

A Comparison between Hybrid and Experimental Extended Polars for the Numerical Prediction of Vertical-Axis Wind Turbine Performance using Blade Element-Momentum Algorithm

Gabriele Bedon, Marco Raciti Castelli, Ernesto Benini

Abstract—A dynamic stall-corrected Blade Element-Momentum algorithm based on a hybrid polar is validated through the comparison with Sandia experimental measurements on a 5-m diameter wind turbine of Troposkien shape. Different dynamic stall models are evaluated. The numerical predictions obtained using the extended aerodynamic coefficients provided by both Sheldal and Klimas and Raciti Castelli et al. are compared to experimental data, determining the potential of the hybrid database for the numerical prediction of vertical-axis wind turbine performances.

Keywords—Darrieus wind turbine, Blade Element-Momentum Theory, extended airfoil database, hybrid database, Sandia 5-m wind turbine.

I. INTRODUCTION

THE aerodynamic design of the vertical-axis wind turbine (VAWT) is still object of debate. The complexity of the unsteady air-blade interaction during a full revolution of the machine has induced several authors to propose different methods in order to predict rotor performances. Two main approaches are generally adopted, leading to different aerodynamic prediction tools:

- Computational Fluid Dynamics (CFD) codes, which offer a very deep understanding of the air dynamics inside the rotor. On the other hand, a great computational effort is requested, due to the long time needed to perform transient simulations.
- Blade Element - Momentum (BE-M) Theory based algorithms, which are very fast to compute a reliable global performance value, but are dependent on the availability of a complete aerodynamic database of the adopted rotor blade section.

Both the approaches have been highly developed and improved.

Carrigan et al. [1] introduced a fully automated process for optimizing the airfoil cross-section of a VAWT, maximizing the torque while enforcing some typical wind turbine design constraints, such as tip speed ratio, solidity and blade

profile. The design system required to maximize rotor torque incorporated both rapid geometry generation and automated hybrid mesh generation tools with viscous, unsteady CFD simulation software.

Sabaeifard et al. [2] performed both experimental and CFD analyses of small scale Darrieus-type straight-bladed VAWTs, describing the effect of some design parameters, including blade number, airfoil type and turbine solidity on rotor performance.

Ferreira et al. [3] [4] presented a systematic CFD analysis of a two-dimensional single bladed rotor configuration, investigating the effect of dynamic stall on VAWT performance and analyzing the influence of the turbulence model in the simulation of the vortical structures spread from the rotor blade.

Raciti Castelli et al. [5] [6] proposed a new CFD based methodology in order to obtain an estimation of both turbine aerodynamic performance and local blade angles of attack, allowing the investigation of any blade profile shape and providing a deep understanding of the flow mechanisms inside the rotor.

Performance prediction tools based on the Blade Element - Momentum (BE-M) Theory have been considered by several authors, thanks to their lower computational requirements [7] [8] [9] [10] [11] [12]. All the BE-M algorithms rely on the knowledge of the machine operative conditions, as well as the aerodynamic characteristics of the blade profile, as discussed by Bak et al. [13].

The most adopted aerodynamic database was provided by Sheldahl and Klimas [14], who performed a series of wind tunnel tests in order to evaluate the lift and drag coefficients over 180° angles of attack for NACA -0009, -0012 and -0015 airfoils and for some Reynolds numbers typical of VAWT blades during operation. These results were successively extended by numerical methods to NACA-0018, -0021 and -0025 and also to a wider range of Reynolds numbers. The availability of such extended databases is one of the main drawbacks of this method: since aeronautical databases (extending up to a limited range of angles of attack) are easily available in literature, several authors investigated the feasibility of their extension by means of numerical methods.

Bak et al. [13] analyzed several methods (based on Inverse Momentum Theory, Actuator Disc Theory, numerical optimization and quasi-3D CFD computations) for the

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derivation of airfoil aerodynamic characteristics, to be used in wind turbine applications.

Raciti Castelli et al. [15] suggested a hybrid database computation methodology, allowing the numerical prediction of the aerodynamic coefficients of a symmetric NACA airfoil characterized by a thickness comprised from 0.09 to 0.25 c. The estimation was based on a combination of the results of the Xfoil numerical code and on the averaged Sandia coefficients (from: [14]), mixed up in order to minimize any possible source of error. This database was validated by Bedon et al. [16] with respect to the experimental results obtained by Sheldahl [17] for a 2-m Darrieus wind turbine, adopting the BE-M code developed by Raciti Castelli et al. [18].

In the present work, the hybrid database proposed by Raciti Castelli et al. [15] is adopted in order to predict the performances of the Sandia 5-meter Darrieus wind turbine studied by Sheldahl and Klimas [19], using a Double Disk Multiple-Streamtube algorithm developed by Raciti Castelli et al. [18]. This code includes a dynamic stall-correction algorithm based on the works of Gormont [22], Strickland et al. [23] and Berg [24].

II. BE-M THEORY

Several BE-M algorithms are available in literature. Some of them include specific corrections to keep both the expansion of the streamtubes and rotor blade dynamic stall into account [25]. As a first operational step, a structured mesh of the rotor - for a given number of vertical and azimuthal subdivisions - is created, as shown in Fig. 1.

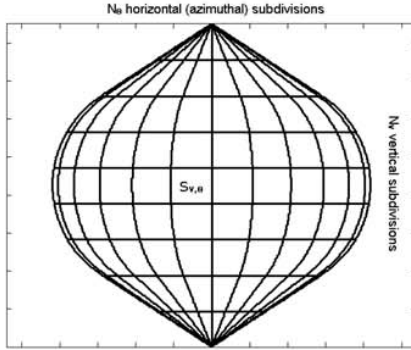


Fig. 1: Schematic view of the mesh generation for a Troposkien VAWT

Fig. 2 shows the Double Disk BE-M model scheme, reporting its six characteristic wind speeds and considering also the expansion of the streamtube, due to rotor blade absorption of kinetic energy in the upwind zone.

Considering the averaged streamwise momentum equation in conjunction with Bernoulli's equation, the non-dimensional streamwise force can be written as:

$$F_x^* = \frac{V_{blade,i}}{U_\infty} \left(1 - \frac{V_{blade,i}}{U_\infty} \right) = \frac{N \cdot F_x}{2 \cdot \pi \cdot r \cdot \Delta h \cdot \sin(\theta) \cdot U_\infty^2} \quad (1)$$

being N the number of rotor blades, F_x the streamwise force, r the rotor radius relative to each considered blade element,

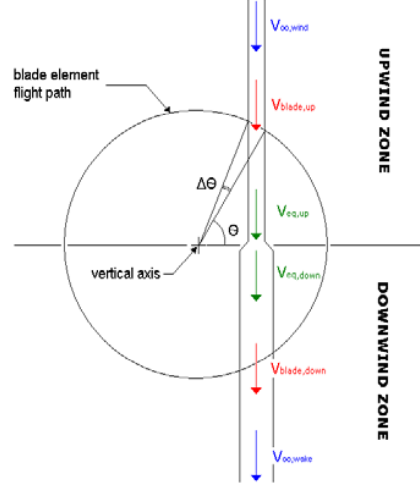


Fig. 2: Plan view of rotor cross-section and visualization of the six streamtube characteristic flow velocities for the adopted Double-Disk Multiple-Streamtube configuration, considering also the correction for streamtube expansion (from: [18])

Δh the height of each single streamtube, U_∞ the unhindered wind speed and θ the azimuthal coordinate of the blade.

The resultant streamwise force acting on each blade section is given by:

$$F_x = -(F_n \sin(\beta) \sin(\theta) + F_t \cos(\theta)) \quad (2)$$

being β the blade element inclination with respect to the horizontal plane, F_t and F_n respectively the aerodynamic forces tangential and normal to the chord line.

Expressing the forces in terms of lift and drag coefficients, the non-dimensional streamwise F_x^* force can be also written as:

$$F_x^* = \frac{Nc}{8\pi r} \left(\frac{W}{U_\infty} \right)^2 \left(C_N - C_T \frac{\cos(\theta)}{\sin(\theta) \sin(\beta)} \right) \quad (3)$$

being c the rotor blade chord, W the relative velocity in the cross-sectional plane of the considered blade element, C_T and C_N respectively the blade element tangential and normal aerodynamic coefficients, in formulas:

$$C_T = C_L \sin(\alpha) - C_D \cos(\alpha) \quad (4)$$

$$C_N = C_L \cos(\alpha) + C_D \sin(\alpha) \quad (5)$$

with C_L and C_D respectively the lift and drag aerodynamic coefficients and α the angle of attack between each blade element chord and the relative wind velocity.

Defining the axial induction factor as:

$$a = 1 - \frac{V_{blade,i}}{U_\infty} \quad (6)$$

where $V_{blade,i}$ is the flow velocity at each blade section (i can be up or down, as shown in Fig. 2), the streamwise momentum equation can be written as:

$$a = F_x^* + a^2 \quad (7)$$

forming the basis for an iterative solution.

Once the streamwise momentum equation is solved, the torque for each streamtube can be computed by means of the following equation:

$$T_s = \frac{1}{2} \rho r C_T \frac{c \cdot \Delta h}{\sin(\beta)} W^2 \quad (8)$$

where ρ is the air density.

Finally, the average power produced by the rotor can be computed as:

$$P = \omega \frac{N}{N_\theta} \sum_{i=1}^{N_\theta} \sum_{j=1}^{N_V} T_{S_{ij}} \quad (9)$$

being ω the angular speed of the rotor, N_θ and N_V respectively the number of horizontal and vertical mesh subdivisions. The rotor power coefficient can be estimated by the formula:

$$C_p = \frac{P}{0.5 \rho U_\infty^3 S} \quad (10)$$

where S is the swept-area of the turbine.

Two different dynamic stall models, based on Gormont works [22], are implemented: Gormont-Strickland [23] and Gormont-Berg [24]. Furthermore, the finite aspect-ratio correction from Viterna and Corrigan studies [26] is adopted.

III. CASE STUDY AND METHODOLOGY

The 5-m Darrieus wind turbine tested by Sheldahl and Klimas [19] is adopted as a case study for the present computations. The rotor is 5.1 m high, with a maximum radius of 2.5 m. The blade shape is straight-circular-straight (SCS): this is considered a good approximation of the Troposkien architecture [27] but cheaper to manufacture. The main geometrical characteristics of the turbine are reported in Table I, while a picture of the described rotor installation in the Sandia test site is shown in Fig. 3.

TABLE I: Main geometrical characteristics of the considered rotor (from: [19])

H	5.1 m
R	2.5 m
N	3
Blade profile	NACA 0015
Blade Shape	Straight-circular-straight (SCS)
c	152.4 mm

The main performance parameter considered in the proposed computations is the Power Coefficient C_p , as defined in (10). The numerical results are presented with respect to the Tip Speed Ratio, defined as:

$$\lambda = \frac{\omega R}{U_\infty} \quad (11)$$

where R is the maximum rotor radius.

Experimental measurements were conducted considering five rotational speeds: 125, 137.5, 150, 162.5 and 175 rpm. A wind speed ranging from 4 to 27 m/s was considered (the open field tests were conducted in the Sandia foreground site: in the present work, the same conditions are considered as inputs for the BE-M algorithm). For the propose computations, the wind

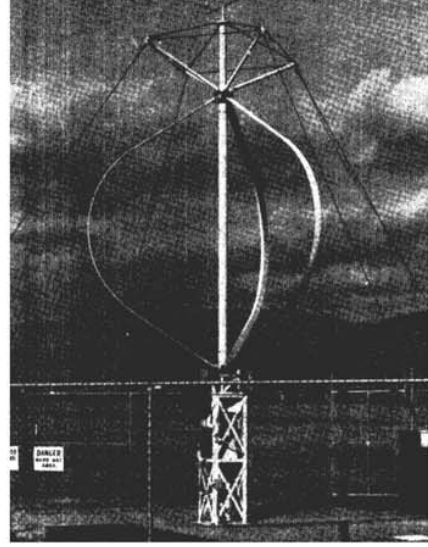


Fig. 3: Sandia 5 m Darrieus wind turbine installation (from: [19])

is considered uniform over the turbine height since it was not possible to collect information about the wind-shear at the test site, while the effect of spokes is neglected.

As a first step, both Gormont-Strickland [23] and Gormont-Berg [24] dynamic stall corrections are adopted in combination with the Sandia coefficients from Sheldahl [14], in order to understand which model is best describing the rotor performance. The best model is successively adopted in combination with the hybrid database provided by Raciti Castelli et al. [15], in order to compare the performance predictions computed on the basis of the two candidate polars. The validity of the proposed hybrid database was already observed by Raciti Castelli et al. [15] for a turbine characterized by a height greater than 5 m and by Bedon et al. [16] for a 2-m high rotor. The aim of the present work is to extend the validity of the hybrid database also to a medium size VAWT architecture.

IV. RESULTS AND DISCUSSION

A. Dynamic stall model comparison

Figs. from 4 to 8 show a comparison between measured and computed rotor performances using both Gormont-Strickland and Gormont-Berg models, for angular rotor velocities ranging from 125 to 175 rpm and adopting the hybrid aerodynamic database [14].

Differently from the conclusions drawn by Bedon et al. [16] for the 2-m Darrieus turbine, both the Gormont-Strickland and the Gormont-Berg models provide a good estimation of the turbine performances. The reliability of the first model was already assessed for small size VAWTs [16], whereas the reliability of the second one was proved for bigger turbines [15]: being a medium size turbine considered in this case, both are reliable. Nevertheless, since the Gormont-Berg model is providing a better estimation of the turbine performances during the blade stall (for tip speed ratios lower than the

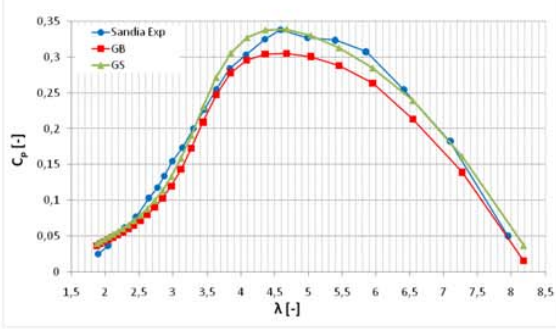


Fig. 4: Rotor performance prediction considering Gormont-Strickland (GS) and Gormont-Berg (GB) models and experimental results for a rotational speed of 125 rpm

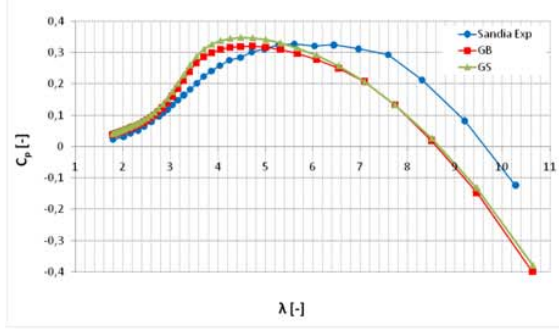


Fig. 7: Rotor performance prediction considering Gormont-Strickland (GS) and Gormont-Berg (GB) models and experimental results for a rotational speed of 162.5 rpm

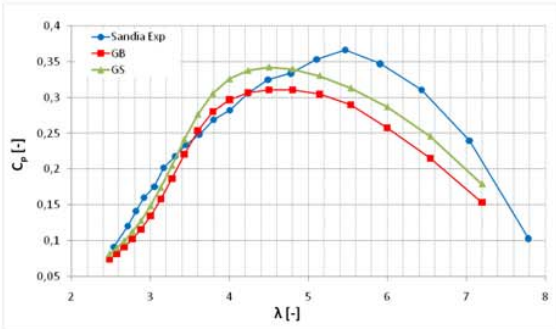


Fig. 5: Rotor performance prediction considering Gormont-Strickland (GS) and Gormont-Berg (GB) models and experimental results for a rotational speed of 137.5 rpm

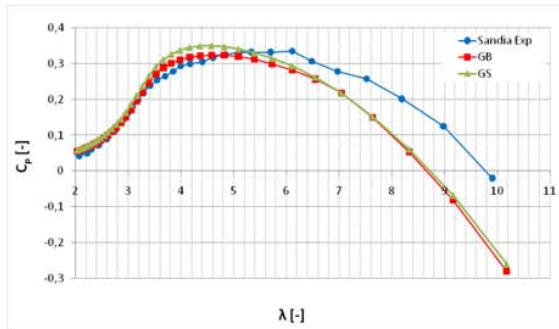


Fig. 8: Rotor performance prediction considering Gormont-Strickland (GS) and Gormont-Berg (GB) models and experimental results for a rotational speed of 175 rpm

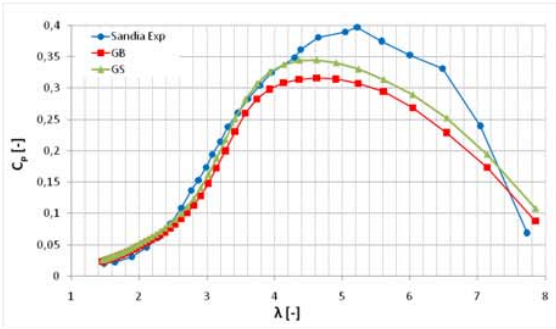


Fig. 6: Rotor performance prediction considering Gormont-Strickland (GS) and Gormont-Berg (GB) models and experimental results for a rotational speed of 150 rpm

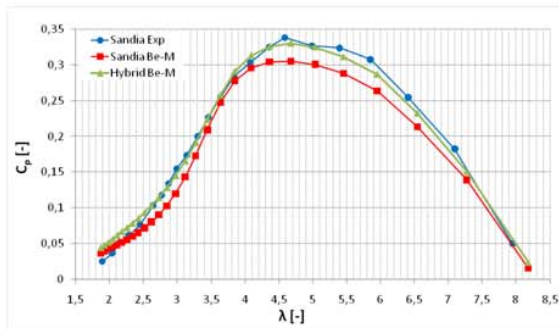


Fig. 9: Rotor performance prediction considering Gormont-Berg model, using both the Sandia database (Sandia BE-M) and the hybrid database (Hybrid BE-M), as well as experimental results for a rotational speed of 125 rpm

one corresponding to the peak power coefficient), this will be considered for the successive computations.

B. Hybrid database results

Figs. from 9 to 13 show a comparison between measured and computed rotor performances using both Gormont-Strickland and Gormont-Berg models, for angular rotor velocities ranging from 125 to 175 rpm and adopting both the hybrid aerodynamic database [15] and the one provided by Sandia [14].

It can be observed that, for low rotational speeds (up to 150 rpm), the hybrid database is providing better results

than the Sandia database. For higher rotational speeds, the hybrid database tends to overestimate rotor performance up to the peak power coefficient, while the performances for higher values of the tip speed ratio result underpredicted. This tendency is also followed by the computations based on the Sandia polars: these limitations can be therefore related to the adopted BE-M algorithm.

V. CONCLUSIONS

The 5-m Darrieus wind turbine manufactured and tested by Sandia has been considered as a case study for the

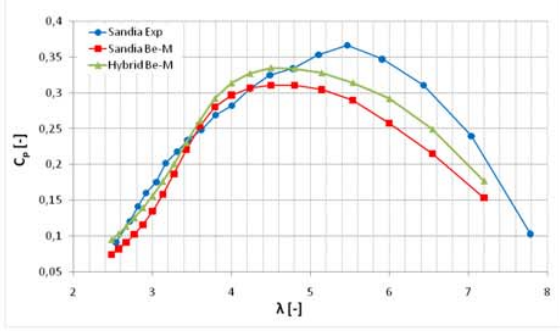


Fig. 10: Rotor performance prediction considering Gormont-Berg model, using both the Sandia database (Sandia BE-M) and the hybrid database (Hybrid BE-M), as well as experimental results for a rotational speed of 137.5 rpm

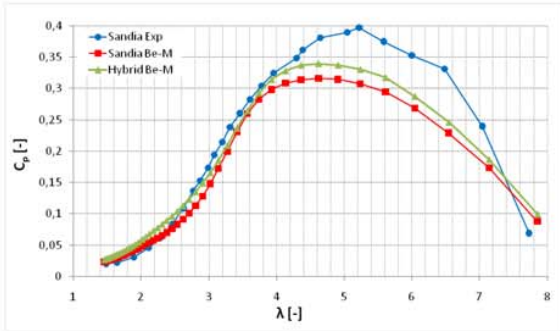


Fig. 11: Rotor performance prediction considering Gormont-Berg model, using both the Sandia database (Sandia BE-M) and the hybrid database (Hybrid BE-M), as well as experimental results for a rotational speed of 150 rpm

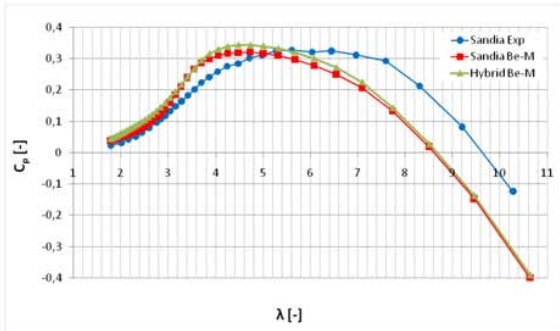


Fig. 12: Rotor performance prediction considering Gormont-Berg model, using both the Sandia database (Sandia BE-M) and the hybrid database (Hybrid BE-M), as well as experimental results for a rotational speed of 162.5 rpm

evaluation of a hybrid database to be used in small-scale VAWT performance prediction codes. Different dynamic stall models have been evaluated using the Sandia aerodynamic coefficients from [14]. Both the Gormont-Strickland and Gormont-Berg models have provided reliable results for the considered rotational speeds.

The Gormont-Berg model has been chosen to be used in combination with the hybrid database from [15]. The hybrid database has provided good results for rotational speeds lower than 150 rpm. For higher rotational speed, the tip speed ratio

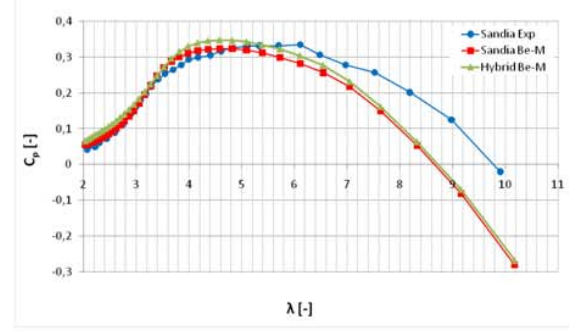


Fig. 13: Rotor performance prediction considering Gormont-Berg model, using both the Sandia database (Sandia BE-M) and the hybrid database (Hybrid BE-M), as well as experimental results for a rotational speed of 175 rpm

of maximum power coefficient has moved towards higher wind speeds. This behavior has been observed using both the Sandia and the hybrid database, leading to the conclusion that the described effect is to be related to the BE-M code.

Further numerical simulations should be performed by considering several blade and rotor architectures, in order to establish the reliability of the hybrid database even for different NACA profiles and rotor blade geometries.

NOMENCLATURE

a [-]	Axial induction factor
c [m]	Airfoil chord
C_D [-]	Airfoil drag coefficient
C_L [-]	Airfoil lift coefficient
C_N [-]	Blade element normal coefficient
C_p [-]	Rotor power coefficient
C_T [-]	Blade element tangential coefficient
F_n [N]	Blade force normal to chord line
F_t [N]	Blade force tangential to chord line
F_x [N]	Resultant streamwise force
F_x^* [N]	Streamwise force exerted by the blade element as it passes through the streamtube
Δh [m]	Streamtube height
H [m]	Rotor total height
N [-]	Number of blades
N_θ [-]	Number of horizontal mesh subdivisions
N_v [-]	Number of vertical mesh subdivisions
P [W]	Power produced by the turbine
r [m]	Rotor radius relative to a blade element
R [m]	Wind turbine maximum radius
T_S [Nm]	Blade element torque for each streamtube
S [m ²]	Rotor swept area
U_∞ [m/s]	Unhindered wind speed
$V_{blade,i}$ [m/s]	Flow velocity at a blade section, i can be up or down
W [m/s]	Relative velocity at a blade element cross-sectional plane

α [rad]	Blade relative angle of attack (between airfoil chord line and relative wind velocity)
β [rad]	Blade element inclination with respect to the horizontal plane
λ [—]	Tip speed ratio
ρ [kg/m ³]	Air density (assumed 1 [kg/m ³])
θ [°]	Blade azimuthal coordinate
ω [rad/s]	Rotor angular velocity

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