimized. The Magnitude response of the interpolation filter and the model filter (in dashed line) is shown in Fig. 2.

### V. EXAMPLE

The cosine modulated filter bank is designed using the Interpolated FIR approach. The filter bank is designed to have 8 channels and an attenuation of about 110dB. The passband and the stopband edges of the image suppressor filter  $I(z)$  is chosen to be  $.05\pi$  and  $0.4\pi$ . This filter is calculated to have an order of 40. The optimal value of  $L$  is calculated to be 3.65 and is rounded off to 4. The passband and the stopband edges of model filter are taken to be  $0.25\pi$  and  $0.5\pi$ , and is determined to have an order of 56. Fig.1 shows the variation of the objective function with the cutoff

frequency. The cut of frequency for the minimum objective function is found to be .27009. Fig.2 shows the magnitude response of filters  $G(z^4)$  and  $I(z)$ . Fig.3 shows the magnitude response of the analysis filters of the 8-channel cosine modulated filter bank. Fig. 5 shows the magnitude response of the distortion function. The maximum deviation from unity is .04dB.

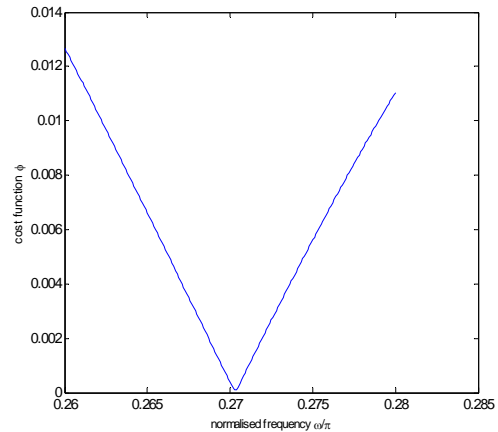


Fig. 1 Plot of cost function vs. normalized frequency

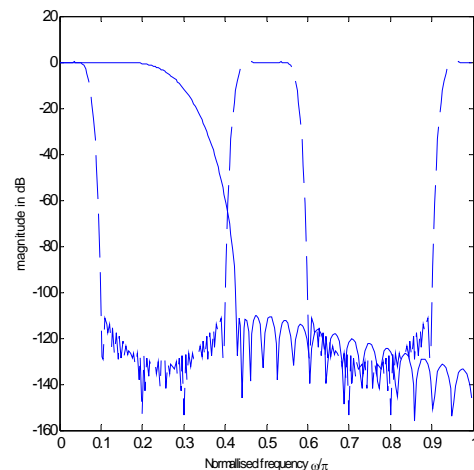


Fig. 2 Plot of magnitude response of  $G(z^4)$  and  $I(z)$

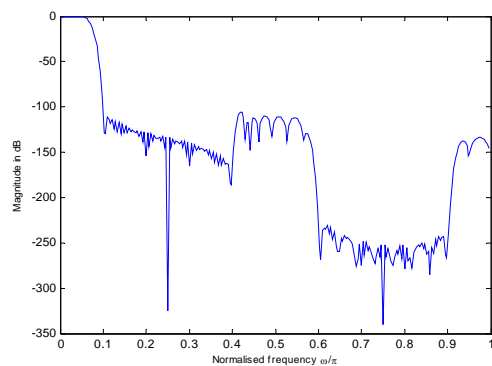


Fig. 3 Plot of magnitude response of the Prototype filter

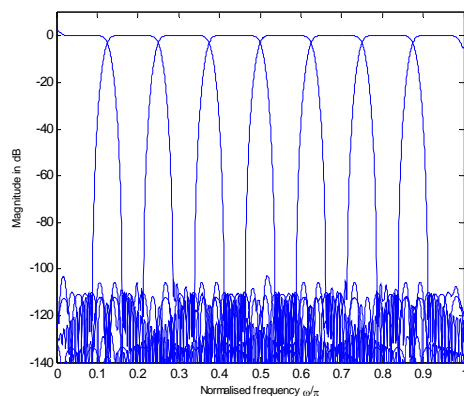


Fig. 4 Magnitude Response of the Analysis Filters

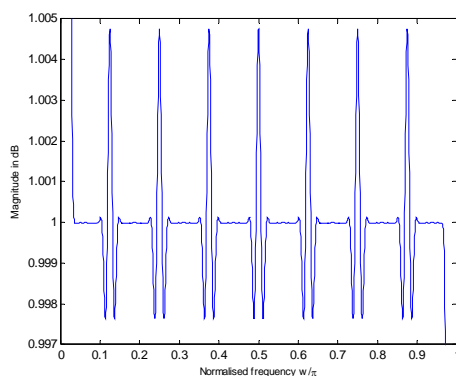


Fig. 5 Amplitude distortion vs. Normalised Frequency

## VI. CONCLUSION

The filter bank has been designed using the IFIR approach. The value of the interpolation factor has been chosen to be the optimal value, on the basis of least number of multipliers required. This value has been found to be significantly different from  $M/2$  as the number of channels is increased. The optimal value is found to be 6 for the case of 16 channels. Because of the smaller number of coefficients 56 in this case the optimisation procedure converges very quickly to the desired value.

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