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A Comparative Study between Discrete Wavelet Transform and Maximal Overlap Discrete Wavelet Transform for Testing Stationarity

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Abstract**—**In this paper the core objective is to apply discrete wavelet transform and maximal overlap discrete wavelet transform functions namely Haar, Daubechies2, Symmlet4, Coiflet2 and discrete approximation of the Meyer wavelets in non stationary financial time series data from Dow Jones index (DJIA30) of US stock market. The data consists of 2048 daily data of closing index from December 17, 2004 to October 23, 2012. Unit root test affirms that the data is non stationary in the level. A comparison between the results to transform non stationary data to stationary data using aforesaid transforms is given which clearly shows that the decomposition stock market index by discrete wavelet transform is better than maximal overlap discrete wavelet transform for original data.

Keywords**—**Discrete wavelet transform, maximal overlap discrete wavelet transform, stationarity, autocorrelation function.

I. Introduction

DURING the last two decades, wavelet analysis to capture volatile behavior in time series data has become increasingly popular in many fields, such as medical and physical sciences. More recently, these methods are also being applied to financial datasets. However, the study of stability data using wavelet analysis is still poorly explored. It is widely known that the financial time series data is a combination of different components operating on different frequencies. Standard time series econometric tools such as Fourier transform usually consider only time or frequency component separately. Whereas, wavelets allow us to study the frequency components of time series with time information simultaneously. Therefore, we can uncover the interactions which are hardly visible using other econometric tools. Consequently, the wavelet transform is very useful in financial time series.

In particular there are two types of wavelet transforms, orthogonal as discrete wavelet transform (DWT) and non orthogonal as maximal overlap discrete wavelet transform (MODWT). DWT is useful in decomposing time series data into an orthogonal set of components with different frequencies by checking the relationship between high frequency fluctuations in stock prices obtained from reconstruction of the series by wavelet crystals. Whereas MODWT is a variant of DWT that can handle any sample size. The smooth and detail coefficients of MODWT multiresolution analysis are associated with zero phase filters and produces a more asymptotically efficient wavelet variance estimator than the DWT. However, the MODWT loses the orthogonality. Some of the studies that compare between the wavelet functions based on DWT or MODWT are given in [1]-[3]. The authors in [1] explored the performance of two wavelet functions namely Daubechies and Haar in extracting the coherent structures from solar wind velocity time series. It was found that both wavelet functions are able to extract coherent structures, however, the coherent time series showed that the Daubechies wavelet function was able to extract more coherent structures than the Haar wavelet. In [2] evaluated the WiMAX traffic prediction accuracy by using five different types of MODWT functions Daubechies, Coiflet, Symmlet, Biorthogonal and Reverse Biorthogonal. Based on Autoregressive Integrated Moving Average (ARIMA), Artificial Neural Network (ANN) and Random Walk (RW) methods, the results indicate that Daubechies and Reverse Biorthogonal produced smallest errors using some statistical measures of error. In [3] a comparison between DWT and MODWT by applying Haar, Daubechies, Symmlet and Coiflet functions using Malaysia stock price was discussed and it was shown that the MODWT performs better than DWT for these functions.

However, in this work the key idea is to investigate the stationarity of time series data using both DWT and MODWT. Five functions namely Haar, Daubechies (db2), Symmlet (sym4), Coiflet (coif2) and Discrete approximation of the Meyer wavelets (dmey) are utilized for original data of US Dow Jones Index (DJIA30). The results reveal that the DWT is better than MODWT to produce more stationary data for original time series data.

The remainder of this paper is organized as follows. Section II briefly discusses the methodology. Section III describes the data, the empirical results and discussion. Finally, Section IV concludes this paper.

II. Methodology

A. Wavelets

The wavelets have two types, the father wavelets ϕ [4] and the mother wavelets ψ where father wavelet ϕ integrates to one and mother wavelet ψ integrates to zero [5]. That is

$$\int \varphi(t)\,dt = 1 \text{ and } \int \psi(t)\,dt = 0. \quad (1)$$

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The mother wavelets are useful in describing the detail and high-frequency components while the father wavelets are good at representing the smooth and low-frequency parts of signal.

Wavelets are derived using a special two-scale dilation equation. Father wavelet $\phi(t)$ and mother $\psi(t)$ are defined as

$$\phi(t) = \sqrt{2}\, \sum \ell\kappa\, \phi(2t-k) \quad (2)$$

$$\psi(t) = \sqrt{2}\, \sum h\kappa\, \phi(2t-k) \quad (3)$$

where $\ell\kappa$ and $h\kappa$ defined in (4) and (5) respectively, are low-pass and high-pass filter coefficients used to pass the original signal as specified in [12].

$$\ell\kappa = \frac{1}{\sqrt{2}} \int \phi(t)\phi(2t-k)dt \quad (4)$$

$$h\kappa = \frac{1}{\sqrt{2}} \int \psi(t)\phi(2t-k)dt \quad (5)$$

The wavelet series approximation to a signal $X(t)$ is defined by

$$X(t) = \sum_k S_{,\kappa}\phi_{,\kappa}(t) + \sum_k d_{,\kappa}\, \psi_{,\kappa}(t) + \sum_k d_{-1,\kappa}\, \psi_{-1,\kappa}(t) + .. + \sum_k d_{1,\kappa}\, \psi_{1,\kappa}(t) \quad (6)$$

where k ranges from 1 to the number of coefficients in the specified components (or crystals) and J is the number of multiresolution levels (or scales). The coefficients $S_{,\kappa}$, $d_{,\kappa}$, ..., $d_{1,\kappa}$ are wavelet transform coefficients given by the projections

$$S_{,\kappa} = \int \phi_{,\kappa}(t)\, X(t) d(t) \quad (7)$$

$$d_{j,\kappa} = \int \psi_{j,\kappa}(t)\, X(t) d(t), \quad j = 1, 2, .J \quad (8)$$

The magnitude of these coefficients gives a measure of the contribution of the corresponding wavelet function to the total signal. The basic functions $\psi_{j,\kappa}$ and $\phi_{j,\kappa}$, $j = 1,2,.J$ are the approximating wavelet functions generated as scaled and translated versions of ϕ and ψ with scale factor 2^j and translation parameter $2^j k$ respectively, defined as:

$$\phi_{j,\kappa} = 2^{\frac{-j}{2}}\phi(2^{-j}\, t - \kappa) = 2^{\frac{-j}{2}}\phi\left(\frac{t-2^j\kappa}{2^j}\right) \quad (9)$$

$$\psi_{j,\kappa}(t) = 2^{\frac{-j}{2}}\psi(2^{-j}\, t - \kappa) = 2^{\frac{-j}{2}}\psi\left(\frac{t-2^j\kappa}{2^j}\right), j = 1,2,..J \quad (10)$$

Translation parameter $2^j\kappa$ is matched to the scale parameter 2^j in a way that as the function $\phi_{j,\kappa}$ and $\psi_{j,\kappa}(t)$ get wider, their translation steps are correspondingly larger.

There are five types of orthogonal wavelet families that are used in practical analysis, Haar, Daubechies, Symmlet, Coiflet, and Discrete approximation of the Meyer wavelets holding the following prominent features:

- The Haar wavelet is a square wave with compact support and is symmetric but it is not continuous unlike the other wavelets.
- The Daubechies are continuous wavelets with compact support and are quite asymmetric.
- The Symmlet have compact support and were built to be as nearly symmetric as possible.
- The Coiflet are symmetric with additional properties that both ψ and φ have vanishing moments.
- The discrete approximation of the Meyer wavelets is symmetric and continuous with compact support.

The aim of discrete wavelet transform is to decompose the discrete time signal to basic functions called the wavelets, to give us a good analytic view of the analyzed signal. DWT is used to calculate the coefficients of approximation in (6) for a discrete signal of final extent $f_1, f_2, ..f_n$ .. It maps the vector f = $(f_1, f_2, ...f_n)'$ to a vector of n wavelet coefficients $\boldsymbol{\omega} = (\omega_1, \omega_2, ..., \omega_n)'$ that contains both the smooth coefficient $S_{,\kappa}$ and the detail coefficients $d_{j,\kappa}$, $j=1,2,..,J$. $S_{,\kappa}$ representing the underlying smooth behavior of the signal at the coarse scale 2^J. On the other hand $d_{j,\kappa}$ describe the coarse scale deviations and $d_{-1,\kappa}$,...,$d_{1,\kappa}$ provide progressively finer scale deviations. When the length of the signal n is divisible by 2^J, there are n/2 coefficients $d_{1,\kappa}$ at the first scale $2^1 = 2$. At the next finest scale $2^2 = 4$, there are $n/4\, d_{2,\kappa}$ coefficients. Similarly, at the coarsest scale, there are $n/2^J d_{j,\kappa}$ and $n/2^J$ $S_{,\kappa}$and coefficients. Altogether there are total of n coefficients: $N= n/2+n/4+...+n/2^{J-1} + n/2^J$.

Maximal overlap discrete wavelet transform (MODWT) [6] is similar to the discrete wavelet transform (DWT) in that low-pass and high-pass filters are applied to the input signal at each level. However, the MODWT does not decimate the coefficients and the number of wavelet and scaling coefficients is same as the number of sample observation at every level of the transform. In other words, MODWT coefficients consider the result of a simple changing in the pyramid algorithm used in computing DWT coefficients through not down sampling the output at each scale and inserting zeros among coefficients in the scaling and wavelet filters. For this reason the MODWT is also called non-decimated DWT, stationary DWT [7], translation invariant DWT [8] and time –invariant DWT [9]. The MODWT loses orthogonality and efficiency in computation.

The MODWT has some advantages over the DWT:

- The MODWT can handle any sample size n .
- The smooth and detail coefficients of MODWT multiresolution analysis are associated with zero phase filters.
- It is transform invariant, since a shift in the signal does not change the pattern of the wavelet transform coefficients.
- Produces a more asymptotically efficient wavelet variance estimator than the DWT.

B. Unit Root Test

Usually financial data often displays non-stationary data or have time varying means, variances and covariance. Therefore in order to avoid the problem of spurious regression, unit root test is used to test for stationarity. Two types of unit root the Augmented Dickey Fuller (ADF) [11] and Phillips Perron (PP) tests [10] are used in this paper. These tests are defined by:

$$\Delta y_t = (P_a - 1)y_{t-1} + \mu_t \quad (11)$$

$$\Delta y_t = a + \beta y_t + \varepsilon_t \quad (12)$$

C. Autocorrelation Function (ACF)

ACF is a mathematical tool that is usually used for analyzing functions or series of values, for example time series signals and to measure the correlation between the signals. ACF is a correlation coefficient. Nevertheless, instead of correlation between two different variables, the correlation is between two values of the same variable at times y_i and y_{t+k} ACF is defined as:

$$p_k = \frac{E((y_t-\mu)(y_{t+k}-\mu))}{\sqrt{E\left((y_t-\mu)^2 E(y_{t+k}-\mu)^2\right)}} = \frac{cov(y_t, y_{t+k})}{Var(y_t)} = \frac{\gamma_k}{\gamma_o}, \; k=0,1,.. \quad (13)$$

Note that by definition $p_0 = 1$.

Furthermore, the ACF is used to detect the stationary or non-stationary data, observing the behavior of the autocorrelation function. A strong and slowly dying ACF well suggests deviations from stationarity. Due to cutting off or tailing off near zero after a few lags the ACF is very persistent , meaning that it decays very slowly and exhibits sample autocorrelations that are still rather large even at long lags. This behavior is characteristic of non-stationary time series. From a time series of finite length, the autocorrelation function is estimated as:

$$r_k = \frac{\sum_{t=1}^{T=k}(y_t-\bar{y})(y_{t+k}-\bar{y})}{\sum_{t=1}^{T=k}(y_t-\bar{y})^2}, k = 0,1,..,K \quad (14)$$

III. Empirical Results and Discussion

In this section we first apply the unit root test to check if our data is stationary or not. After that, we choose five wavelet families to apply DWT and MODWT with original data.

A. Unit Root Test

Fig. 1 shows the stock market index behavior over the time and we can see the data are not stationary and it appear that (DJIA30) rise to the peak in the end of 2007 , but steep down to lowest point in the beginning of 2009 .

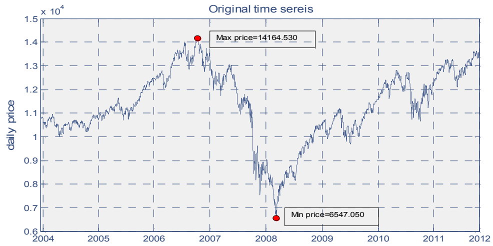

Fig. 1 Time series plot of DJIA30

The results two tests using unit root test are given in Table I. These results show that P-value > 0.05, indicating that the data has unit root and therefore is non stationary at level.

TABLE I
Unit Root Tests of Daily Stock Price

	ADF	PP
Level	-1.608800	-1.525140
P-value	0.7896	0.8210

B. DWT and MODWT

In this section we apply DWT and MODWT using five functions, Daubcheis2 (db2), Haar (haar), Symmlets4 (sym4), Coifltes2 (coif2), and discrete approximation of the Meyer wavelets (dmey) for original series. It is to note that we are composing until level 7 to get the smooth data.

C. Autocorrelation

With regard to comparison between DWT and MODWT for stability, the ACF is the best method to use to analyze the fluctuation in data. Consequently, if the autocorrelation is high (low) then the stationarity will be high (low). Table II gives the autocorrelation values for the original data after decomposition by DWT and MODWT using the said five functions until lag 20.

TABLE II
AUTOCORRELATION VALUES FOR APPROXIMATION

lags	ACF for DWT					ACF for MODWT				
	Haar	db2	dmey	sym4	coif2	Haar	db2	dmey	sym4	coif2
0	1	1	1	1	1	1	1	1	1	1
1	0.997	0.999	0.9995	0.999	0.9995	0.9991	0.9988	0.9982	0.9986	0.9986
2	0.994	0.999	0.9989	0.999	0.9989	0.9973	0.9963	0.9944	0.9956	0.9955
3	0.991	0.998	0.9982	0.998	0.9982	0.995	0.993	0.9904	0.9917	0.9917
4	0.988	0.997	0.9975	0.997	0.9975	0.9922	0.9894	0.987	0.9879	0.9878
5	0.985	0.996	0.9968	0.997	0.9968	0.9892	0.9859	0.984	0.9844	0.9844
6	0.982	0.995	0.996	0.996	0.9959	0.9862	0.9826	0.981	0.9814	0.9814
7	0.979	0.994	0.9951	0.995	0.9951	0.9832	0.9796	0.9779	0.9787	0.9786
8	0.976	0.993	0.9942	0.994	0.9942	0.9802	0.9768	0.975	0.976	0.9759
9	0.973	0.991	0.9932	0.993	0.9932	0.9772	0.974	0.9724	0.9732	0.9732
10	0.971	0.99	0.9922	0.992	0.9922	0.9742	0.9711	0.9698	0.9704	0.9703
11	0.968	0.989	0.9912	0.991	0.9911	0.9711	0.9681	0.9667	0.9673	0.9672
12	0.965	0.987	0.99	0.99	0.9899	0.968	0.9649	0.9633	0.964	0.964
13	0.962	0.986	0.9889	0.988	0.9888	0.9648	0.9616	0.9599	0.9607	0.9607
14	0.959	0.984	0.9876	0.987	0.9875	0.9616	0.9583	0.9566	0.9574	0.9573
15	0.956	0.982	0.9864	0.986	0.9862	0.9583	0.955	0.9534	0.9541	0.954
16	0.953	0.981	0.985	0.984	0.9849	0.9551	0.9518	0.9502	0.9509	0.9508
17	0.95	0.979	0.9837	0.983	0.9835	0.9519	0.9486	0.947	0.9477	0.9476
18	0.947	0.977	0.9822	0.982	0.982	0.9487	0.9454	0.9438	0.9445	0.9445
19	0.944	0.975	0.9807	0.98	0.9805	0.9455	0.9422	0.9406	0.9413	0.9413
20	0.941	0.974	0.9792	0.978	0.979	0.9424	0.9391	0.9375	0.9382	0.9382

From Table II it can be observed that after decomposition by families based on DWT and MODWT, the ACF of MODWT is less than ACF of DWT for different lags of the data. Since the difference between ACF is not significant, so we use t-test to ascertain that which of the two wavelet transforms is better to be used for decomposition time series data in order to get more stationary data. We define

$$H_{0:}\ \mu\ DWT = \mu\ MODWT,\ H_1: \mu\ DWT > \mu\ MODWT$$

From t-test P value < 0.05 for all functions accept Haar, revealing that the difference is there between DWT and MODWT of time series data decomposition. From Table III we can see that the mean of DWT is bigger than the mean of MODWT except Haar, which is quite similar. Consequently, the DWT is better than MODWT to decompose time series data to get more stationary data.

TABLE III
MEAN TABLE

Wavelet functions	Wavelet transforms	N	Mean
Haar	DWT	21	0.970457
	MODWT	21	0.973390
db2	DWT	21	0.988743
	MODWT	21	0.970595
dmey	DWT	21	0.991262
	MODWT	21	0.969005
sym4	DWT	21	0.990895
	MODWT	21	0.969714
coif2	DWT	21	0.991186
	MODWT	21	0.969667

In addition, from Fig. 2 it can easily be seen that the mean ACF for functions by DWT is higher than the mean ACF for functions by using MODWT accept Haar function which cross each other because the mean is quite similar . Therefore, the DWT is better than MODWT to decomposition time series data.

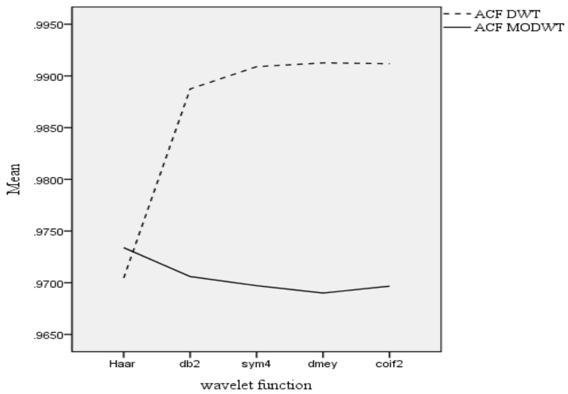

Fig. 2 ACF of DWT and MODWT

IV. CONCLUSION

The DJIA30 data of closing index were decomposed using five families of discrete wavelet transform and maximal overlap discrete wavelet transform. The results clearly show the decomposition stock market index by DWT is better than MODWT for original series to get more stationary data. Despite that the MODWT has some advantages over the DWT but it loses orthogonality. Therefore the information in the transform is not equivalent to the information in the original series. In addition, the problem lies while using the

MODWT because the crystals exist at roughly half the length of the wavelet basis into the series at any given scale and the crystal coefficients start more along the time axis as the scale level increases.

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